Jet observables and energy-momentum tensor

P.S.Cherzor and $\boxed{\text{N.A.Sveshnikov}}$

Department of Physics, Moscow State University, Moscow, 119899

Abstract. We clarify and extend the theorem of Sveshnikov and Tkachov [1], [2], which gives an explicit connection between jet observables and the energy-momentum tensor. We check the relation between jet observables and the energy-momentum tensor for non-scalar (spinor and vector) fields, give a correct treatment of the light-cone singularity for massless particles, and extend the theorem of [1], [2] to the massless case. We also discuss the issue of gauge invariance.

1 Introduction

Modern QCD increasingly emphasizes the actuality of precision measurements and perturbative calculations of higher order corrections (cf. measurements of α_s and other parameters of the Standard Model).

It was argued in [3] that in the context of precision measurements a central role is played by a special class of observables — the so-called C-correlators — that contain all information about multijet structure and possess optimal stability properties with respect to experimental errors. As was shown in [1], [2], the C-correlators possess another property that makes them extremely attractive from theoretical point of view. Namely, the Sveshnikov and Tkachov theorem (ST)[1], [2] expresses C-correlators (and, consequently, a vast class of other jet observables [3]) in terms of the energy-momentum tensor in such a manner that no information about hadron bound states is used.

In the arguments of [1], [2] there are some gaps. The purpose of this work is to fill in them:

(i) The ST theorem was proved in [1], [2] only for scalar fields. So one needs to check it for non-scalar spinor and vector fields.

(ii) The ST theorem was accurately proved only for massive particles. The massless case exhibits some subtleties due to the light-cone singularity that have to be clarified.

(iii) The issue of gauge invariance in the case of gluons has to be clarified.

2 Setup

The C-correlators have the following form:

$$\left\langle \sum_{i_1} \dots \sum_{i_N} E_{i_1} \dots E_{i_N} \quad f_N \; (\, \widehat{\mathbf{p}}_{i_1}, \dots \widehat{\mathbf{p}}_{i_N} \;) \right\rangle_P . \tag{1}$$

Here i_k are indexes that run over all particles produced in an event, N is the order of the correlator ($N = 1, 2, \dots$). Note that the definition is entirely in terms of observable quantities — particles' energies (E_i) and angles ($\widehat{\mathbf{p}}_i$).

One can rewrite it in the Fock space formalism as follows:

$$\int d\mathbf{n}_1 \dots \int d\mathbf{n}_N \;\; \langle in \,|\varepsilon \; (\mathbf{n}_1) \dots \varepsilon \; (\mathbf{n}_N)|\, in\rangle \times f_N \; (\mathbf{n}_1, \dots \mathbf{n}_N), \tag{2}$$

where $\varepsilon \; (\mathbf{n}_i)$ is an operator-valued distribution on the unit sphere:

$$\varepsilon \; (\mathbf{n}) \equiv \sum \int \frac{d\mathbf{p}}{2p_0} \; |\mathbf{p}| \; a^+(\mathbf{p}) \; a^-(\mathbf{p}) \, \delta(\; \widehat{\mathbf{p}}, \mathbf{n}). \tag{3}$$

In [1], [2] the following relation between $\varepsilon\ (\mathbf{n})$ and the energy-momentum tensor was found:

Theorem (Sveshnikov and Tkachov, 1995):

$$\varepsilon\ (\mathbf{n}) = \lim_{t\to\infty}\ t^3 \int_0^1 \rho^2 d\rho\ n_i T_{i0}(\rho\mathbf{n}t, t)\,, \tag{4}$$

where T_{i0} is the energy-momentum tensor (the weak limit is implied here).

In the context of high energy QCD we can regard gluon, quark and ghost fields as massless. This corresponds to high energy limit $(\mathbf{p},\mathbf{n}) = |\mathbf{p}| \simeq p_0$.

In order to compare the massive and massless cases, we will reverse the procedure of [1], [2] and from the field representation of T_{i0} obtain operator representation for both cases.

