Radiative decays of mesons in the extended model with the heavy singlet quark

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Abstract

It is shown that the unitary gauge is convenient for loop calculations in the Standard Model and its extensions with heavy singlet quarks. Rare radiative decays of B-mesons studied in the extended SM, the results agree with the known experimental data and differ from the SM by 10-15 per cent.

At the last time the Standard Model (SM) is on the stage of detailed studying of the rare processes, that can be in particular caused by the Flavour Changing Neutral Currents (FCNC). These processes are determined in the SM by loop contributions and have high sensitivity to the SM extension type, where they can manifest at the tree level. Thus thorough investigation of the FCNC effective vertices is in need in the SM framework and beyond. Namely the rare radiative decays of B-mesons (together with other types of rare transitions for neutral and charged mesons) are good probes of the FCNC and loop effects. The processes under consideration are connected with the following type of neutral transitions: $q_i \to q_j X$, where $X = \gamma, Z, G$, and analysis of them in the SM is permanent and intensive during several last years. Here we consider only some of the rare radiative B-decays in the SM extension that contain so-called singlet quarks, and for definiteness we consider the model with one down heavy singlet quark [1, 2, 3].

The goal of this work is to calculate straightforwardly in the framework of SM and singlet-quark model effective FCNC vertices for the transitions under consideration at the one-loop level. At the same time we would like to discuss not only specific details of calculations, but some experimental consequences for the rare radiative decays in the extended model too.

The general form of the effective Hamiltonian for processes we are interested in is [4, 5, 6, 7]:

$$H_{eff} = -\frac{4G_F}{\sqrt{2}}\lambda_t \sum_{j=1}^{8} C_j(\mu) O_j(\mu)$$

and, for instance, for the $b \to s\gamma$ transition $\lambda_t = V_{tb}V_{ts}^*$. The coefficients C_j are computed from the standard electro-weak Lagrangian with QCD contributions, that can be significant for these processes. The dominant operators in the sum are dipole–type operators:

$$O_7 = \frac{e}{16\pi^2}\bar{s}_\alpha \sigma^{\mu\nu}[m_b(\mu)R + m_s(\mu)L]b_\alpha F_{\mu\nu};$$

$$O_8 = \frac{g_s}{16\pi^2}\bar{s}_\alpha \sigma^{\mu\nu}[m_b(\mu)R + m_s(\mu)L]b_\beta G_{\mu\nu}\lambda_{\alpha\beta}/2,$$

and $L = \frac{1-\gamma_5}{2}$, $R = \frac{1+\gamma_5}{2}$. At the one-loop level processes that we are interested in, are described by known penguin diagrams and diagrams with "self-energy" insertions. In the extended model there are additional diagrams that are generated by tree level FCNC vertices $d_i d_j Z$ due to mixing of singlet quark with the ordinary ones. Thus it appears "Z-mediated" penguin diagram and there are "self-energy" and tadpole extra diagrams (Fig. 1, Fig. 2). The singlet quark diagrams contribute to coefficients C_7 and C_8 only.

It should be noted that corresponding effective vertices in the SM have been considered in numerous papers in the unphysical gauges. There is generally accepted point of view that in the unitary gauge in spite of the absence of extra-diagrams with unphysical degrees of freedom there are too high powers of momenta in tensor structures. As a consequence numerous divergencies arise that are difficulty controlled. However from our straightforward calculations it follows that in the unitary gauge (at least in the case under consideration) the highest powers of tensor expressions effectively decrease due to their cancellation at the initial stage. Moreover, on the mass-shell (for external particles) considerable simplification of all expressions takes place and there is simply controlled full compensation of all divergencies.

It is important to note that in unphysical gauges the structure of Ward- Slavnov-Teylor (WST)-identities complicates because of presence of ghost states.Besides, for the different SM extensions, where new fields occur, number of diagrams drastically increases. On the contrary, in the unitary gauge this rise is not so large, namely, in the model under consideration there are only three non-zero extra diagrams - tadpole diagrams are zero and the diagram with the insertion of $d_i d_j Z$ vertex into γ-line is zero for the on-shell case. Moreover, the check-up of WST-identities is simplified.

Because of dimensional regularization procedure using it appears a problem of choice of γ_5-definition. There are two accepted approaches to this problem, namely [8]:

1. so-called "naive" scheme, where for all μ $[\gamma_5, \gamma_\mu]_+ = 0$;

2. Breightenlohner-Maison scheme, where $[\gamma_5, \gamma_\mu]_+ = 0$ for $\mu = 0, 1, 2, 3$, and $[\gamma_5, \gamma_\mu]_- = 0$ for the rest of μ.

