QCD Running Coupling: Freezing Versus Enhancement in the Infrared Region

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Abstract

We discuss whether the "freezing" of the QCD running coupling constant in the infrared region is consistent with the Schwinger–Dyson (SD) equations. Since the consistency of the "freezing" was not found, a conclusion is made that the "analytization" method does not catch the essential part of nonperturbative contributions. Proceeding from the results on consistency of the infrared-enhanced behaviour of the gluon propagator with SD equations the running coupling constant is modified taking into account the minimality principle for the nonperturbative contributions in the ultraviolet region. It is shown that the requirements of asymptotic freedom, analyticity, confinement and the value of the gluon condensate are compatible in the framework of our approach. The possibilities to find agreement of the enhanced behaviour of the running coupling constant with integral estimations in the infrared region are also discussed.

The phenomenon of asymptotic freedom [1] called forth the impressive success of perturbative QCD in the description of experimental data plethora. However, there is a wide region of phenomena which is intractable in the framework of perturbation theory. Nonperturbative effects modify the infrared behaviour of the quark and gluon Green's functions. With decreasing q^2, the renormalization group improved one-loop running coupling constant

$$\bar{\alpha}_s(q^2) = \frac{4\pi}{b_0 \ln(q^2/\Lambda^2)}. \tag{1}$$

($b_0 = 11C_2/3 - 2N_f/3$) increases, which may indicate a tendency of unlimited growth of the interaction at large distances, leading to confinement of coloured objects. However, at $q^2 = \Lambda^2$ in expression (1) the pole is present, which is nonphysical at least due to the failing of the perturbation theory, and the account of nonperturbative effects becomes obligatory.

In recent works [2] a solution of the problem of a ghost pole was proposed with imposing a condition of analyticity in q^2. The idea of "forced analyticity" goes back to works [3, 4] of the late fifties, which were dedicated to the problem of Landau-Pomeranchuk pole [5] in QED. Using for $\bar{\alpha}_s(q^2)$ a spectral representation without subtractions, in papers [2] was obtained the following expression for the running coupling constant

$$\bar{\alpha}_s^{(1)}(q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln(q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - q^2}\right]. \tag{2}$$

This expression is analytic in the infrared region due to nonperturbative contributions and it has a finite limit at zero (although the derivative is infinite).

Nowadays a possibility of "freezing" of the coupling constant at low energies was discussed [6] in the framework of some scheme of approximate calculations ($(16\frac{1}{2} - N_f)$ - expansion). In the approach [7] with confining background field "freezing" was also obtained,

$$\bar{\alpha}_s(q^2) = \frac{4\pi}{b_0 \ln((m_B^2 + q^2)/\Lambda^2)}. \tag{3}$$

Here m_B is process-dependent constant of the order of $1GeV$.

In the present note we follow the approach of Refs. [8, 9]. We discuss a problem of consistency of the constant behaviour of the running coupling constant in the infrared region with the Schwinger-Dyson (SD) equation for the gluon propagator. Further we include into consideration additional nonperturbative terms, those singular in the infrared region term $\sim 1/q^2$ in particular, the necessity of the renormalization invariance being taken into account. Then we consider possibilities to fulfill the demands of confinement,

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asymptotic freedom, analyticity, correspondence with estimates of the gluon condensate and integral estimates for $\bar{\alpha}_s(q^2)$ in the infrared region.

Let us consider the integral SD equation for the gluon propagator in ghost-free axial gauge [10] $A^a_\mu \eta_\mu = 0$, η_μ is a gauge vector, $\eta^2 \neq 0$. In this gauge the running coupling constant is directly connected with the gluon propagator and the Slavnov-Taylor identities [11] have the simplest form. The important advantage of the axial gauge consists in a possibility to exclude from the SD equation the term, which contains the full four-gluon vertex by means of contraction of the equation with the tensor $\eta_\mu \eta_\nu / \eta^2$. We work in the Euclidean momentum space, where smallness of the momentum squared means smallness of all its components. For the gluon propagator $D_{\mu\nu}(p)$ we suppose the approximation $D_{\mu\nu}(p) = Z(p^2) D_{(0)\mu\nu}(p)$ to be appropriate for studying the infrared region. $D_{(0)\mu\nu}(p)$ is free gluon propagator. In the first of Refs. [9] the possibility of the infrared behaviour for renormalized function $Z_R(p^2) = Z(p^2)/Z(\mu^2)$