3 Massive case

3.1 Scalar field

We already mentioned the result for scalar massive fields. Let us start from the well-known massive scalar Lagrangian and represent fields in the operator form:

$$\varphi\ (x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{2p_0}(e^{-ipx}\ a^-(\mathbf{p}) + H.C.)\,. \tag{5}$$

Following [1], [2] one gets the following expression (in the non-commutative case):

$$\varepsilon\ (\mathbf{n}) = \int_0^\infty \frac{p^3 dp}{4p_0} \left(a^+a^-(p\mathbf{n}) + a^-a^+(p\mathbf{n})\right)\,. \tag{6}$$

3.2 Spinor field

Now we turn to massive spinor fields for which we will obtain a similar result. We start with free asymptotic fields

$$\psi\ (x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{2p_0}(e^{-ipx}\ u_s(\mathbf{p})\ b_s(\mathbf{p})\ + e^{ipx}\ v_s(\mathbf{p})\ d_s^+(\mathbf{p})\ ). \tag{7}$$

The expression for the energy-momentum tensor for free fields is well-known (see e.g. [4]). So let us substitute (7) into (4) and use the stationary phase method. In the stationary phase equations only case of opposite signs contributes to $\varepsilon\ (\mathbf{n})$, because the same-sign contribution gives asymptoticaly $\varepsilon\ (\mathbf{n}) = 0$. By straightforward substitution (4) one obtains the following formula:

$$\lim_{t\to\infty}\ t^3 \int_0^1 \rho^2 d\rho\ n_i \frac{i}{2}\left[\overline{\psi}\gamma_i\partial_0\psi - \partial_0\overline{\psi}\gamma_i\psi\right] = \lim_{t\to\infty} \frac{1}{(2\pi)^3} \int_0^1 \rho^2 d\rho\ n_i \int \frac{d\mathbf{p}}{2p_0} \int \frac{d\mathbf{q}}{2q_0}$$

$$\times \exp\{\pm\frac{i\sqrt{1-\rho^2}}{2m}[(\mathbf{p})^2 - \rho^2(\mathbf{n},\mathbf{p})^2] \mp \frac{i\sqrt{1-\rho^2}}{2m}[(\mathbf{q})^2 - \rho^2(\mathbf{n},\mathbf{q})^2]\} \cdot F, \tag{8}$$

$$F = \frac{q_0}{2}[\overline{u}_s(\mathbf{p}')b_s^+(\mathbf{p}')\gamma_i u_s(\mathbf{q}')b_s(\mathbf{q}') - \overline{v}_s(\mathbf{p}')d_s(\mathbf{p}')\gamma_i v_s(\mathbf{q}')d_s^+(\mathbf{q}')]$$
$$+\frac{p_0}{2}[\overline{u}_s(\mathbf{p}')b_s^+(\mathbf{p}')\gamma_i u_s(\mathbf{q}')b_s(\mathbf{q}') - \overline{v}_s(\mathbf{p}')d_s(\mathbf{p}')\gamma_i v_s(\mathbf{q}')d_s^+(\mathbf{q}')], \tag{9}$$

where

$$\mathbf{p}' = \frac{m\rho}{\sqrt{1-\rho^2}}\mathbf{n} + \frac{\mathbf{p}}{\sqrt{t}}, \qquad \mathbf{q}' = \frac{m\rho}{\sqrt{1-\rho^2}}\mathbf{n} + \frac{\mathbf{q}}{\sqrt{t}}. \tag{10}$$

The difference from the scalar case is only in the operator part of the expression. After rewriting ε ($\mathbf{n}$) in terms of

$$p \equiv |\mathbf{p}| = \frac{m\rho}{\sqrt{1-\rho^2}} \tag{11}$$

and using for the operator part (8) the following formulae:

$$\overline{u}_s \gamma_\mu u_s = 2p_\mu, \qquad \overline{v}_s \gamma_\mu v_s = 2p_\mu, \qquad [d_s, d_s^+]_+ = 0 \tag{12}$$

one obtains the final result:

$$\int\limits_0^\infty \frac{p^3 dp}{2p_0} (\, b_s^+ b_s(p\mathbf{n}) + d_s^+ d_s(p\mathbf{n})) \,, \tag{13}$$

in agreement with the scalar case.