However, it had been shown [9] that for the last case WST- identities break down for these vertices, at the same time the "naive" scheme holds the identities. Due to this fact we use the "naive" γ_5-definition and add some extra arguments to this choice. Our arguments stem from the generalizaton of binary structure of the full Poincare group representation for n-dimensional space (for details see [10]).

It should be noted that in the most of papers where the effective vertices calculations were made, standard renormalization procedure carries out. For the case under consideration there is no need to introduce corresponding counterterms due to full compensation of divergencies on mass shell, where results have physical meaning. Of course for off-shell values it is nessesary to take into account certain counterterms, but it seems natural that for any complete set of on-shell diagrams in every fixed order full cancellation of divergencies will take place, so in the case, when the bare term is absent, on mass shell the renormalization procedure is not compulsory.

Thus we have computed the full set of on- and off-shell expressions for the effective FCNC vertices, but complete results have rather cumbersome form so for the most interesting physical cases it is sufficiently to use some approximation. Namely, we consider the approach: $p_i^2, m_i^2 \ll m_{W,Z}^2$, where $p_i(m_i)$ - are momenta (masses) of external quarks. For the $q_i q_j \gamma(Z, G)$ transitions both for the SM, and the singlet quark model first approximation's expressions are equal to zero exactly. For the second approximation we have following results:

$$\Gamma_\gamma^\mu(\beta)_{2-nd} = \frac{e g_s^2}{9 \cdot 2^7 \pi^2} \frac{i\sigma^{\mu\nu} k_\nu}{m_W} \Big(\sqrt{\beta_i} R + \sqrt{\beta_j} L\Big) \Big\{ A_{ij}^k A_\gamma^W(\alpha_k) + u_{ij} \Big[-A_\gamma^W(0) + \frac{1}{2} A_\gamma^Z(\alpha_h) - \frac{1+v}{2} A_\gamma^Z(0) \Big] \Big\},$$

where $\alpha_h = m_h^2/m_W^2$; $\alpha_k = m_k^2/m_W^2, k = u, c, t$; $1 + v = 2 - 4\sin^2\Theta_W/3$; $\beta_{i,j} = m_{i,j}^2/m_W^2$; $m_h = 500 - 600$ GeV [3, 11].

$$A_\gamma^W(\alpha_k) = 3\alpha_k \Big[\frac{7 - 5\alpha_k - 8\alpha_k^2}{(\alpha_k - 1)^3} + \frac{6\alpha_k(3\alpha_k - 2)}{(\alpha_k - 1)^4} \ln \alpha_k \Big];$$

$$A_\gamma^W(0) = 46; A_\gamma^Z(0) = -8;$$

$$A_\gamma^Z(\alpha_h) = \frac{8 - 30\alpha_h + 9\alpha_h^2 - 5\alpha_h^3}{(\alpha_h - 1)^3} + \frac{18\alpha_h^2}{(\alpha_h - 1)^4} \ln \alpha_h.$$

$$\Gamma_G^\mu(\beta)_{2-nd} = \frac{g_s g^2}{3 \cdot 2^8 \pi^2} \frac{i\sigma^{\mu\nu} k_\nu}{m_W} \Big(\sqrt{\beta_i} R + \sqrt{\beta_j} L\Big) \Big\{ A_{ij}^k A_G^W(\alpha_k) + u_{ij} \Big[-A_G^W(0) - \frac{1}{2} A_G^Z(\alpha_h) + \frac{1+v}{2} A_G^Z(0) \Big] \Big\} \lambda_n,$$

where

$$A_G^W(0) = -A_G^Z(0) = 8; A_G^Z(\alpha_h) = A_\gamma^Z(\alpha_h);$$

$$A_G^W(\alpha_k) = 3\alpha_k \Big[\frac{2 + 5\alpha_k - \alpha_k^2}{(\alpha_k - 1)^3} - \frac{6\alpha_k}{(\alpha_k - 1)^4} \ln \alpha_k \Big].$$

Our results in the unitary gauge for the SM contributions $A_{\gamma,G}^W$ coincide with the ones for unphysical gauges [12, 13]. The terms in brackets that are proportional to the matrix element u_{ij}, are generated by singlet quark extra contributions. Here $u_{ij} = U_{hj}^* U_{hi}$, and $U_{hi,j}$ - the element of the generalized mixing matrix. It describes mixing of the singlet h-quark with down $d_{i,j}$ quarks. ¿From above formula for $\Gamma_\gamma^\mu(\beta)$ it is evident that the dominant contribution of the h-quark caused by the non- unitarity of the 3×3 Kobayashi-Maskawa submatrix in the whole 4×4 mixing matrix. For the gluon vertex $\Gamma_G^\mu(\beta)$ all h-quark contributions have the same order. Our results for "Z-mediated" contributions differ from the ones in [14], where, incidentally, there are only numerical estimations for these diagrams.

