Supersonic Flow of a Nonequilibrium Gas Past a Thin Body

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Abstract

A nonlinear structure of the acoustic field radiated by a thin body in the supersonic flow of a nonequilibrium gas up to the third order of perturbation theory is investigated. The integral characteristics of a flowover process (drag and lift force coefficients $C_{x;y}$) was received.

A nonequilibrum gas represents the media with incomplete investigated hydrodynamical properties. The classical problem of the fluid mechanics is the problem of a supersonic motion around a thin body [1]. This problem, in particular, was investigated in [2], [3] in the linear approach in a case of inviscid flow of the nonequilibrum gas. In [4], [5] the lift and drag force coefficiens for bodies of the various form in the supersonic flow of nonequilibrum partially ionized gas was measured experimentally. The discharge was created at a face or on a lateral surface of the body. The essential reduction of the drag force was marked.

The present paper considers the supersonic motion of the nonequilibrium gas around a thin body up to the order of ε^3, where ε is a small parameter. Drag and lift forces, as is known, connected with the impulse which is carried away by shock waves. However in acoustically active media the structure of a acoustic field is defined by the second viscosity coefficient $\mu \gg \nu$, where ν is the first (shift) viscosity coefficient, [6]. Since the acoustic field structure around a body is sufficiently changed.

We consider a plain supersonic flow ($M = v_0/u_s > 1$ is the лach number , v_0 is the flow velocity at infinity, u_s is the sound velocity) around a thin body ($l_x/l_y \gg M$) with a small incidense angle $\delta = l_y/l_x \ll 1$. Here l_y is effective thickness, l_x is the length of the body.

Futher we examine the integrated characteristics of a flow - drag and lift force coefficients $C_{x;y}$:

$$C_{x;y} = \left(\frac{1}{2}\rho_0 v_0^2\right)^{-1} \int P n_{x;y} ds$$

where ρ_0 is a gas density at infinity, P is a gas, $n_{x;y}$ are components of a normal vector. The integration is conducted along an airfoil surface $y = y(x)$. Pressure on the airfoil surface can be represented in the form $P = \sum_{j=0}^{1} P_j$, where P_0 is the gas pressure at infinity, $P_1 = -\rho_0 v_0 v_x - \rho_0 \left(v_y^2 + v_x^2\right)/2$ is disturbance of pressure, $v_{x;y}$ are components of the flow velocity pertubations.

The assumption formulated permit to use of the equations which describe the fluid motion in form

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v}\right) = -\nabla P \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho\vec{v}) = 0 \tag{2}$$

It is possible to search the decision of system (1), (2) as: $\vec{v} = \vec{v}_0 + \nabla\phi(x,y)$, $\rho = \rho_0 + \rho'$, considering ϕ and ρ' as small parameters. This system must be completed with the equation of state and energy. In th nonequilibrium gas for perturbances of parameters these equations up to the of ε^2 order lead to the following relations between the pressure and density disturbances

$$P' = u_s^2\rho' + \frac{1}{2}\left(\frac{\partial^2 P_0}{\partial \rho_0^2}\right)\rho'^2 + \frac{m u_s^2}{\tau}\rho' + \mu_\Sigma \nabla\vec{v}$$

where μ_Σ is the total viscosity.

After substitution of $\vec{v}$ and ρ in system (1), (2) and standart transformations we have:

$$\frac{\partial^2\phi}{\partial y^2} - (M^2-1)\frac{\partial^2\phi}{\partial x^2} - \frac{M}{u_s}\left[[2+\gamma M^2)]\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x^2} + 2\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y}\right] -$$
$$- \frac{M}{u_s}\frac{\partial^2\phi}{\partial x\partial t} - \frac{M}{u_s^3}(\gamma-1)\frac{\partial\phi}{\partial t}\frac{\partial^2\phi}{\partial x\partial t} + \frac{M^2(\gamma-1)+1}{u_s^2}\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x\partial t} - \gamma\frac{M^2}{u_s^2}\frac{\partial\phi}{\partial t}\frac{\partial^2\phi}{\partial x^2} - \tag{3}$$
$$- \frac{1}{u_s^2}\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial y\partial t} - \mu_\Sigma\frac{M^3}{u_s}\frac{\partial^3\phi}{\partial x^3} - \mu_\Sigma M^2\frac{\partial^3\phi}{\partial x^2\partial t} + M\alpha_\infty\frac{\partial\phi}{\partial x} + \frac{\alpha_\infty}{u_s}\frac{\partial\phi}{\partial t} = 0$$

