The Nonlinearities in Nuclear Reactions

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Abstract

The nonlinear Schroedinger equation, the Kortveg de Vries equation and the Kadomtsev–Petviashvili equation are applying for the description of the interaction of heavy ions with nuclei. The problems of dynamical instability, anomalous large angular scattering, dynamical nuclear compressibility and hot spot are considered from the point of view of the soliton concept.

The aim of the present lecture is to apply the Nonlinear Schroedinger Equation (NoSE), the Korteveg de Vries Equation (KdVE) and the Kadomtsev–Petviashvili Equation (KPE) to an interpretation of some effects in nuclei.

1. Nonlinear Schroedinger Equation (NoSE)

Earlier we applied NoSE for the explanation such phenomena as the ALAS (Anomalous Large Angular Scattering) in interactions of α-particles with light nuclei [1]. Also we used the NoSE for the description of α-particle spectra in ^{20}Ne [2]. In both cases we exploitated the NoSE in such a form:

$$-(\hbar^2/2m)\nabla^2 X + VX + C \mid X \mid^2 X = \text{ EX} \; , \tag{1}$$

where $C = K/9$ and K being the compressibility constant. It turned out that Eq. (1) is a particular solution of a general equation suggested by Bialynsky-Birula and Mysielsky [3]. Using instead of the cubic term $\epsilon \log(a \mid \psi \mid^2)\psi$ they formulated the nonlinear quantum mechanics without any contradiction in classical quantum mechanics. By the way, Gaehler, Klein and Zeilinger [4] attempted to find the constant ϵ according to the predictions made in the work [3]. But it was made without any success. Our approach supposes the constant $C = K/9$ or ϵ (here both constants are the same) to be the compressibility constant. It was confirmed from the values of the constant K, obtained from different experiments with the help of this approach ($K \approx 270 MeV$) and also by the recent work of J.Braecher [5]. J.Braecher considered the nonlinear term in the NoSE from the point of view of the information theory. According to the work of Shannon and Weaver [6] the information I acquired upon measurements of the state$\mid \psi >$ is proportional to the logarithm of the probability $I = -\log_2(a \mid \kappa \mid^2)$ bits, where $a = \mid \psi_0 \mid^{-2}$.

The certain amount ϵ of energy per bit is expended, transferred, stored, associated with the information encoded. That is why it is necessary to add the term ϵI to the standard Hamiltonian:

$$H = H_0 + \epsilon I = \sum_{j=1}^{N} \frac{1}{2m} \nabla_j^2 + U - \epsilon \log_2(a \mid \psi \mid^2) \tag{2}$$

Now it is possible to show that this constant ϵ is very close to our compressibility constant. The total energy as the expectation value of the Hamiltonian (2) is:

$$E =< \psi \mid H >=< \psi \mid H_0 \mid \psi > +\epsilon < \psi \mid H_0 \mid \psi >=$$
$$= E_0 \epsilon < \psi \mid \log_2(a \mid \psi \mid^2) \mid \psi > \tag{3}$$

Define the entropy for the subsystem:

$$S = K < \psi \,|\, \ln(a \,|\, \psi \,|^2) \,|\, \psi > \tag{4}$$

where K is the Boltzmann constant. Now we obtain the free energy E_0, the temperature T and the constant ϵ :

$$E_0 = E - TS, \quad T = \left(\frac{\partial E}{\partial S}\right)_V, \quad \epsilon = KTln(2) \tag{5}$$

The value $\epsilon = KTln(2)$ is very close to the constant C. If we suppose the value KT for the average kinetic energy $\approx 30MeV$ we obtain the reasonable value of the compressibility constant.

In the work [7] was obtained linear rise of the entropy with energy: for heavy-ions beams in the energy range $6 \div 9$ Gev for the cases of central and peripherical collisions of ions $^{32}S, ^{16}O$, ^{28}Si with nuclei of emulsion. The authors could explain this experiment based on the results of the Prague group using the "information entropy":

$$S = -\epsilon P_n \ln P_n,$$

where P_n is probability of the production of "n" particles in the final state ($\sum_n P_n = 1$). This linear rise of the entropy with energy is in an agreement with nonlinear theory.