$$Z_R(x) = Z_R(0) + o(1), \quad x \to +0, \tag{4}$$

has been studied ($Z_R(0)$ is nonzero constant) in the framework of the nonperturbative approach of Baker - Ball - Zachariasen (BBZ) [12]. The approximations of the BBZ approach as well as the condition $y = 0$ imposed on the gauge parameter $y = (p\eta)^2 / p^2 \eta^2$ seems to be adequate to studying the possibility of the infrared behaviour which is not too singular. With the assumption (4) the equation for the function $Z_R(p^2)$ takes the form

$$Z_R^{-1}(p^2) = 1 + Z_R(0) \frac{g^2 C_2}{16\pi^2} \frac{11}{3} \ln p^2 + o(\ln p^2), p^2 \to 0. \tag{5}$$

We see that behaviour $Z_R(p^2) \simeq Z(0) \neq 0$ for $p^2 \to 0$ does not agree with the SD equation.

This conclusion stimulate us to look for the possibilities different from the assumption of the finiteness of the coupling constant at zero. Recently a possibility of the soft singular power infrared behaviour of the gluon propagator has been discussed [13], $D(q) \sim (q^2)^{-\beta}$, $q^2 \to 0$, where β is a small positive non-integer number. In Ref. [14] the consistency of such behaviour with the same equation was studied. A characteristic equation for the exponent β was obtained and this equation was shown to have no solutions in the region $0 < \beta < 1$. The authors of Ref. [15] also came to the conclusion on the inconsistency of the soft singular infrared behaviour of the gluon propagator. The case of possible interference of power terms was studied in Ref. [16], and it was shown that in a rather wide interval $-1 < \beta < 3$ of the non-integer values of the exponent the characteristic equation had no solutions [2]. At present a more singular, in comparison with the free case, infrared behaviour of the form $D(q) \simeq M^2/(q^2)^2$, $q^2 \to 0$ seems to be best justified [17, 12, 18]. The models based on this assumption are widespread and rather attractive. On this way it is possible to describe the dynamical chiral symmetry breaking, to solve the $U(1)$ problem, to evaluate the topological susceptibility [19, 20], to calculate the condensates of gluon and quark fields [21], etc. The physical consequences of such enhancement of zero modes are discussed in reviews [22, 23]. Bearing in mind the remarks made above, let us turn to the problem of nonperturbative contributions. The "analytized" expression (2) without nonphysical singularity will be a starting point of our further consideration. We see that this expression has nonperturbative tail with the behaviour $1/q^2$ at $q^2 \to 0$. To answer the question if this behaviour is admissible let us consider the important physical quantity, namely, the gluon condensate $K = < vac \mid \alpha_s/\pi : G^a_{\mu\nu} G^a_{\mu\nu} :\mid vac >$. According to the definition (see e.g., [22]) up to the quadratic approximation in the gluon fields, after the Wick rotation one has

$$\begin{aligned} K \;&=\; -\frac{\delta^{aa}}{\pi} \int \frac{d^4k}{(2\pi)^4} (\delta_{\mu\nu} k^2 - k_\mu k_\nu) D^{(0)}_{\mu\nu}(k) \frac{g^2}{2\pi} (Z(k) - Z^{pert}(k)) = \\ &=\; \frac{48}{\pi} \int \frac{d^4k}{(2\pi)^4} \left(\bar{\alpha}_s(k^2) \;-\; \bar{\alpha}_s^{pert}(k^2) \right) \;=\; \frac{3}{\pi^3} \int_0^\infty \bar{\alpha}_s^{np}(y)\, y dy\,, \end{aligned} \tag{6}$$

where $\bar{\alpha}_s^{np}$ is nonperturbative part of the running coupling constant. The one-loop "analytized" behaviour Eq. (2) leads to the quadratic divergence in Eq. (6) at infinity and this is true for two- and three-loop expressions [2] of the analytization approach. Note that "freezed" behaviour (3) at large q^2 and known enhanced Richardson's running coupling constant in the infrared region [24] (coinciding formally with (3)

