Precise Analysis of Data on Electromagnetic Structure of Nucleon

A.F. Grashin, I.B. Lukasevich

Moscow Engineering Physics Institute,
115409 Moscow, Russia

Abstract

Data on electromagnetic structure of nucleon and pion are analyzed with form factors derived in the framework of supernonequilibrium thermodynamics, and a precise description of the data is obtained as a result. The electric and magnetic radii are extracted with a high accuracy.

During fourty years a lot of experimental data on electromagnetic structure of nucleon has been obtained, and the problem has arisen to combine the data from various experiments for revealing systematic errors and reducing statistical uncertainties. Precise analysis of the data can be performed in the framework of supernonequilibrium thermodynamics. The foundation of this approach was reported in refs.[1, 2] where the temperature continuum $0 \le \tau < \infty$ with a large variance

$$\left\langle (\Delta\,\tau)^2 \right\rangle = T^2/a\,, \tag{1}$$

was introduced, $T = \langle \tau \rangle$ and $a \sim 1$ being the average temperature and a nonequilibrium parameter.

This approach has resulted in the form factor

$$F(Q^2) = N \cdot K_1(\eta)/\eta\,, \tag{2}$$

$$\eta = a\,(1 + Q^2/m^2)^{1/2}\,, \tag{3}$$

$$N = a/K_1(a)\,. \tag{4}$$

Here K_1 is the modified Bessel function of the first order, $Q^2 = -q^2$ is the negative square of the four–momentum transfer and m is a model parameter. The form factor (2) has a pole in the crossing channel (time-like region) at $t = -Q^2 = m^2$ as in the vector dominance model, so that m may be interpreted as the mass of an intermediate meson.

For adequate description of the data we take into account the width of the isovector resonance shifting the pole to the complex region by the substitution

$$t \to t + i\,f(t)\,(t - 4m_\pi^2)^{3/2}, \tag{5}$$

as it was done e.g. in works [3, 4], with analytical function $f(t)$ decreasing at infinity (m_π is the pion mass). In this investigation we choose $f(t)$ in the form:

$$f(t) = \gamma/(1 - \delta t)\,, \tag{6}$$

where γ, δ are free parameters.

Hence, the vector form factor is given by eq.(2) with

$$\eta = \frac{a}{m}\left[m^2 + Q^2 - \frac{\gamma(Q^2 + 4m_\pi^2)^{3/2}}{1 + \delta Q^2}\right]^{1/2}, \tag{7}$$

$$N = a_0/K_1(a_0), \quad a_0 = a\left[1 - 8\gamma m_\pi^3/m^2\right]^{1/2} \tag{8}$$

instead of (3) and (4).

The electromagnetic structure can be described by functions (2) assuming

$$G_E^{(p,n)} = F_E^{(p,n)} = \frac{1}{2}(F_{SE} \pm F_{VE})\,, \tag{9}$$

$$G_M^{(p,n)} = \mu_{(p,n)} F_M^{(p,n)} = \frac{1}{2}\,(\mu_p + \mu_n) F_{SM} \pm \frac{1}{2}\,(\mu_p - \mu_n) F_{VM} \tag{10}$$

with (3),(4) for isoscalar (S) and (7),(8) for isovector (V) terms. Appropriate parameters are lettered as a_{SE}, m_{SE}, a_{VE}, m_V for the electric (charge) form factors and a_{SM}, m_{SM}, a_{VM}, m_V for the magnetic form factors. Electromagnetic radii are determined by

$$\langle r^2 \rangle = -\frac{1}{6}\, \frac{dF(Q^2)}{dQ^2}\bigg|_{Q^2=0} \tag{11}$$

for various types of form factors. The nine model parameters have one constraint

$$dG_E^n/dQ^2\big|_{Q^2=0} = \frac{1}{12}\,\left[\langle r_{VE}^2 \rangle - \langle r_{SE}^2 \rangle\right] = (0.0190 \pm 0.0004)\ fm^2 \tag{12}$$

due to precise experiment [5], therefore, we have eight independent parameters for fitting of the data.

To obtain an adequate set of the proton data we have adopted a compilation of Höhler et al. [6] for $Q^2 < 0.15\ GeV^2$ and a global extraction of data for $0.15 \le Q^2 < 10\ GeV^2$ from the work of Walker et al. [7]. In addition we have taken the data from refs. [8] and [9] covering low Q^2 and high Q^2 regions. Four groups of data are renormalized by adjustable factors N([6]), N([7]), N([8]) and N([9]).