3.3 Vector field

The massive vector field is treated similarly. One starts with the standard representation:

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{p}}{2p_0} (e^{-ipx} A_\mu^-(\mathbf{p}) + H.C.). \tag{14}$$

The vector field Lagrangian in a general covariant gauge is as follows:

$$L = -\frac{1}{2} \partial^\mu A^\nu \partial_\mu A_\nu + \frac{m^2}{2} A^\mu A_\mu + \frac{\alpha}{2} \partial^\mu A^\nu \partial_\nu A_\mu. \tag{15}$$

As a consequence of the Noether theorem it is always possible to add a 4-divergence to $T_{\mu\nu}$, which allows us to make a convenient choice of the Lagrangian. We starts with the Stückelberg Lagrangian ($\alpha = 1$ in (15)) and use the fact that the theory in this case is gauge invariant. The Lorentz condition is applied (for $\alpha = 1$ it appears automatically from the Lagrangian) in the operator form (which arises from fields structure). In this way one can avoid non-physical states. In this way there is no contradiction: one retains Lorentz invariance and local commutativity. But the components $A_\mu^\pm(\mathbf{p})$ are no longer independent.

The negative-sign contributions to the T_{00} can be eliminated by representing operators $A_\mu^\pm(\mathbf{p})$ in (14) in a local frame of reference in momentum space (cf. e.g. [4]). To obtain T_{0i} one can choose any gauge for example $\alpha = 0$ for calculational simplicity. The result is gauge independent and has the form of (3), as expected. In the final formula only physical components survive:

$$\int\limits_0^\infty \frac{p^3 dp}{4p_0} (a_i^+ a_i^-(p\mathbf{n}) + a_i^- a_i^+(p\mathbf{n})). \tag{16}$$

4 Massless case

The result (4) has to be interpreted carefully in the massless case. From the accurate derivation (given below) it follows that the integration from 0 to 1 should be spread (formally integration with θ-function over ρ) over a region A (see Fig.1).

At infinitesimal ρ we can neglect contribution of particles in the $\Delta-$region, because of limited experimental sensitivity and hence such slow particles could not be registered by detectors. In the massive case, the region Δ_1 simply does not contribute as follows from the formula for stationary phase manifold: in the denominator square root of negative value does not have any physical meaning. But presence of the region Δ_1 plays an important role when we discuss massless fields.

