Quantum correlation functions in the problem of collective spontaneous emission

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Abstract

Dynamics of collective spontaneous emission for a system of two-level atoms interacting with quantum electromagnetic field is studied. A closed chain of equations for atomic correlation functions is obtained with the use of a new decoupling for three-particle correlators. It is shown that these equations yield a better description of experimental results.

In quantum theory of the superradiance the method of elimination of the boson variables was usually applied to obtain the chain of kinetic equations for atomic time correlation functions [1]. The fundamental problem which arises in the studies of such a hierarchy lies in the choice of chain breaking by means of decoupling of many-particle atomic correlators. In the problems of kinetics of many-particle systems the method of decoupling of the correlation functions may be suggested by analysing a particular form of the Hamiltonian in a special case when the particle interaction can be characterized by a small parameter. But in the case of superradiance one must obtain the kinetic equations which are suitable for all stages of the process. In the region of the superradiant peak the reasoning on exact breaking becomes invalid owing to the absence of a small parameter. At present, any performance criteria of the decoupling of the many-particle correlators are unknown. However, it is obvious that any method of decoupling can not break the kinematical relations for quasispin atomic operators. To appreciate the advantage of one or another decoupling method one can analyze the time behavior of dynamical observables and compare the results with an experiment.

In the framework of the method of elimination of boson variables one can usually use the traditionally decoupling of the three-particle atomic correlators which is analogous to the Tyablyakov's decoupling of the two-time three-particle Green functions in the theory of ferromagnetism. This decoupling does not take into account the fluctuations of the population inversion operator which is incorrect in the region of the superradiant peak where the mean value of the population inversion operator vanishes. To clear up this trouble, A.V.Andreev and others [2] have suggested more symmetrical decoupling for three-particle atomic correlators by analogy with the Vick theorem for fermi- and bose-operators. This decoupling takes into account some symmetry properties of kinetic equations for two-level systems interacting with quantum electromagnetic field and leads to wider and less symmetric curve for intensity of superradiance. This fact ensures better agreement between theory and experiment. But one can easily demonstrate that this decoupling leads to nonphysical behavior of the mean value of the population inversion at large times.

In this paper more complicated and consistent decoupling for three-particle atomic correlators has been suggested which properly accounts for kinematical properties of the quasispin atomic operator and improves the correspondence between the theoretical and experimental results for parameters of superradiant pulse but does not lead to nonphysical behavior of dynamical observables. The analogous decoupling for Green functions in the theory of ferromagnetism was used earlier by Callen [3].

Let us consider the N two-level emitters interacting with quantum electromagnetic field. To focus attention on the problem of the decoupling of the many-particle correlators and simplify our calculations, the wavelength of the electromagnetic radiation is taken to be much larger then the characteristic dimensions of the sample with emitters (so-called "point" Dicke model). In the framework of such approximation one can not take into account the effects of propagation and interference of the electromagnetic waves in our sample. The hamiltonian of considered system in dipole and RWA approximations can be written in the following form

$$H = H_A + H_F + H_{AF}, \tag{1}$$

where $H_A = \sum\limits_f \hbar\Omega R^z$ is the hamiltonian of free two-level atoms, $H_F = \sum\limits_k \hbar\omega_k a_k^+ a_k$ is the hamiltonian of free electromagnetic field and $H_{AF} = \sum\limits_k \sum\limits_k g_k \left(R_f^+ a_k + a_k^+ R_f^- \right)$ is the interaction hamiltonian. Here the index f enumerates two-level emitters in the sample, Ω is atomic frequency, R_f^z is population inversion operator of f-th atom and $R_f^{\pm}$ are operators describing transitions between the ground and exited levels in f-th emitters obeying the commutation relations

$$[R_f^{\pm}, R_{f'}^z] = \mp R_f^{\pm}\,\delta_{ff'}, \qquad [R_f^{+}, R_{f'}^{-}] = 2\,R_f^z\,\delta_{ff'},$$

$a_k^{+}(a_k)$ is operator of creation (annihilation) of a photon with frequancy ω_k, wave-vector $\vec{k}$ and polarization $\vec{e}_\lambda$, g_k is the coupling constant.