[2] Note that this interval contains values $-1 < \beta < 0$ for which the propagator vanishes at zero.

at $m_B = \Lambda$) do not ensure the convergence of the integral (6) at infinity. Accordingly to the results of Refs. [17, 12, 18] let us add in Eq. (2) an isolated infrared singular term of the form $1/q^2$. This term is harmless at zero and it can improve the behaviour of the integrand at infinity and make the integral logarithmically divergent. To make integral (6) convergent at infinity it is sufficient to add one more isolated singular term of pole type with parameters chosen appropriately. In this sense the model we come to is minimal. For the running coupling constant we obtain the following expression:

$$\bar{\alpha}_s(q^2) \;=\; \frac{4\pi}{b_0}\left(\frac{1}{ln(q^2/\Lambda^2)} \;+\; \frac{\Lambda^2}{\Lambda^2-q^2} \;+\; \frac{c\Lambda^2}{q^2} \;+\; \frac{(1-c)\Lambda^2}{q^2+m_g^2}\right), \tag{7}$$

with mass parameter m_g,

$$m_g^2 \;=\; \Lambda^2/(c-1). \tag{8}$$

It is worth to note that an account of nonperturbative contributions in Eq. (7) preserves perturbative time-like cut of Eq. (2). With the given value of the QCD scale parameter Λ, the parameter c can be fixed by the string tension κ or the Regge slope $\alpha' = 1/(2\pi\kappa)$ assuming the linear confinement $V(r) \simeq \kappa r = a^2 r$ at $r \to \infty$. We define the potential $V(r)$ of static $q\bar{q}$ interaction [25, 26] by means of the three-dimensional Fourier transform of $\bar{\alpha}_s(\vec{q}^2)/\vec{q}^2$ with the only contribution of the dressed gluon exchange taken into account. This gives the following relation

$$c\Lambda^2 = (3b_0/8\pi)a^2 = (b_0/16\pi^2)g^2M^2. \tag{9}$$

Taking $a \simeq 0.42\,GeV$, one obtains $c = \Lambda_1^2/\Lambda^2$ where $\Lambda_1^2 = 3b_0\kappa/8\pi \simeq 0.434\,GeV$ ($b_0 = 9$ in the case of 3 light flavours). From Eq. (8) one obtains

$$m_g \;=\; \frac{\Lambda^2}{\sqrt{\frac{27}{8\pi}a^2-\Lambda^2}}, \tag{10}$$

and tachion absence condition limits the parameter Λ, $\Lambda < 434MeV$. With Eq. (9) taken into account the only parameters in Eq. (7) are Λ and b_0. For the colour group $SU_c(3)$ ($C_2 = 3$), $N_f = 3$ and $\Lambda = 300MeV$ the behaviour of running coupling constant is represented in Fig. 1 for the cases under discussion. It should be noted that at sufficiently large q the curves in Fig. 1 seem to be indistinguishable, but the presence of the amplifying multipler y in Eq. (6) allows one to distinguish the ultraviolet behaviour of the nonperturbative contributions.

Let us represent the expression (7) in explicitly renormalization invariant form. It can be done without solving the differential renormalization group equations. In this order we write $\bar{\alpha}_s(q^2) = \bar{g}^2(q^2/\mu^2,\ g^2)/4\pi$ and use the normalization condition $\bar{g}^2(1, g^2) = g^2$. Then we obtain the equation for dependence of the parameter Λ^2 on g^2 and μ^2:

$$g^2/4\pi = \frac{4\pi}{b_0}\left[\frac{1}{\ln(\mu^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2-\mu^2} + c\frac{\Lambda^2}{\mu^2} + \frac{(1-c)\Lambda^2}{\mu^2+m_g^2}\right].$$

From dimensional reasons $\Lambda^2 = \mu^2 exp\{-\varphi(x)\}$, where $x = b_0 g^2/16\pi^2 = b_0\alpha_s/4\pi$, and we obtain for the function $\varphi(x)$ the equation:

$$x = \frac{1}{\varphi(x)} + \frac{1}{1-e^{\varphi(x)}} + ce^{-\varphi(x)} - \frac{(c-1)^2}{(c-1)e^{\varphi(x)}+1}.$$