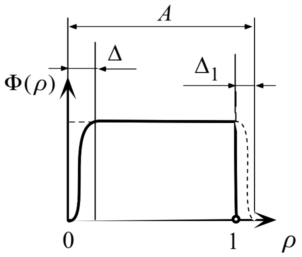

Fig.1

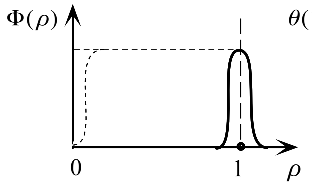

Fig.2

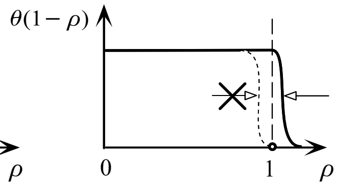

Fig.3

4.1 Scalar and spinor fields

Consider a massless case. Here situation becomes rather intricate. Integration over interval [0,1] in (4) — if done formally — leads to a result that differs by coefficient $1/2$ from what is to be expected from analogy with the massive case. The difficulty is due to the fact that all massless states are sitting on the light cone so that their distribution $\Phi(\rho) \equiv \rho^2 n_i T_{i0}(\rho \mathbf{n} t, t)$ is a $\delta-$function. However, the position of the latter corresponds to the boundary of the integration region at $\rho = 1$. Therefore, care is needed in how one defines the $\delta-$function. The trick we use ensures a natural connection of the massive and massless cases. Namely, let us extend the integration region into positive direction over unit interval to $1+\varepsilon$ and decrease it near zero, obtain: $\rho = [1-\varepsilon', 1+\varepsilon]\ ,\forall\varepsilon' \in (0,1)\ ,\forall\varepsilon \in (0,\infty)$. The non-zero part of the integral (4) comes from the infinitesimal region near 1 symmetrically on both sides.

All massless particles occupy the infinitesimally narrow δ-region at $\rho = 1$. The stationary phase method gives the equations:

$$\frac{\mathbf{p}}{\mid \mathbf{p} \mid} = \rho \mathbf{n}, \quad \frac{\mathbf{q}}{\mid \mathbf{q} \mid} = \rho \mathbf{n}, \tag{17}$$

$$\epsilon \cdot \epsilon' \mathbf{n p} + \mathbf{n q} = 0, \tag{18}$$

where ϵ and ϵ' independently takes the values ± 1. For the positive sign of $\epsilon \cdot \epsilon'$, the stationary manifold is $\mathbf{p} = \mathbf{q} = 0, |\mathbf{p}| = p_0 = 0$. In the stationary region asymptotically $\varepsilon\ (\mathbf{n}) = 0$.

For the negative $\epsilon \cdot \epsilon'$ sign the stationary manifold is: $\rho = 1$, $\mathbf{p} = \mathbf{q} = \varpi \mathbf{n}$, in the stationary region

$$\mathbf{p}' = \varpi \mathbf{n} + \frac{\mathbf{p}}{\sqrt{t}}, \quad \mathbf{q}' = \varpi \mathbf{n} + \frac{\mathbf{q}}{\sqrt{t}}, \qquad \rho' = 1 - \frac{\rho}{\sqrt{t}}. \tag{19}$$

Here after representing $\mathbf{p}, \mathbf{q}$ as a series we use change of variables $\mathbf{p} \leftrightarrow \mathbf{p}', \mathbf{q} \leftrightarrow \mathbf{q}'$. As was announced above, let us introduce ε into integration limits over ρ $(0 < \varepsilon < 1)$. After substituting the above into (4), for scalar fields one obtains the following formula:

$$\varepsilon\ (\mathbf{n}) = \lim_{t \to \infty} \frac{t^3}{(2\pi)^3} \int\limits_{\varepsilon}^{1+\varepsilon} \rho'^{\,2} d\rho'\ n_i \int \frac{d\mathbf{p}'}{2p_0'} \int \frac{d\mathbf{q}'}{2q_0'} p_0' \cdot q_i' \cdot \epsilon \cdot \epsilon' \cdot a^{\epsilon}(\mathbf{p}') a^{\epsilon'}(\mathbf{q}')$$

$$\times \exp\left\{it\left[\epsilon\ (p_0' - \mathbf{p}'\mathbf{n}\rho') + \epsilon'\ (q_0' - \mathbf{q}'\mathbf{n}\rho')\right]\right\} \tag{20}$$

Let us insert into this expression $1 = \int\limits_0^{\infty} \delta(\varpi - p)\ d\varpi$. One can represent the absolute value of $\mathbf{p}$ as series and retain only contributions of order $O(\frac{1}{\sqrt{t}})$. It yields the argument of the δ-function: $\left(-\frac{(\mathbf{p},\mathbf{n})}{\sqrt{t}}\right)$. One must take into account

$$(\mathbf{p}_{\perp}, \mathbf{n}) = 0, \quad \mathbf{p} = \mathbf{p}_{\perp} + \mathbf{p}_{\parallel}. \tag{21}$$

The integrals over $\mathbf{p}_{\perp}, \mathbf{q}_{\perp}$ are trivial and yield the factor $(2\pi)^2 \varpi^2$. Now one can take the integrals over $q_{\parallel}, \rho$ in two independent ways with the same final result:

The first way is to take the integral over ρ first and apply the formula:

$$\int_{-\infty}^{\infty} dq_{\|} \frac{(e^{ibq_{\|}} - e^{-iaq_{\|}})}{iq_{\|}} = 2\pi\,. \tag{22}$$

Note that the result is independent of $a, b > 0$.