2. Korteveg de Vries Equation

Besides NoSE there is another interesting equation, which was applied many times for the prediction of new forms of motion in nuclei [8]. This is the KdV equation. The KdV equation, was applied many times in nuclear physics [8,9,10,11,12,13].

In the work [8] the connection was established between the solution of KdVE and the nuclear potential in Schroedinger equation. At the certain circumstances we can see the evolution of the nuclear potential with time.

G.N.Fowler, S.Raha and R.M.Weiner [11] attempted to predict the localized stable surface excitations in peripheral heavy ion collisions like in Rossby waves.

In the work [12] A.Sandulesky and W.Greiner obtained a new coexistence model consisting of the usual shell model and a cluster model, describing a soliton moving on the nuclear surface. They got the excellent description of experimental spectroscopic factors for cluster decays.

In its turn we considered the perturbation of the nuclear surface as a quadrupole deformation of the nuclear potential:

$$U = U_0 \mathrm{sech}^2((r-R)/a) = 2aU_0 \frac{df(2(r-R)/a)}{dr} \tag{22}$$

The evolution of this perturbation having the soliton form with time is given by the KdV equation [14]:

$$U_t - 6UU_x + \beta U_{\mathrm{XXX}} = 0, \qquad (23)$$

where the function U is the potential of the Schroedinger equation

$$\beta\psi_{xx} - U\psi = 0, \qquad \beta = \hbar^2/2m \tag{24}$$

On the other hand this perturbation in the form of soliton can decay on the certain quantity of solitons with the small amplitude [19]. If we fix KdVE in the form

$$U_t + UU_x + \beta U_{\mathrm{XXX}} = 0, \tag{25}$$

its solution will be in the form:

$$U(x) = U_0 \mathrm{sch}^2(U_0/12\beta)^{1/2} X). \tag{26}$$

As an example we consider the interference of Coulomb and Nuclear excitations in the frame of DWBA. The differential cross section, corresponding to the transition with angular momentum λ is given by this way:

$$\frac{dz}{d\Omega} = \frac{1}{4\pi}\left(\frac{2\mu}{\hbar^2}\right)\frac{k'}{k}\frac{2I'+1}{(2I+1)(2\lambda+1)}\sum_{\mu=-\lambda}^{\lambda}|T_\mu^\lambda(\Theta)|^2,$$

where amplitude T^λ_μ is determined by this way:

$$T^\lambda_\mu = \sum_{l,l'} i^{l-l'} \exp[i(\sigma_l + \sigma_{l'})] < l'\lambda 00|l0 > \times$$

$$\times < l'\lambda - \mu\mu|l0 > [(l'-\mu)!/(l+\mu)!]^{1/2} P^\mu_{l'}(\cos\Theta)[C^\lambda_{ll'} + N^\lambda_{ll'}],$$

where $C^\lambda_{ll'}$ and $N^\lambda_{ll'}$ are Coulomb and Nuclear matrix elements:

$$C^\lambda_{ll'} = (kk')^{-1}\int_0^\infty dr f_l(kr) F^\lambda_c(r) f_{l'}(k'r)$$

$$N^\lambda_{ll'} = (kk')^{-1}\int_0^\infty dr f_l(kr) F^\lambda_N(r) f_{l'}(k'r)$$

$f_l(kr)$ is a solution of the Schroedinger equation with a potential the optical model. $F^\lambda_c(r)$ and $F^\lambda_N(r)$ are Coulomb and Nuclear form-factors:

$$F^\lambda_c(r) = \begin{cases} \frac{4\pi Z_p e[B(E_\lambda)]^{1/2}}{(2\lambda+1)r^{(\lambda+1)}}; & r \geq R_c \\ \beta_\lambda = \frac{3Z_pZ_Te^2}{2\lambda+1}\frac{Z^\lambda}{R_c^{\lambda+1}}; & r < R_c \end{cases}$$

$$F^\lambda_N = \beta^N_\lambda R\frac{dU}{dr}, \quad \beta_\lambda = \frac{4\pi[B(E\lambda)]^{1/2}}{3Z_T e R^\lambda_c}.$$