The solution of this equation at $c > 1$ is the function $\varphi(x)$, which has the behaviour $\varphi(x) \simeq 1/x$ at $x \to 0$ and $\varphi(x) \simeq -\ln(x/c)$ at $x \to +\infty$. The obtained relation ensures the renormalization invariance of $\bar{\alpha}_s(q^2)$. At low g^2, we obtain $\Lambda^2 = \mu^2 \exp\{-4\pi/(b_0\alpha_s)\}$, which indicates the essentially nonperturbative character of three last terms in Eq. (7) and these terms are absent in the usual perturbation theory.

The acceptance in Eq. (1) of the cancellation mechanism for the nonphysical singularity of perturbation theory by the nonperturbative contributions leads to the necessity of supplementary definition of the integral (6) near the point $k^2 = \Lambda^2$. This problem can be reformulated as a problem of dividing perturbative and nonperturbative contributions in $\bar{\alpha}_s$ resulting from the introduction of some parameter

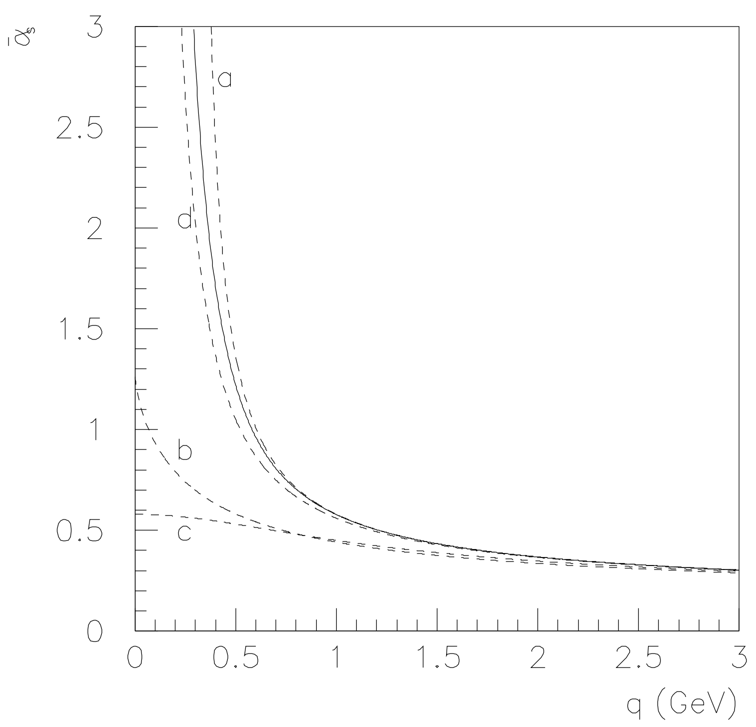

Figure 1: Momentum behaviour of the running coupling constant. Continuous curve corresponds to our Eq. (7); the curve (a) is one-loop Eq. (1); the curve (b) shows "analytized" Eq. (2); the curve (c) is "freezed" behaviour Eq. (3) where m is taken 1 GeV; the curve (d) corresponds to Richardson parameterization for the running coupling constant (Eq. (3) with $m_B = \Lambda$). Here $\Lambda = 300$ MeV, $b_0 = 9$.

$k_0 \approx 1$ GeV. This results in the absence of the pole at $k^2 = \Lambda^2$ in both perturbative and nonperturbative parts [3]. Nonperturbative contributions in Eq. (7) decrease at infinity as $1/q^6$, the integral in Eq. (6) converges, and we obtain

$$K(\Lambda, k_0) = \frac{4}{3\pi^2} \left\{ \Lambda^4 \, ln \left[\left(\frac{\Lambda_1^2}{\Lambda^2} - 1 \right) \left(\frac{k_0^2}{\Lambda^2} - 1 \right) \right] + k_0^2 \Lambda^2 \right\} . \tag{11}$$