Large discrepancies in neutron data prevented from obtaining unambiguous results in previous investigations. Our parameterization allows to discriminate neutron data when processing in common with the proton data. In this context, we consider the neutron structure determined by the data from refs.[10–20]. Experimental ratios σ_n/σ_p from refs.[10–12,14] are used for calculation of G_M^n with the help of proton and electric neutron form factors extracted from our treatment.

The values of G_E^n obtained by Gari and Krümpelmann [21] via analysis of deuteron form factors are incompatible with other data providing $\chi^2 = 36$ for 13 points and, therefore, are not included in fitting procedure, contrary to our work [22].

The analysis gives the best fit parameters listed in tab.1 and $\gamma = 0.137 \pm 0.011\ GeV^{-1}$, $\delta = 0.0071 \pm 0.0024\ GeV^{-2}$; $\chi^2/DF = 101.3/116$. They correspond to the nucleon radii

$$\begin{aligned} \langle r_{Ep} \rangle &= (0.8572 \pm 0.0056)\ fm\,, \\ \langle r_{Mp} \rangle &= (0.8479 \pm 0.0046)\ fm\,, \\ \langle r_{Mn} \rangle &= (0.798 \pm 0.014)\ fm\,. \end{aligned} \tag{13}$$

Having averaged $\langle r_{Ep} \rangle$ and $\langle r_{Mp} \rangle$ we obtain the proton radius

$$\langle r_p \rangle = (0.8526 \pm 0.0036)\ fm\,. \tag{14}$$

The best fit is obtained with following renormalization factors:

$$\begin{aligned} N([6]) &= 0.9956 \pm 0.0015\,, \\ N([7]) &= 1.0100 \pm 0.0073\,, \\ N([8]) &= 1.0000 \pm 0.0009\,, \\ N([9]) &= 1.0226 \pm 0.0092\,, \end{aligned} \tag{15}$$

Figs.1–3. The ratios of calculated form factors to the dipole fit (16). Data are from refs.[6-16]. Fig.4. The calculated electric neutron form factor. Data are from refs.[14,17-20,24]. Hatched areas correspond to Saclay data [23] extracted with Nijmegen and RSC potentials. Dashed lines restrict regions of uncertainties coming from standard errors in the model parameters.

Table 1

The best fit parameters

	SE	SM	VE	VM
a	0.540 ± 0.036	0.420 ± 0.055	2.11 ± 0.11	1.344 ± 0.065
r (fm)	0.7879 ± 0.0066	0.9468 ± 0.0331	0.9214 ± 0.0050	0.8281 ± 0.0051
m (GeV)	0.6593 ± 0.0082	0.5377 ± 0.0235	0.6971 ± 0.0091	

Table 2 and figs.1–3 give the ratios G_E^p, G_M^p and G_M^n to the dipole parametrization

$$F_D = [1 + Q^2/0.71(GeV^2)]^{-2}\,. \tag{16}$$

Large uncertainties in magnetic form factors for $Q^2 > 30\ GeV^2$ are caused by the errors in parameters γ and δ. The electric and magnetic proton form factors have a difference at small Q^2, but this difference raises with Q^2 and becomes more than the sum of standard errors in G_E^p and G_M^p for $Q^2 > 0.01\ GeV^2$. Magnetic form factor of the neutron considerably differs from G_E^p and G_M^p over the whole interval of Q^2 exhibiting a significant violation of the scaling rule.

In fig.4 the electric neutron form factor is given, and the data from refs. [23, 24] are shown in addition. These data were obtained in quasielastic electron-deuteron scattering and are not included in the fitting procedure because of strong dependence upon the n–p potential. It is seen from fig.4 that our theoretical form factor G_E^n is in agreement with data obtained on the basis of the Reid Soft Core potential at low Q^2, however at higher Q^2 the form factor is closer to the data of ref.[23] obtained with the Nijmegen potential.

The proton radii and Q^2–dependence of the proton form factors are in a good agreement with results of analysis from ref.[25], in which VDM model was used. However, there is a significant discrepancy with a QCD–VM formula of Gari and Krümpelmann [21] and with the improved formula from ref.[25].

The structure of the pion can be described by the expression (2) with (7),(8) and γ and δ as in the isovector nucleon form factor. Experimental data in the region $0.015 \le Q^2 \le 10\ GeV^2$ were presented in refs.[26–28]. The data have been renormalized by factors for which we have obtained the values 0.998 ± 0.023, 1.0047 ± 0.0052, 1.0056 ± 0.0019 respectively. For the charge radius we have obtained

$$\langle r_\pi \rangle = 0.6475 \pm 0.0087\ fm \tag{17}$$

for $a = 1.00 \pm 0.16$, $\chi^2/DF = 80/76$. Fig.5 represents Q^2–dependence of the pion form factor. The data from ref.[27] obtained for the range $Q^2 < 0.1\ GeV^2$ are not shown in the figure. A good fitting of the neutron data and small errors in the radii (13),(17) demonstrate the advantage of processing experimental data by our method.