Furthermore, the WST identities fulfil for these complete expressions, and this fact is straightforwardly checked. It should be added that the simple replacement of corresponding coefficients in the vertices leads to formulas for up-quarks.

Obtained results can be used for the analysis of different rare processes, but here we consider only some radiative B-meson decays, namely: inclusive $B \to (X_s, X_d)\gamma$ and exclusive $B \to (K^*, \rho)\gamma$ branchings.

For the inclusive transitions there is known expression ([15], for example):

$$Br(b \to s\gamma) = \frac{6\alpha_{em} R_{sl} Br(b \to cl\bar{\nu})}{\pi g(m_c/m_b)(1 - 2\alpha_s(m_b)/(3\pi) f(m_c/m_b)} |\frac{V_{tb}V_{ts}^*}{V_{cb}}|^2 \left[\eta^{16/23} A_\gamma + \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) A_g + \sum_{i=1}^{6} h_i^{\eta_i} C_i\right]^2.$$

Here V_{ij} - are corresponding mixing matrix elements, parameter $\eta = \dfrac{\alpha_s(M_Z)}{\alpha_s(m_b)}$ results from the leading-log renormalization group scaling down to $\mu \sim m_b$, R_{sl} is the semileptonic branching fraction, $g(k) = 1 - 8k^2 + 8k^6 - k^8 - 24k^4 \ln k$ - is the phase space function for $Br(b \to cl\bar{\nu})$, function $f(m_c/m_b)$ accounts for QCD corrections to the semileptonic decay, the sum of $h^{\eta_i} C_i$ accounts for admixtures of operators $O_i, i = 1, .., 6$ at the leading-log level. The terms $A_{\gamma,G}$ are:

$$A_\gamma = A_\gamma^{SM} - \frac{u_{ij}}{A_{ij}}[C_\gamma^W + C_\gamma^Z],$$

$$A_G = A_G^{SM} - \frac{u_{ij}}{A_{ij}}[C_g^W + C_g^Z],$$

where

$$u_{ij} = A_{ij}^h = -\sum_{k=1}^{3} A_{ij}^k$$

due to unitarity of 4×4 mixing matrix. Coefficients $C_{\gamma,g}^{W,Z}$ can be extracted from the above expressions for the effective vertices.

For the exclusive decays we use the formula:

$$Br(B \to \rho^0(K^*)\gamma) \sim M_B^5 |C_7^{eff}|^2 |V_{td(s)}|^2 |F_{\rho(K)}(O_7)|^2.$$

With the model of formfactors from [16] and having quark distribution amplitudes in the vector mesons (see [17]) it is possible to find some exclusive radiative branchings. But it should be noted that we consider here only neutral mesons radiative decays since short distance contributions strongly dominate in the case. We neglect next-to-leading logs corrections too as well as corrections to the spectator model. That is why we are interested in the main effects of heavy singlet quarks.

In our numerical calculations matrix elements from generalized mixing matrix were used (see [18]), and $m_t = 180$ GeV. Final results for different values of renormalization scale μ are given in the following Table:

Branchings	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$	the SM	Exp.
$B \to X_s\gamma \cdot 10^{-4}$	2.75 ± 0.31	2.29 ± 0.27	1.86 ± 0.22	2.55 ± 1.28	2.32 ± 0.67
$B \to X_d\gamma \cdot 10^{-5}$	0.61 ± 0.15	0.49 ± 0.12	0.40 ± 0.11	1.0 ± 0.8	-
$\dfrac{B \to X_d\gamma}{B \to X_s\gamma}$	0.023 ± 0.008	0.022 ± 0.007	0.023 ± 0.008	0.06 ± 0.02	-
$\dfrac{(B \to X_s\gamma)_{singlet}}{(B \to X_s\gamma)_{SM}}$	0.92 ± 0.13	0.89 ± 0.10	0.88 ± 0.11	-	-
$B \to K^*\gamma \cdot 10^{-5}$	4.10 ± 0.85	3.96 ± 0.82	3.87 ± 0.80	4.5 ± 1.5	$4.3 \pm 1.0 \pm 0.6$
$B \to \rho^0\gamma \cdot 10^{-5}$	0.11 ± 0.04	0.10 ± 0.03	0.09 ± 0.03	0.085 ± 0.065	-
$\dfrac{(B \to K^*\gamma)_{singlet}}{(B \to K^*\gamma)_{SM}}$	0.87 ± 0.14	0.90 ± 0.12	0.86 ± 0.14	-	-
$\left(\dfrac{B \to K^*\gamma}{B \to X_s\gamma}\right)_{singlet}$	$0.14 \div 0.5$	$0.18 \div 0.6$	$0.21 \div 0.7$	-	$0.15 \div 0.3$

In addition to this for the ratio $R = \dfrac{Br(B \to \rho\gamma)}{Br(B \to K^*\gamma)}$ we find the value: $R_{singlet} = 0.026 \pm 0.022$. The experiment gives only the upper limit: $R_{exp} < 0.34$.