where ϕ is the potential of velocity, $\alpha_\infty = mu_s/\tau$ is spatial increment of acoustic instability. Let's notice that the characteristic lengths of energy dissipation for low-frequency and high-frequency perturbances $\sim \alpha_0^{-1}\ \alpha_\infty^{-1}$ must br small in comparison with length l_x.

The kinematic boundary condition on the airfoil has a form

$$\left(v_0 + \frac{\partial\phi}{\partial x}\right)n_x + \frac{\partial\phi}{\partial y}n_y\bigg|_{y\to\pm 0} = 0 \tag{4}$$

In space above a body ($y > 0$) it is possible to assume $n_x \approx -\partial\zeta_2(x))\partial x$, and under a body $n_x \approx \partial\zeta_1(x))\partial x$, $n_y \ll 1$, where $y = \zeta_{2;1}(x)$ are equations of the wing surfaces.

The most interest is represented a problem of a stationary flow. The nonlinear acoustic theory for nonequilibrum gas [6] predicts an opportunity of stabilization of acoustic instability developing with $\alpha_\infty < 0$ in the second order of the theory of perturbances. For a stationary wave $\phi = \phi(x - wt, y)$ where $w \to 0$ equation (3) is reduced to a form:

$$\frac{\partial^2\phi}{\partial y^2} - (M^2-1)\frac{\partial^2\phi}{\partial x^2} - \frac{M}{u_s}\left[[2+\gamma M^2)]\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x^2} + 2\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y}\right] -$$
$$- \frac{\mu_\Sigma M^3}{u_s}\frac{\partial^3\phi}{\partial x^3} + \alpha_\infty M\frac{\partial\phi}{\partial x} = 0 \tag{5}$$

The similar equations are well known in the theory of nonlinear ion-sound waves, gravitational waves on a surface of a liquid etc.

For the further simplification of considered model the method of the consecutive approaches is used. Decision of the equation (6) in the firsr order is a simple wave: $\partial\phi/\partial y = F_{2;1}(u)$, where $u = \partial\phi/\partial x$, and $F_{2;1}(u) = \pm\beta u \pm \gamma M^3u^2/4u_{s0}\beta$, $\beta = \sqrt{M^2-1}$.

In the following order we search the decision in a form

$$\frac{\partial\phi}{\partial y} = F_{2;1}(u) + \varepsilon g(x,y)\ ,\ \pm\beta\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} = 0$$

where $\partial^2\phi/\partial x\partial y = \partial u/\partial y$. In result we have

$$\frac{\partial u}{\partial y} \pm \beta\frac{\partial u}{\partial x} \pm \frac{\gamma M^3}{2\beta u_s}u\frac{\partial u}{\partial x} = \pm\mu_\Sigma\frac{M^3}{u_s}\frac{\partial^2 u}{\partial x^2} \mp \frac{M\alpha_\infty}{2\beta}u \tag{6}$$

Boundary condition (5) after substitution of functions $u, F_{2;1}, g$ are transformed to a form

$$u(y=\pm 0) = \pm\frac{Mu_s}{\beta}\frac{\partial\zeta_{2;1}}{\partial x} \pm \mu_\Sigma\frac{M^4}{2u_s\beta^3}\frac{\partial^2\zeta_{2;1}}{\partial x^2} \mp \frac{\alpha_\infty M^2u_s}{2\beta^3}\zeta_{2;1} \tag{7}$$

The equation (6) has the limited decisions in variable $\eta = x - \beta y$ in the form of the sawtooth wave, [6]. Another types of the limited decisions with α_∞ 0 this equation does not admit.