The second way is to take the integral over $q_{\|}$ first and then the integral over ρ. Take the limit and use the formula:

$$\int_{-\infty}^{\infty} d\rho\, \delta(\rho) = 1. \tag{23}$$

In both cases one obtains the following result:

$$\frac{1}{4}\int_{0}^{\infty} \varpi^2 d\varpi\, (a^+a^-(\varpi\mathbf{n}) + a^-a^+(\varpi\mathbf{n})). \tag{24}$$

For massless spinors the calculations are similar. The result is

$$\frac{1}{2}\int_{0}^{\infty} \varpi^2 d\varpi\, \left(b_s^+ b_s(\varpi\mathbf{n}) + d_s^+ d_s(\varpi\mathbf{n})\right)\,. \tag{25}$$

4.2 Gauge field

The massless vector case is slightly more interesting because it is necessary to consider the problem due to broken Lorentz invariance when one deals with physical states. It is impossible to use the local frame of reference in momentum space as in the massive case because of the denominator that is singular at ($m = 0$). To circumvent the problem one can use the standard Gupta-Bleuler quantization procedure (see e.g. [4]). For quantum fields the Lorentz condition can be rewritten as a condition for physical states:

$$\partial_\mu A_\mu^- \Phi = 0, \quad \Phi^* \partial_\mu A_\mu^+ = 0. \tag{26}$$

Now one turns to the local frame of reference [4]. The non-physical zeroth and third components cancel each other as expected. In the average value over a physical state Φ only the physical components remain. (Average value of observable over physical states is the same as over all states). Let us turn to the Lagrangian in Lorentz gauge ($\alpha = 0$). One takes into account the above arguments, repeats the calculation of $\varepsilon\,(\mathbf{n})$, and obtains:

$$\frac{1}{4}\int_{0}^{\infty} \varpi^2 d\varpi\, (a_1^+a_1^-(\varpi\mathbf{n}) + a_2^+a_2^-(\varpi\mathbf{n}) + a_1^-a_1^+(\varpi\mathbf{n}) + a_2^-a_2^+(\varpi\mathbf{n})). \tag{27}$$

5 Conclusion

We have seen that the expression (4) of C-correlators in terms of the energy-momenum tensor $T_{\mu\nu}$ is correct for non-scalar particles as well. The distribution of massive particles lies inside the integration region for ρ (see Fig.1).

In the massless case the essence of calculation is that all particles are sitting on the light cone. Therefore the operator-valued distribution Φ is δ-function (see Fig.2 above). It follows from our analysis that the θ-function that describes the integration region must be defined as a limit "from the right" of smooth regulators (Fig.3).

This definition for the massless case should be important: (i) in axiomatic proofs; (ii) in proofs of a generalization of the KLN theorem for jet observables *.

*This work was planned by N.A. Sveshnikov.

For gauge particles we verified the formula (4) checked for Lorentz gauge using in the Gupta-Bleuler formalism. It would be interesting to develop a treatment for general (not necessarily covairant) gauges. There is systematic formalism for treatment of arbitrary gauges for the case of Green's functions (see e.g. [5]) but not for the Fock space. It would be useful to develop such a formalism in order to clarify calculational issues enocountered when there are loop and phase space integrals simultaneously.

6 Acknowledgements

P.C. thankful to F.V. Tkachov for a help with preparation of this paper and the organizers of the XII Intern. Workshop QFTHEP'97 for a financial support.

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