Fig.5. The calculated charge form factor of the pion. Data are from refs.[26,28].

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Table 2

Theoretical values of form factors for the nucleon and pion, errors being given in parentheses

Q^2 (M^2)	G_E^p/G_D	$G_M^p/\mu_p G_D$	G_E^n	$G_M^n/\mu_n G_D$	F_π
0.010	0.99684(40)	0.99754(32)	0.00471(10)	1.00088(92)	0.98234(46)
0.020	0.99390(78)	0.99534(61)	0.00909(18)	1.0018(18)	0.96525(89)
0.030	0.9912(12)	0.99338(88)	0.01317(26)	1.0026(26)	0.9487(13)
0.040	0.9886(15)	0.9916(11)	0.01697(34)	1.0035(33)	0.9326(16)
0.050	0.9863(18)	0.9901(14)	0.02051(41)	1.0044(40)	0.9171(20)
0.060	0.9841(22)	0.9887(16)	0.02382(48)	1.0053(46)	0.9020(23)
0.070	0.9820(25)	0.9875(18)	0.02691(55)	1.0061(52)	0.8873(26)
0.080	0.9801(28)	0.9865(20)	0.02980(61)	1.0070(58)	0.8730(28)
0.090	0.9783(31)	0.9856(21)	0.03249(67)	1.0079(63)	0.8592(31)
0.100	0.9767(34)	0.9848(23)	0.03501(73)	1.0087(68)	0.8457(33)
0.200	0.9655(58)	0.9821(35)	0.0528(12)	1.017(11)	0.7292(48)
0.300	0.9610(77)	0.9849(43)	0.0617(16)	1.025(13)	0.6380(56)
0.400	0.9602(93)	0.9903(48)	0.0658(19)	1.032(15)	0.5648(60)
0.500	0.962(11)	0.9967(52)	0.0671(20)	1.037(16)	0.5048(63)
0.600	0.965(12)	1.0035(54)	0.0667(21)	1.042(16)	0.4549(65)
0.700	0.968(13)	1.0102(56)	0.0654(22)	1.046(17)	0.4127(67)
0.800	0.973(14)	1.0165(58)	0.0635(22)	1.048(17)	0.3767(69)
0.900	0.977(15)	1.0225(59)	0.0613(22)	1.050(17)	0.3456(71)
1.000	0.982(16)	1.0280(61)	0.0590(22)	1.051(17)	0.3186(72)
2.000	1.028(25)	1.0573(68)	0.0380(18)	1.026(16)	0.1676(79)
3.000	1.060(34)	1.0563(71)	0.0254(14)	0.976(16)	0.1059(75)
4.000	1.077(44)	1.0414(72)	0.0178(11)	0.921(19)	0.0740(68)
5.000	1.083(53)	1.0205(73)	0.0130(9)	0.870(25)	0.0550(61)
6.000	1.080(62)	0.9974(75)	0.0098(8)	0.824(32)	0.0428(55)
7.000	1.070(70)	0.9740(80)	0.0076(6)	0.785(39)	0.0345(49)
8.000	1.055(77)	0.9513(86)	0.0060(5)	0.752(46)	0.0284(45)
9.000	1.037(83)	0.9297(93)	0.0048(5)	0.725(52)	0.0240(41)
10.000	1.015(89)	0.909(10)	0.0040(4)	0.702(57)	0.0206(38)
20.000	0.756(110)	0.775(14)	8.3(1.3)E-04	0.638(89)	0.0075(20)
30.000	0.537(103)	0.726(37)	2.7(0.6)E-04	0.70(11)	0.0044(14)
40.000	0.381(88)	0.72(10)	1.1(0.3)E-04	0.78(17)	0.0031(11)
50.000	0.274(74)	0.73(20)	4.8(1.5)E-05	0.87(30)	0.0024(9)
60.000	0.199(63)	0.73(33)	2.4(0.9)E-05	0.92(48)	0.0019(7)
70.000	0.146(54)	0.72(48)	1.3(0.5)E-05	0.94(70)	0.0015(6)
80.000	0.108(47)	0.69(62)	7.0(3.6)E-06	0.92(90)	0.0013(5)
90.000	0.081(40)	0.63(73)	4.1(2.5)E-06	0.86(1.1)	0.0010(5)
100.000	0.060(34)	0.56(79)	2.5(1.7)E-06	0.77(1.1)	0.0008(4)