To obtain the kinetic equations for system under consideration we shall follow the standard procedure of elimination of boson variables developed by N.N.Bogolubov (Jr.) and collaborators (see for instance [4]). Let ρ_t be the statistical operator of the $A - F$ system (the A - subsystem consists of N - emitters and the F - subsystem is the electromagnetic field) which obeys the Liouville equation

$$i\,\hbar\,\frac{\partial \rho_t}{\partial t} = [H_t, \rho_t]$$

with the initial conditions

$$\rho_{t_0} = \rho_A\,\rho_B, \qquad \rho_F = \exp[-\beta \sum_k \hbar\,\omega_k a_k^{+} a_k]/sp\ \exp[-\beta \sum_k \hbar\,\omega_k a_k^{+} a_k],$$

where $\beta = 1/kT$.
Here t_0 is the initial point, ρ_A and ρ_F are the equilibrium statistical operators for A- subsystem, for the quantum electromagnetic field subsystem. These conditions describe the situation when the interaction between the initially equilibrium subsystems starts at the moment $t = t_0$. We restrict ourselves the problem of spontaneous collective radiation, and therefore, assume that $\rho_F = |0 >< 0|$, i.e. at the moment t_0 the radiation is absent. We also suggest that at initial time all atoms are excited and the correlations between emitters are absent.

Let $O(t)$ be an operator of a A - subsystem in the Heisenberg representation. This means that O acts on the eigenfunctions of the hamiltonian (1) as on functions of variables only of the A - subsystem. Using the method of elimination of boson variables we can obtain the exact kinetic equation for O - operator in the form

$$\frac{\partial < O(t) >}{\partial t} + < [H_A(t), o(t)] > / i\,\hbar =$$

$$= \sum_k \frac{g_k^2}{\hbar^2} \int_{t_0}^{t} d\tau \left(e^{i\omega_k (t-\tau)} \sum_f^N \sum_{f'}^N \langle [R_{f'}^{+}(t), O(t)] R_f^{-}(\tau) \rangle \right) + .A. \qquad (2)$$

The right-hand side of (2) contains variables that depend on different times t and τ. But if the coupling between the quantum field and emitters is weak, we can use the following "zero" approximation for the atomic operators

$$R_f^{\pm}(\tau) = R_f^{\pm}(t)\,exp[\pm\, i\,\Omega\,(t - \tau)].$$

Using the transition to the limit $t_0 \to -\infty$ we can perform integration in the right-hand side of (2) and obtain the markovian kinetic equation

$$\frac{\partial < O(t) >}{\partial t} + < [H_A(t), O(t)] > / i\hbar =$$

$$\frac{1}{\tau_0} \sum_f^N \sum_{f'}^N \langle [R_{f'}^{+}(t), O(t)] R_f^{-}(t) \rangle + \ .A., \qquad (3)$$

where τ_0 is the time of spontaneous emission.

Let O be the collective population inversion operator $R^z = \sum_f R_f^z$. For this operator one can easily obtain from equation (3)

$$\langle \dot{R}^z(t) \rangle = -\frac{1}{\tau_0}(\frac{N}{2} + \langle R^z(t) \rangle) - \frac{1}{\tau} \sum_g^N \sum_{g' \neq g}^N \langle R_{g'}^{+} R_g^{-} \rangle. \qquad (4)$$

The right-hand side of the equation (4) contains more complicated two-particle correlation functions of the $< R_f^+(t) R_{f'}^-(t) >$ type. For collective two-particle correlators $K(t) = \sum_f \sum_{f'} < R_f^+(t) R_{f'}^-(t) >$ which defines the macroscopic polarization of the atomic system we can obtain from (3) the following equation

$$\dot{K}(t) = \frac{2}{\tau_0} \sum_f^N \sum_{f'}^N \sum_{f'' \neq f'}^N \langle R_{f''}^z, R_{f'}^+ R_f^- \rangle =$$

$$= \frac{2}{\tau_0} \sum_{f'}^N \sum_{f'' \neq f'}^N \langle R_{f''}^z R_{f'}^z \rangle + \frac{2}{\tau_0} \sum_f^N \sum_{f' \neq f}^N \sum_{f'' \neq f'}^N \langle R_{f''}^z, R_{f'}^+ R_f^- \rangle. \tag{5}$$

Now we restrict our consideration to dynamics of one- and two-particle atomic correlation functions and therefore the decoupling procedure for three-particle correlators of type

$$\langle R_f^z R_{f'}^+ R_{f''}^- \rangle \tag{6}$$

should be adopted.