Thus, there are some conclusions:

1. In connection with precision tests of the SM at the loop level it is nessesary to compare expressions for the effective $q_i q_j \gamma, G, Z$ vertices computed in a different way. Our results for these vertices obtained in the unitary gauge are in agreement at one loop level with the SM results for unphysical gauges.

2. The unitary gauge is more convenient for the loop calculations in the framework of extended models. In this case the number of extra diagrams is not so large, the power of tensor structures effectively decreases, and the procedure of cancellation of the divergencies is under the control due to some "block" structure of the cancellation.

3. In the model with the heavy singlet quark the dominant h-quark effect caused by the violation of the 3×3 mixing matrix unitarity and is significant when $A_{ij} \sim u_{ij}$.

4. There are no contradictions between known experimental data for rare radiative decays and predictions of the model with the heavy singlet quark.

5. For the considered above B-decay branchings differences from the SM results are only less 10%.

6. There is the one point where the discrepancy between the SM and the extended model is large as 100%, namely, the decay channel $B \to X_d\gamma$ (see in Table of numerical results). So more analysis is in need for the case.

7. The results for the considered branchings agree with the definition of generalized 4×4 mixing matrix and corresponding numerical values of its elements [3, 18].

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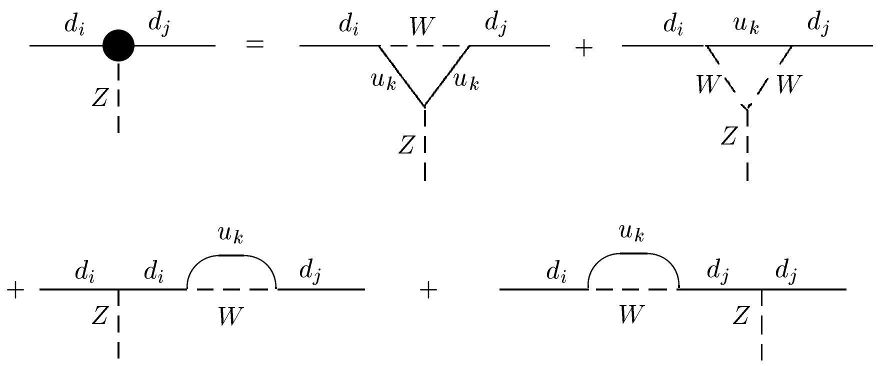

Figure 1: The effective vertex $d_i d_j Z$ in the SM

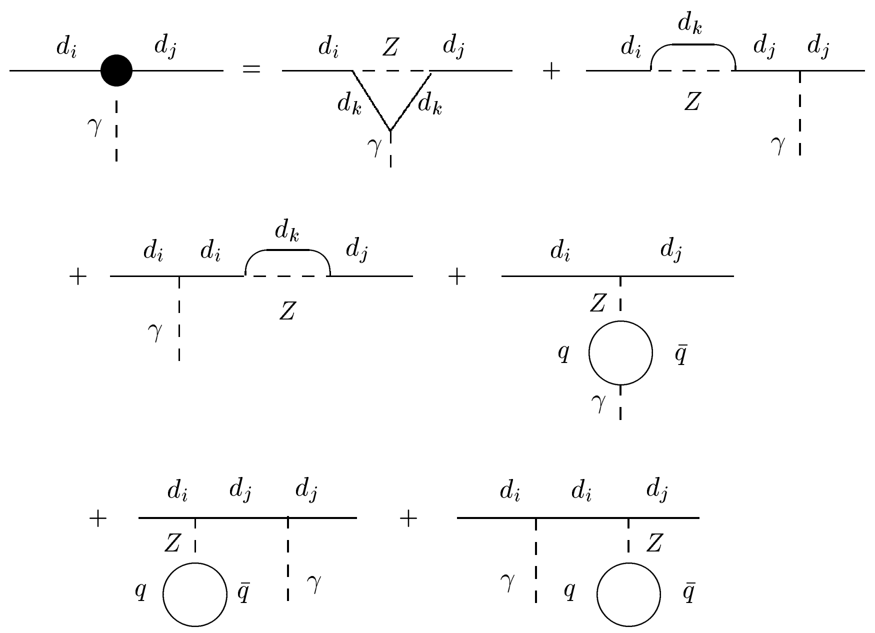

Figure 2: Extra contributions to the effective vertex in the model with singlet quark