In the considered problem the expression for P_1 can be transformed to a form: $P_1 = -\rho_0 v_0 \partial\phi/\partial x$, [1]. Coefficients $C_{x;y}$ are expressed through coefficient of pressure $C_p = P_1/\rho_0 v_0^2/2$ on a the airfoil surface:

$$C_x = \int_0^{l_x} \left(C_{p2} \frac{\partial \zeta_2(x)}{\partial x} - C_{p1} \frac{\partial \zeta_1(x)}{\partial x} \right) dx \ , \ C_y = \int_0^{l_x} (C_{p2} - C_{p1}) \, dx$$

The value C_p on both surfaces up to the accepted accuracy is defined by expression:

$$C_{p2;1} = \frac{2}{\beta} \left[\mp \frac{\partial \zeta_{2;1}}{\partial x} \pm \frac{M\alpha_\infty}{\beta^2} \zeta_{2;1} \right]$$

Let's define angles of an inclination of the top and bottom lines of a wing to it chorde by expressions $\theta_{2;1}(x); = \partial\zeta_{2;1}(x)/\partial x - \delta$. Let's notice, that here $\zeta_{2;1}(0) = l_y$, $\zeta_{2;1}(l_x) = 0$.

Under condition of the homogeneous distribution of the excited particles up to the order of ε^2 we receive

$$C_x = \frac{2}{\beta} \left[2\delta^2 + < \theta_1^2 >_x + < \theta_2^2 >_x \right] , \ C_y = \frac{2}{\beta} \left[2\delta - \frac{M\delta}{\beta^2} \alpha_\infty l_x \right] . \tag{8}$$

Let's notice, that by replacement of a variable

$$\xi = \pm \frac{\beta y - x}{l_x} \ ; \ \tau = \frac{\epsilon y M^4 l_y}{\beta^2 l_x^2} \ ; \ \eta = \mp \frac{u \beta l_x}{M u_s l_y} \tag{9}$$

equation (6) can be reduced to form

$$\frac{\partial \eta}{\partial t} + \eta \frac{\partial \eta}{\partial \xi} = \mu_{1\infty} \frac{\partial^2 \eta}{\partial \xi^2} + \nu_{1\infty} \eta \tag{10}$$

This equation is similar under the form with received in [6] for acoustic perturbances in the nonequilibrum media. The equations (6) and (10) were received by means of the paerturbation methons, use the same small parameters. Since the description of the acoustic field structure around the body can be received by means of the variables transformations in the basic equations, represented in [6]. In approach $\omega\tau \ll 1$, where τ is the relaxation time we have th equation

$$\begin{aligned} \frac{\partial u}{\partial y} \pm \beta \frac{\partial u}{\partial x} \pm \frac{\epsilon M^3}{\beta u_0} u \frac{\partial u}{\partial x} \mp \frac{k_3 M^2}{\beta u_0^3} \frac{l_x}{l_y} \frac{\partial u^3}{\partial x} &= \pm \beta_0 \frac{M^4}{\beta u_0} \frac{\partial^3 u}{\partial x^3} \pm \mu_\Sigma \frac{M^3}{u_0} \frac{\partial^2 u}{\partial x^2} \\ &\pm k_1 \frac{M^3}{\beta u_0^2} u \frac{\partial^2 u}{\partial x^2} \pm k_2 \frac{M^3}{\beta u_0^2} \frac{\partial^2 u^2}{\partial x^2} \end{aligned} \tag{11}$$

with boundary condition

$$\begin{aligned} u(y = \pm 0) = \pm \frac{M u_0}{\beta} \frac{\partial \zeta_{2;1}}{\partial x} + \mu_\Sigma \frac{M^3}{\beta^2 u_0} \frac{\partial u}{\partial x} - \frac{\epsilon M^3}{\beta^2 u_{s0}} u^2 + \beta_0 \frac{M^4}{\beta^2 u_0} \frac{\partial^2 u}{\partial x^2} + \\ k_1 \frac{M^3}{\beta^2 u_0^2} \int u \frac{\partial^2 u}{\partial x^2} dx + k_2 \frac{M^3}{\beta^2 u_0^2} \frac{\partial u^2}{\partial x} + \frac{k_3 M^2}{\beta^2 u_0^3} \frac{l_x}{l_y} u^3 \end{aligned} \tag{12}$$