The well-known traditional decoupling of three-particle correlators is

$$\langle R_f^z R_{f'}^+ R_{f''}^- \rangle = \langle R_f^z \rangle \langle R_{f'}^+ R_{f''}^- \rangle \qquad (f \neq f' \neq f''). \tag{7}$$

The physical meaning of this approximation is that we neglect the fluctuations of the population inversion by substituting the operator of the population inversion by its mean value. This procedure does not take into account the quantum fluctuations of the population inversion R_f^z.

To improve the formula (7), one can employ more symmetrical expression for z- component of the quasispin operator; to do this, the well-known kinematical properties of the atomic quasispin operators can be used [4])

$$R_f^z = \frac{1}{2} - R_f^- R_f^+ , \tag{8}$$

$$R_f^z = \frac{1}{2}(R_f^+ R_f^- - R_f^- R_f^+). \tag{9}$$

Let us multiply the relation (8) by α and the relation (9) by $(1-\alpha)$, where α is an arbitrary c - number, and combine the obtained equations. As a consequence, we derive the most symmetrical expression for population inversion operator R_f^z.

$$R_f^z = \frac{\alpha}{2} + \frac{1}{2}(1-\alpha)\, R_f^+ R_f^- - \frac{1}{2}(1+\alpha) R_f^z R_f^+. \tag{10}$$

Let substitute the expression (10) for R_f^z in to the correlator $\langle R_f^z R_{f'}^+ R_{f''}^- \rangle$. As a consequence, from such manipulation results the following set of the correlation functions

$$\langle R_f^+ R_f^- R_{f'}^+ R_{f''}^- \rangle, \langle R_f^+ R_f^- R_{f'}^+ R_{f''}^- \rangle.$$

These correlation functions may be decoupled in symmetrical way by using the Vick theorem

$$\langle R_f^+ R_f^- R_{f'}^+ R_{f''}^- \rangle = \langle R_f^+ R_f^- \rangle \langle R_{f'}^+ R_{f''}^- \rangle + \langle R_f^- R_{f'}^+ \rangle \langle R_f^+ R_{f''}^- \rangle,$$

$$\langle R_f^- R_f^+ R_{f'}^+ R_{f''}^- \rangle = \langle R_f^- R_f^+ \rangle \langle R_{f'}^+ R_{f''}^- \rangle + \langle R_f^- R_{f'}^+ \rangle \langle R_f^+ R_{f''}^- \rangle.$$

As a consequence, for correlators (6) we have

$$\langle R_f^z R_{f'}^+ R_{f''}^- \rangle. = \langle R_f^z \rangle \langle R_{f'}^+ R_{f''}^- \rangle - \alpha \langle R_f^- R_{f'}^+ \rangle \langle R_f^+ R_{f''}^- \rangle. \tag{11}$$

Let us note that in the case $\alpha = 0$ from (11) one can obtain the traditional decoupling (7). By setting $\alpha = 1$ we can reduce the decoupling (11) to the Hartree-Fock approximation which has been used by A.V.Andreev et al [3].

Following this method of decoupling (formula (11)) and using the approximation

$$\langle R^z_f R^z_{f'} \rangle = \langle R^z_f \rangle \langle R^z_{f'} \rangle$$

one can obtain the following set of closed equations describing kinetics of considered system

$$\frac{\partial R^z(t)}{\partial t} = -\frac{1}{\tau_0}(\frac{N}{2} + R^z) - \frac{1}{\tau_0} K(t),$$

$$\frac{\partial K(t)}{\partial t} = \frac{2}{\tau_0}\langle R^z \rangle^2 + \frac{2}{\tau_0}\langle R^z \rangle K(t) - \frac{2}{\tau_0}\alpha \frac{K(t)^2}{N}. \tag{12}$$

Solving the system (12) one can obtain not only the time behavior of the mean value of the atomic level population inversion, but the evolution of the intensity of the superradiant impulse

$$I(t) = \frac{\partial}{\partial t} \sum_k \hbar\, \omega_k a^+_k a_k,$$

which connects with the mean value of the population inversion by the well-known relations

$$I(t) = -\hbar\, \Omega \frac{\partial \langle R^z(t) \rangle}{\partial t}. \tag{13}$$