For us the nuclear form-factor is a point of interest. Usually the form-factor is the derivative from the total optical potential which reminds the solitonic shape:

$$F^\lambda_N(r) = \beta^N_\lambda R\frac{dU}{dr} = \beta^N_\lambda\frac{RV_0}{2a}sech^2\frac{r-R}{a}$$

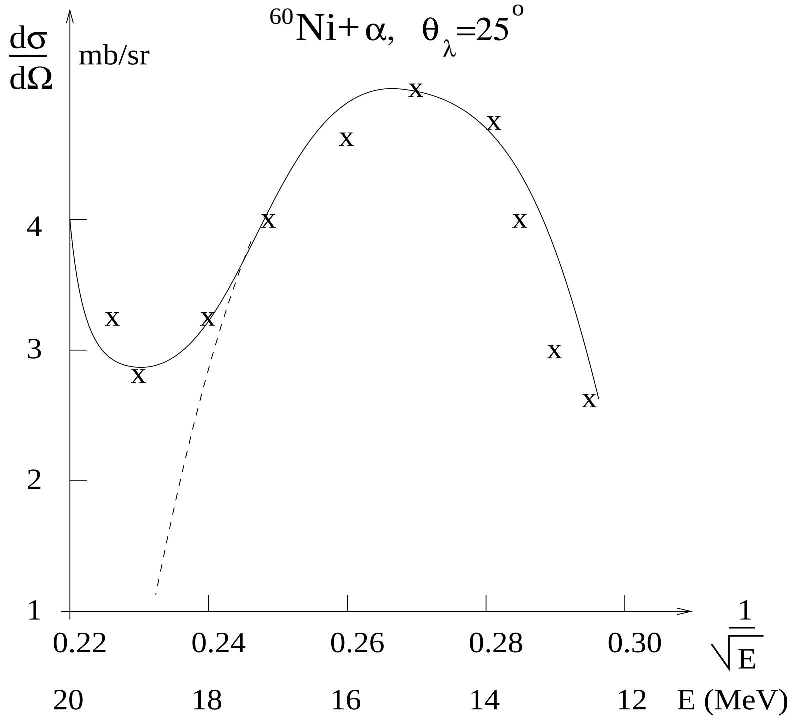

Fig. 1

In Fig.1 we can see the excitation function for the inelastic scattering of α-particles with the excitation of the first exiting state of ^{60}Ni. Experimental points were taken from the work [15]. To demonstrate the solitonic character of the Coulomb-Nuclear interference we used instead of energy scale the time scale.

There is another interesting effect connecting with the KdVE. This wonderful fact [16] that energies of some low-lying levels of given nucleus exited in inelastic scattering are multiple to the energy of its first excited state can be understood classically as the so called Huygens clock synchronization effect due to nonlinear interaction of some clocks placed at the same beam. Namely the observed experimental energy (frequency) distribution can be described by the system for q oscillators located in one nucleus and therefore coupled nonlinearly.

This problem has a link with Fermi, Pasta and Ulam (FPU) problem when they considered many years ago the behavior of oscillations in a chain of 64 mass points interacting through nonlinear forces [17]. The problem of FPU can be considered in terms of KdVE. The result of FPU was unexpected: if the initial data assigned all the energy to the lowest mode, only a few modes were exited as time went on and almost all the energy was eventually given back to the lowest mode.