Phenomenology gives the positive value of the gluon condensate K in the interval $(0.32\,GeV)^4$ — $(0.38\,GeV)^4$ [28, 29]. If one takesfor gluon condensate the value $K^{1/4} = 0.33 GeV$ and consider the formula (11) as equation for Λ at different values of k_0, the picture will be the following. At $k_0 < \bar{k}_0 = 0.777\,GeV$ there are no solutions in the interval $\Lambda = 0.1\,GeV - 0.434\,GeV$. At $k_0 = \bar{k}_0$ there is a single solution $\Lambda = 375 MeV$ corresponding to $m_g = \bar{m}_g \simeq 0.6\,GeV$. At $k_0 > \bar{k}_0$ for Λ two solutions appear to which correspond two values m_g, one of them increases with increase of k_0 and other decreases with increase of k_0. This situation is illustrated in Fig. 2 where we used Eq. (10) to connect the parameters Λ and m_g.

It is seen from Eq. (7) that the pole singularities are located at two points $q^2 = 0$ and $q^2 = -m_g^2$. This corresponds to the two effective gluon masses, 0 and m_g. Therefore, the physical meaning of the parameter m_g is not the constituent gluon mass, but rather the mass of the exited state of the gluon.

[3]See also Ref. [27] where the problem of perturbative and nonperturbative contributions to $\bar{\alpha}_s$ is discussed and the definition of infrared finite regularized perturbative part of $\bar{\alpha}_s$ is suggested.

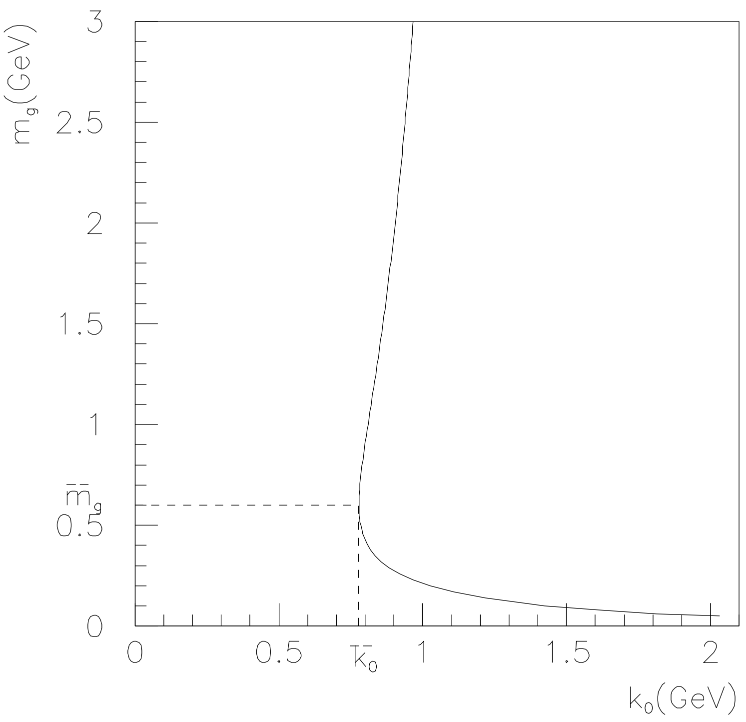

Figure 2: The dynamical gluon mass m_g corresponding to the nonperturbative scale k_0 for fixed value of the gluon condensate $K^{1/4} = 0.33$ GeV.

In a number of the QCD calculations it is necessary to estimate the integrals of the form

$$F(q^2) = \int\limits_0^{q^2} \bar{\alpha}_s(k^2) f(k^2) dk, \tag{12}$$

where $f(k^2)$ is a smooth function and integration includes the infrared region where the usual perturbation theory is inapplicable. In this case the infrared matching scale μ_I can be introduced ($\Lambda << \mu_I << q$) and contribution to the integral (12) from the region $k > \mu_I$ can be calculated perturbatively. To estimate the contribution of the region $k < \mu_I$ in Ref. [30] the running coupling constant is assumed to be regular at zero and power terms are extracted using dimensional arguments. In this connection the nonperturbative parameters α_p are introduced,

$$\alpha_p(\mu_I) = \frac{p+1}{\mu_I^{p+1}} \int\limits_0^{\mu_I} dk \bar{\alpha}_s(k) k^p, \tag{13}$$