In approach $\omega\tau \gg 1$ the appropriate system has a form

$$\begin{aligned} \frac{\partial u}{\partial y} \pm \beta \frac{\partial u}{\partial x} \pm \frac{\epsilon M^3}{\beta u_\infty} u \frac{\partial u}{\partial x} \mp k_3 \frac{M^2}{\beta u_\infty^3} \frac{l_x}{l_y} \frac{\partial u^3}{\partial x} = \pm \frac{3 M \alpha_\infty^2}{2} \int u dx \pm \mu_\Sigma \frac{M^3}{u_\infty} \frac{\partial^2 u}{\partial x^2} \\ \pm k_1 \frac{M^3}{\beta u_\infty^2} u \frac{\partial^2 u}{\partial x^2} \pm k_2 \frac{M^3}{\beta u_\infty^2} \frac{\partial^2 u^2}{\partial x^2} \mp \frac{M^3 \epsilon \alpha_\infty}{\beta u_\infty} \frac{\partial u}{\partial x} \int u dx \mp \frac{M \alpha_\infty^2}{\beta u} \pm \frac{M^3 \nu_\infty}{\beta u_\infty} u^2 \end{aligned} \tag{13}$$

$$u(y=\pm 0) = \pm\frac{Mu_{s0}}{\beta}\frac{\partial\zeta_{2;1}}{\partial x}\mu_\infty\frac{M^3}{\beta^2 u_\infty}\frac{\partial u}{\partial x} - \frac{\Psi_\infty M^3}{\beta^2 u_{s0}}u^2 + \frac{3M\alpha_\infty^2}{2}\int(\int udx)dx +$$
$$k_{1\infty}\frac{M^3}{\beta^2 u_{s0}^2}\int u\frac{\partial^2 u}{\partial x^2}dx + k_{2\infty}\frac{M^3}{\beta^2 u_{s0}^2}\frac{\partial u^2}{\partial x} + k_{3\infty}\frac{M^2}{\beta^2 u_{s0}^3}\frac{l_x}{l_y}u^3 - \frac{M^3\Psi\infty\alpha_\infty}{\beta^2 u_\infty}$$
$$\int\frac{\partial u}{\partial x}\int udxdx - \frac{M\alpha_\infty}{\beta^2}\int udx + \frac{M^3\nu_\infty}{\beta^2 u_\infty}\int u^2 dx \qquad (14)$$

Let's consider, for simplicity a plain wing $\theta_{1;2} = 0$. The calculations, completely similar mentioned above, allow to receive expressions for drag ang lift force coefficients. In approach $\omega\tau \ll 1$

$$C_x = \frac{2}{\beta}\delta^2\left[2 + \delta\frac{k_{30}M^3}{\beta u_0}\right], \quad C_y = \frac{2}{\beta}\delta\left[2 - \delta\frac{k_{30}M^3}{\beta u_0}\right] \qquad (15)$$

In approach $\omega\tau \gg 1$

$$C_x = \frac{2}{\beta}\delta^2\left[2 - \frac{M}{2\beta}\alpha_\infty l_x + \delta\frac{k_{3\infty}M^3}{\beta u_\infty}\right], \quad C_y = \frac{2}{\beta}\delta\left[2 - \frac{M}{2\beta}\alpha_\infty l_x - \frac{M}{4\beta}\alpha_\infty^2 l_x^2 - \delta\frac{k_{3\infty}M^3}{\beta u_\infty}\right] \qquad (16)$$

The given results demonstrate dependence of the drag and lift forces in the nonequilibrum gases from the degree of non-equilibrium and distribution of the exited particles. These parameters has a significant influence on the structure of an acoustic field around airfoil surface.

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