Now we apply the similarity principle. The initial condition we take in the form:

$$U(x,0) = U_0 \phi(x,l). \tag{27}$$

With new variables

$$\eta = U/U_0, \quad x = X/l, \quad \tau = U_0 t/l, \tag{28}$$

we obtain a new KdVE:

$$\eta_t + \eta\eta_x + \sigma^{-2}\eta_{\mathrm{XXX}} = 0, \tag{29}$$

where $\sigma = l(U_0/\beta)^{1/2}$.

In the work [18] the critical value $\sigma = \sqrt{12}$ was obtained. With increase of the number σ the corresponding perturbation breaks up into a larger number of solitons. The number of this solitons was determined by Karpman [19]

$$N = 2a\sqrt{U_0 6\beta}. \tag{30}$$

For the scattering of α–particles with kinetic energies in the range 4–12 MeV we can get for the nucleus ^{12}C the value $N \sim 5$. It corresponds to the number of existing states generated by the surface potential well: $K^+ = 0, 2, 4, 6, 8$. Gardner et al. [20] showed, that the amplitude of solitons determined by the eigenvalues: $A = 2E_n$.

The potential having the soliton shape gives the following spectrum [21]:

$$E_n = -\frac{\hbar^2}{8m\Delta}\left(-(1-2n) + (1 + 8m\Delta U_0 \hbar^{-2})^{1/2}\right). \tag{31}$$

For our example ($n = 0, 2, 4, 6, 8$) we obtain from (31) the following values (in MeV): $-15.57, -8.65, -1.75, 5.19, 12.11$, respectively. The smaller eigenvalue the lager a soliton amplitude. Knowing the eigenvalues one can construct the total potential [22].

Thus we interpreted two equations NoSE and KdVE respectively to the problem of clustering. This work is only the first approximation to this problem because we considered only one dimension case. To our mind the next step should be the application of Kadomtsev-Petviashvili equation to this problem.

The most promising thing is the application as the similarity principle to the linear alpha-chain states in nuclei [23].

3. Nuclear instability, a soliton theory, the Kadomtsev - Petviashvili Equation

An existence of solitary waves is determined by two essential factors, namely, nonlinearity and dispersion. Both the factors, which are responsible for the stability of a wave, are connected in their turn with two different types of instability. A localized pulse will tend to spread out due to dispersion terms of the equations of motion. The nonlinearity which is responsible for the formation of solitary waves, on the other hand, automatically leads to their destruction, if it is alone. Both instabilities may compensate each other and lead to stable solutions (solitons).

Let us look from these points of view at multi fragmentation phenomena. The formation and breakup of a highly excited and compressed nuclear system is the most striking process observed in intermediate-energy heavy ion reactions. How does such a system expand and finally disassemble when passing through a regime of dynamical instabilities? What is the mechanism of the clustering (stable light and intermediate mass fragment production)? Quite a variety of models have been developed to discuss this question (see, for instance, the recent review [25] and Proceedings [26]). The dynamical clusterization in the presence of instabilities is the focus of attention of the intermediate energy heavy ions physics [27], [28]. Ten years ago multifragmentation has been associated with the onset of the spinodal instability [29]. This instability is associated with the transit of a homogeneous fluid across a domain of negative pressure, which leads to its breaking up into droplets of denser liquid embedded in a lower density vapor. Since the spinodal instability can occur in an infinite system, it can be called the bulk or volume instability. On the other hand, it physically means that pressure depends on density, that is just a nonlinearity in terms of density [30].

Recently [31], it has been pointed out that a new kind of instability (sheet instability) may play an important role in multifragmentation. This new instability can be assigned to the class of surface instabilities of the Rayleigh-Taylor kind [32]. System escapes from the high surface energy of the intermediate complex by breaking up into a number of spherical fragments with less overall surface. At the same time, it physically means the existence of the gradient terms of the equations of motion, i.e the dispersion [30].

The spinodal instability and the Rayleigh-Taylor instability may compensate each other and lead to stable quasi-soliton type objects. In the next Sec. we present this physical picture using a simple analytical model proposed to describe the time evolution of compressed nuclear systems.