which should be found from experiment. In the approach connected with infrared renormalons [31] the analogous ambiguity arises when evaluating the integral (12). This ambiguity can be understood in the following way. Expanding $\bar{\alpha}_s(k^2)$ in the series of powers of $\bar{\alpha}_s(q^2)$ and integrating over k^2 one obtains an asymptotic series with terms growing as factorial which can be estimated by the finite number of terms up to power corrections. The characteristic integral $\bar{A}(\mu) \equiv \alpha_0(\mu)/\pi$ (zero moment) has been estimated [30, 32] with the result $\alpha_0(2GeV)/\pi = 0.18 \pm 0.02$. This integral turned out to be not only stable with respect to the choice of the infrared regularization (fit-invariance) but also relative to different approximations of the high-energy tail of the running coupling (number of active quarks, one- or two-loop approximation). It is shown in Refs. [2] that the behaviour (2) comply with integral estimations of Refs. [30, 32]. Let us see whether the behaviour (7) is compatible with this estimations. In this case we also encounter the problem of definition in expressions of the form (12) and necessity to fix the corresponding ambiguity. The following method seems to be highly convenient and universal,

$$\frac{1}{(k^2)^m} \Longrightarrow \lim_{\epsilon' \to 0} \frac{d}{d\epsilon'} \frac{\epsilon'}{(k^2)^m} \left(\frac{\mu^2}{k^2}\right)^{\epsilon'} . \tag{14}$$

The finite arbitrariness described by the constant μ^2 arises only if we integrate the logarithmic singularities (which are usually connected with physics). In the case of local integrability ($n > 2m$, here n is the dimension of integration space) the rule (14) runs idle and at $n < 2m$ it corresponds [33] to analytic continuation in m. In accordance with the rule (14) the contribution of the singular term of Eq. (7) to the moments (13) is the following:

$$\Delta^{sing}\alpha_p(\mu_I) = \frac{p+1}{p-1}\frac{3a^2}{2\mu_I^2}, p \neq 1, \ \Delta^{sing}\alpha_1(\mu_I) = -\frac{3a^2}{2\mu_I^2}\ln\frac{\mu^2}{\mu_I^2}. \tag{15}$$

Thus at $p \leq 1$ the singular term of Eq. (7) should be considered as a distribution, and fit-invariance of the lowest moment $\bar{\alpha}_0(\mu_I)$ for different variants of infrared behaviour can probably indicate the absence of "freezing" of the running coupling constant and universality of ambiguity fixing. In favour of "freezing" of the coupling could indicate the equality of the moments $\alpha_p(\mu_I)$ for different p. For contribution of the last two terms of Eq. (7) to zero moment one obtains:

$$\Delta\alpha_0(\mu_I)/\pi = -\frac{4}{9}\frac{c\Lambda^2}{\mu_I^2} - \frac{4}{9}\frac{\Lambda}{\mu_I}(c-1)^{3/2}arctg(\frac{\mu_I}{\Lambda}\sqrt{c-1}). \tag{16}$$

Taking $a \simeq 0.42GeV$, $\mu_I = 2GeV$ one finds rather small contribution of the singular term, $\Delta^{sing}\alpha_0(\mu_I)/\pi$ $= -(3a^2)/(2\pi\mu_I^2) \simeq -0.021$. At $\Lambda \to \Lambda_1$ the second term of Eq. (16) vanishes as $-(16/9)(\Lambda_1 - \Lambda)^2/\Lambda_1^2$ and it can be made small by corresponding choice of Λ. For example, at $\Lambda = 0.375GeV$ (in this case $c = 1.3476$) this term equals to -0.0216. Having in mind the results of Refs. [2] where the contribution of first two terms of Eq. (7) has been evaluated we conclude that the running coupling constant Eq. (7) can be consistent with integral estimations in the infrared region. Note, that if we compare Eq. (5) and Eq. (12) of the second of Refs. [2] we find that "analytization" procedure in two-loop case leads at large momentum to the half nonperturbative contribution of one-loop case. It can point to the tendency of minimization of nonperturbative contributions in the ultraviolet region.

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