Solution of equations (12), (13) may be found only numerically. As discussed above, to choose the decoupling the many-particle correlators (the parameter α in the framework of our approximations) one can analyze theoretical and experimental behavior of the dynamical observables. It is well known that the harnessing of the traditional decoupling (7) on examination of the dynamics of the superradiance leads to more narrow and symmetrical impulse of the intensity of collective radiation in comparison with the experimental data. For instance, for experiment in cesium vapor[5]-[6] the width of the experimental superradiant impulse of intensity ($\approx$ 0.4 nc) is approximately three times greater than that of the theoretical curve calculated in the framework of the assumption (7).

To improve the agreement between the experimental and theoretical data, we suggest to use new decoupling of the three-particle correlators (11) by choosing

$$\alpha = n < R^z_f(t) >,$$

where n is the positive integer.

The results of calculations are shown in Figs 1-2. In Fig. 1 is plotted the time dependence of the superradiant intensity $i = I * 10^{-10}$ and the mean value of the population inversion $R_z(t) =< R^z(t) > /N$ for three different types of decoupling for the model with the effective number emitters $N = 10^6$ (here $\tau = t/\tau_R$). Curves a, b and c correspond to solutions of (12) and (13) with the use of the standard decoupling (7), the new decopling (11) with $\alpha =< R^z_f >$ (or $n = 1$ in (14) and the Hartree-Fock decouplings. One can easily see from Fig.1 that new decoupling does not lead to nonphysical behaviour of the inversion population in contrast with that for Hartree-Fock approximation. On the other hand this decopling leads to wider and assymetrical superradiant impulse in comparison to the traditional approach. These results are in better agreement with the experiment. In Fig.2 the time behaviour of the intensity $i(t)$ for different values of the parameter n is shown. By increasing the n, the delay time and the width of the intensity impulse also increase.

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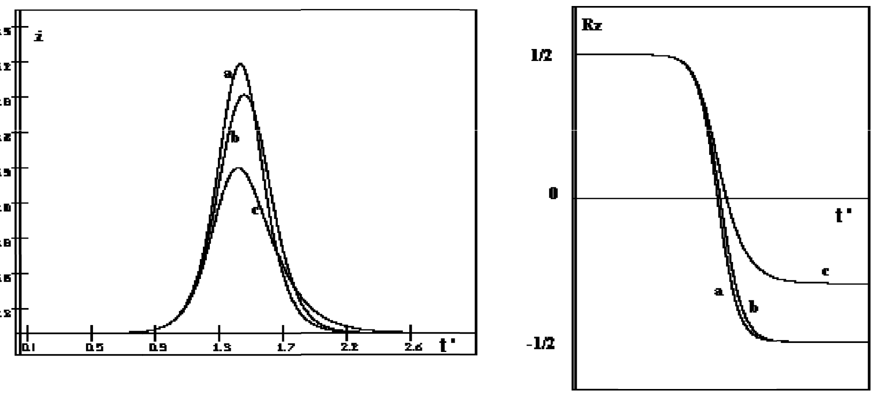

Fig.1 Time behaviuor of the intensity i and mean value of the population inversion R_z. Curves a,b and c correspond to the standard, new and Hartree-Fock decouplings.

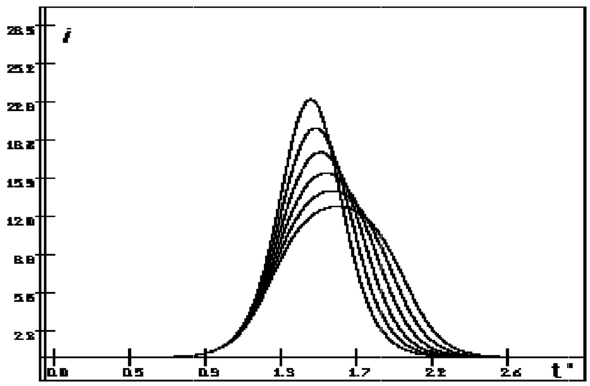

Fig.2 Time behaviour of the intensity i for different values of the parameter n.