An analysis of stability of nonlinear dynamical systems and an analysis of nonlinear evolution of initial complex states is a traditional goal of Soliton Theory. The inverse methods to integrate nonlinear evolution equations are often more effective than a direct numerical integration. Let us demonstrate this statement for a very simple case [35].

The type of systems under our consideration are uncharged slabs of nuclear matter. The slabs are finite in the z coordinate and infinite and homogeneous in two transverse directions.

The basic equations for the static mean-field description of the slabs are the following

$$
\begin{gathered}
\psi_{\mathbf{k}_\perp n}(\mathbf{x}) = \frac{1}{\sqrt{\Omega}}\psi_n(z) exp(i\mathbf{k}_\perp \mathbf{r}), \qquad \epsilon_{\mathbf{k}_\perp n} = \frac{\hbar^2 k_\perp^2}{2m} + e_n, \\
-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\psi_n(z) + U(z)\psi_n(z) = e_n\psi_n(z), \\
\rho(\mathbf{x}) \Longrightarrow \rho(z) = \sum_{n=1}^{N_0} a_n \psi_n^2(z), \\
A \Longrightarrow \mathcal{A} = (6A\rho_N^2/\pi)^{1/3} = \sum_{n=1}^{N_0} a_n, \qquad a_n = \frac{2m}{\pi\hbar^2}(e_F - e_n), \\
\frac{E}{A} \Longrightarrow \frac{\hbar^2}{2m\mathcal{A}}\Big(\sum_{n=1}^{N_0} a_n \int_{-\infty}^{\infty} (\frac{d\psi_n}{dz})^2 dz + \frac{\pi}{2}\sum_{n=1}^{N_0} a_n^2\Big) + \frac{1}{\mathcal{A}}\int_{-\infty}^{\infty} \mathcal{E}[\rho(z)]dz,
\end{gathered}
\tag{1}
$$

where $\mathbf{r} \equiv (x, y), \mathbf{k}_\perp \equiv (k_x, k_y)$, and Ω is the transverse normalization area. N_0 is the number of occupied bound orbitals.

The dynamical description will be done in the framework of the inverse mean field method. One can found the details of this approach in [36],[37]. We concentrate here only on essentials.

The evolution of a system is given by the famous hydrodynamical Korteveg-de Vries equation (KdV) for the mean-field potential $U(z,t)$

$$
\sum_{n=1}^{N} \frac{\partial U}{\partial(S_n t)} = 6U\frac{\partial U}{\partial z} - \frac{\hbar^2}{2m}\frac{\partial^3 U}{\partial z^3}, \tag{2}
$$

where S_n are constants which are determined by the initial conditions.

General solution of KdV Eq. (2) can be derived in principle via direct methods numerically. This way is to assign a functional of interaction $\mathcal{E}$ (as usual an effective density dependent Skyrme force), a total number of particles (or a thickness of a slab $\mathcal{A}$) and to solve Hartree-Fock equations to derive a spectrum of the single particle states e_n and wave functions $\psi_n(z,0)$, the density profile $\rho(z,0)$ and the one-body potential $U(z,0)$ for the initial compressed nucleus. Then, one calculate an evolution of the one-body potential with the help of Eq. (2).

However, there is an inverse method to solve KdV Eq. (2). The main advantage of this way is to reduce the solution of the nonlinear KdV Eq. (2) to the solution of the linear Schrödinger - type equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\psi_n(z,t) + U(z,t)\psi_n(z,t) = e_n\psi_n(z,t), \tag{3}$$

and linear integral Gelfand - Levitan - Marchenko equation to derive the function $K(x,y)$

$$K(x,y) + B(x+y) + \int_x^\infty B(y+z)K(x,z)dz = 0. \tag{4}$$

The kernel B is determined by the reflection coefficients $R(k)(e_k = \hbar^2k^2/2m)$. and by the N bound state eigenvalues $e_n = -\hbar^2\kappa_n^2/2m$. N is the total number of the bound orbitals.

$$B(z) = \sum_{n=1}^{N} C_n^2(\kappa_n) + \frac{1}{\pi}\int_{-\infty}^{\infty} R(k)exp(ikz)dk.$$

The coefficients C_n are uniquely specified by the boundary conditions

$$C_n(\kappa_n) = \lim_{z\to\infty}\psi_n(z)exp(\kappa_n z),$$

and the wanted single particle potential is given by

$$U(z,t) = -\frac{\hbar^2}{m}\frac{\partial}{\partial z}K(z,z).$$

The time t is included in Eqs. (3,4) only as a parameter, so it has been omitted in the above formulas.

The general solution, $U(z,t)$, should naturally contain both, contributions due to the continuum of the spectrum and to its discrete part. There is no way to obtain the general solution $U(z,t)$ in a closed form. Eqs. (3,4) have to be solved only numerically.

However in the case of reflectless ($R(k) = 0$) symmetrical ($U(-z,0) = U(z,0)$) potentials one can derive the following basic relations

$$\begin{aligned} U(z,t) &= -\frac{\hbar^2}{m}\frac{\partial^2}{\partial z^2}ln(det\|M\|) = -\frac{2\hbar^2}{m}\sum_{n=1}^{N}\kappa_n\psi_n^2(z,t), \\ \psi_n(z,t) &= \sum_{n=1}^{N}(M^{-1})_{nl}\lambda_l(z,t), \\ \lambda_n(z,t) &= C_n(\kappa_n)exp(-\kappa_n z + 2\hbar^2\kappa_n^3 S_n t/m)), \\ M_{nl}(z,t) &= \delta_{nl} + \frac{\lambda_n(z,t)\lambda_l(z,t)}{\kappa_n+\kappa_l}, \\ C_n(\kappa_n) &= \left(2\kappa_n\prod_{l\neq n}^{N}\frac{\kappa_n+\kappa_l}{\kappa_n-\kappa_l}\right)^{1/2}. \end{aligned} \tag{5}$$

So, the wave functions, potential and the density profile are completely defined by the bound state eigenvalues. The first step is to solve the Schrödinger Eq. (3) for the initial potential $U(z,0)$, which is suitable to simulate compressed nuclear system or to simulate this state with the help of spectrum. Then one calculates the evolution of $\rho(z,t)$ and $U(z,t)$ with the help of Eqs. (5).

Although there is definitely some progress in the application of the inverse methods to nuclear physics they are not yet too popular. As illustration of these methods, we consider a one-dimensional three-level system in details [38].

A three-level system may be useful for modeling the evolution of light nuclei, for instance, oxygen [39].

Let us present the main formulas to calculate wave functions of a three-level system $\kappa_3 > \kappa_2 > \kappa_1$

$$\begin{aligned}
\psi_1(z,t) &= \Big(2\kappa_1\big(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)\Big)^{1/2} D^{-1}(z,t)\big(ch(\xi_2+\xi_3)-\big(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big)ch(\xi_3-\xi_2)\big),\\
\psi_2(z,t) &= \Big(2\kappa_2\big(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big)\Big)^{1/2} D^{-1}(z,t)\big(sh(\xi_3+\xi_1)-\big(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)ch(\xi_3-\xi_1)\big),\\
\psi_3(z,t) &= \Big(2\kappa_3\big(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big)\big(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)\Big)^{1/2} D^{-1}(z,t)\big(ch(\xi_1+\xi_2)-\big(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)ch(\xi_2-\xi_1)\big),\\
D(z,t) &= ch(\xi_1+\xi_2+\xi_3)+\big(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)ch(\xi_3+\xi_2-\xi_1)\\
&+ \big(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big)ch(\xi_3+\xi_1-\xi_2)+\big(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big)ch(\xi_2+\xi_1-\xi_3),
\end{aligned}$$

$$\xi_n(z,t) \equiv \kappa_n z - 2\hbar^2\kappa_n^3 S_n t/m \qquad n=1,2,3. \tag{6}$$

The asymptotic behavior of the wave functions and the mean - field potential have the following form (ξ_n is fixed)

$$\lim_{t\to\infty}\psi_k = \delta_{kn}\sqrt{\kappa_n}sech(\xi_n-\xi_n^0),$$

$$\lim_{t\to\infty}U(z,t) = -\frac{\hbar^2\kappa_n^2}{m}sech^2(\xi_n-\xi_n^0),$$

$$\xi_n^0 = \frac{1}{2}ln\Big(\prod_{k=1}^{n-1}\big(\frac{\kappa_n-\kappa_k}{\kappa_n+\kappa_k}\big)^2\prod_{m\neq n}^{n}\big|\frac{\kappa_n+\kappa_m}{\kappa_n-\kappa_m}\big|\Big).$$

So, for large z and t the time - dependent one - body potential and the corresponding density distributions are represented by a set of stable solitary waves. The energy spectrum of an initially compressed system completely determines widths, velocities and the phase shifts of the solitons.

The number of waves is equal to the number of occupied bound orbitals. The thickness ('number' of particles) of an n - wave is equal to a_n.

Reflecting terms ($R(k) \neq 0$) of $U(z,t)$ cause ripples (oscillating waves of a small amplitude) in addition to the solitons in the final state.

The initially compressed system expands so that for large times one can observe separate density solitons and ripples ('emissions'). This picture is in accordance with the TDHF simulation of the time evolution of a compressed O^{16} nucleus [39]. The disassembly shows collective flow and clusterization. It is important to note that the clusterization was not observed in the absence of the self-consistent mean-field potential, i.e this confirms our supposition that the nonlinearity is very important for the clustering.

It is necessary to note that the present model is too primitive in order to describe a real breakup process. However this model can be used to illustrate an inverse mean-field method scheme, a nonlinear principle of superposition and the idea that nonlinearity and dispersion terms of the evolution equation can lead to clusterization in the final channel.

Certainly there are a lot of open questions relative to the presented approach. The most crucial concerns the generalization to 3+1 dimensional model with a finite temperature. One possible way to do it keeping simplicity of the approach would be to use an information theory [40],[41] and the Kadomtsev-Petviashvili equation (KPE). Motivation: from one side KPE was applied for the explanation of a movement of the Jupiter red spot , from the other hand the hot spot model was applied in nuclear physics for an explanation of the fragmentation processes.

A two dimensional soliton is a solution of KPE of the form

$$\frac{\partial^2}{\partial x^2}(u_t - 6uu_x - u_{xxx}) = -3u_{yy}$$

The exact solution of this equation is

$$u(x, y, t) = 2\frac{\partial^2}{\partial x^2}\ln[4(\nu + \nu^*)^2 + |x - i\nu y - 3\nu^2 t|^2],$$

where ν is an arbitrary parameter.

There is difference between one dimensional solitons and two-dimensional ones: the phase shifts in the last case are equal zero. A Ssoliton solution of KPE looks like

$$u_{sol} = 2\kappa^2 sech^2[\kappa(x + ky - (4\kappa^2 + k^2)t]$$

where a parameter κ determines an orientation of solitons. From this it is clear that a particular solution of KPE in one dimension is the solution of KdVE.

The problem of dynamical instability and clusterization (stable fragments formation) in a breakup of excited nuclear systems are considered from the points of view of the soliton concept. It is noted that the volume (spinodal) instability can be associated with nonlinear terms, and the surface (Rayleigh-Taylor type) instability, with the dispersion terms in the evolution equations. The both instabilities may compensate each other and lead to stable solutions (solitons).

To conclude, the simple analytical model is presented to describe the time evolution of cold compressed nuclear systems in the framework of the inverse mean-field method. It is demonstrated that the nonlinearity and dispersion terms of the evolution equations can lead to clusterization in the final channel.

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