Instant–Form Approach to Two–Body Systems: Electroweak Properties of Pseudoscalar Mesons

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Abstract

We present a relativistic treatment of the problem of electroweak structure of composite quark systems in the framework of instant form of relativistic Hamiltonian dynamics (RHD). Integral representations for electromagnetic form factors and semileptonic decay form factors of the pseudoscalar mesons are obtained. Our approach gives the well description of pion and kaon electromagnetic structure and semileptonic decay of kaon at the same values of model parameters.

The relativistic description of bound states was always an important problem in nuclear physics and particle physics. This problem became particularly topical in connection with the development of quark physics, in which the relativistic properties of the light quarks play a fundamental role.

In the relativistic theory of the description of composite systems, it is possible to identify two main but very different approaches.

The first is the method of field theory. Based on the principles of quantum field theory – quantum chromodynamics (QCD) – it is rightly regarded as the most consistent approach to the solution of this problem. However, standard perturbative QCD gives sufficiently reliable computational prescriptions only for the description of so–called "hard" processes, which are characterized by large momentum transfers, and it does not permit the calculation of characteristics determined by "soft" processes. Moreover, there are strong indications [1] that perturbative QCD is not valid for the description of the currently existing experimental facts in exclusive processes. This applies, in particular, to the description of the elastic form factors of such well–studied composite systems as the pion, kaon, nucleon and deuteron. Of course, in the framework of field theory itself there exist various approaches to overcoming these difficulties. For example, there are the well–known approaches associated with the use of the Bethe–Salpeter equation (see *e.g.* [2]– [4]) , and quasipotential approaches (see *e.g.* [4] –[8]).

The second method in the relativistic theory of composite systems, in the framework of which we shall operate, is based on the direct realization of the algebra of the Poincaré group on the set of dynamical observables on the Hilbert state space of the system. This approach is called the theory of direct interaction, or relativistic Hamiltonian dynamics (RHD) (for review see [9] and references therein). RHD unifies the potential approach to composite systems and a condition of the Poincaré–invariance. It should be noted that the establishment of the connection between RHD and field theory is a difficult and as yet unresolved problem. The idea of RHD goes back to a paper of Dirac [10], in which he considered the different methods of describing the evolution of classical relativistic systems – differing in the evolution parameter: point form (PF), instant form (IF), and light–front (LF) dynamics.

There now exists a large number of studies of the use of LF dynamics (see, for example, [11]–[16] and the references given there). Some studies also contain investigations using other form of dynamics.

However, most of quantitative investigations of specific systems that have so far been made are associated with LF dynamics. In particular, this is because this form of dynamics has the smallest number (only 3) of generators that contain interaction. There are some other advantages that caused the fact that LF dynamics is widely used. For example it is the possibility of interpreting the results with the help of Feynmann diagrams calculated in the infinite–momentum frame; the antiparticle contributions to Feynman diagrams are suppressed. However, the use of LF dynamics leads to certain difficulties that are associated with a loss of the rotational invariance [17, 18], since the generator of the total angular momentum contains an interaction. Moreover, the space reflection and time reversal operators necessarily depend on interactions [19].

Some time ago it was proved that S matrices are equivalent in the different dynamics forms [20]. This fact is interesting but it does not mean the equivalence of the forms. First, there are problems which can not be reduced to S matrix, e.g. the calculation of form factors. Second, one has to keep in mind that any concrete calculation uses some approximations; the approximations usually used in different forms of dynamics are nonequivalent.

Our point of view is the following. One must not be conservative, one must choose the form adequate to the problem in question and to the approximation to be done. It seems us that this is in the spirit of RHD – the choosing of the adequate degrees of freedom.

Now we present a relativistic treatment of the problem of electroweak meson structure in the framework of IF of RHD [21] – [25]. IF of relativistic dynamics, although not widely used, has some advantages. The calculations can be performed in a natural straightforward way without special coordinates. IF dynamics is particularly convenient to discuss a nonrelativistic limit of relativistic results. This approach is obviously rotational invariant, so IF dynamics is the most suitable for spin problems.

Our approach to an electroweak structure of two–particle composite systems has the following advantages.

- The electromagnetic current matrix element satisfies the current conservation law automatically [24].
- We use the relativistic modified impulse approximation (MIA). It is constructed in a relativistic invariant way. This means that our MIA does not depend on the choose of the coordinate frame, and this contrasts principally with the "frame–dependent" impulse approximation usually used in IF dynamics.
- Our approach provides a correct and natural nonrelativistic limit ("the correspondence principle" is fulfilled).
- For composite systems (including the spin 1 case) the approach guarantees the uniqueness of the solution and it does not use such concepts as "good" and "bad" current components.
- The approach describes correctly the spin Wigner rotation and so gives the correct (QCD) asymptotic.

It is also worth to notice that our approach gives the adequate description of concrete composite systems: $\pi-, K-$ mesons (quarks systems) and deuteron (nucleons system).

Let us consider π meson and K meson as quark (q) – antiquark $(\bar{Q})$ composite system. We shall use different quark masses M_q and $M_{\bar{Q}}$ as in K meson. The results for π meson can be obtained if $M_q = M_{\bar{Q}}$.

We shall act following the basic assumptions, valid for all forms of dynamics in RHD [9]. RHD is based on the including of the operator, describing $q\bar{Q}$ interaction in the generators of the Poincaré group while the commutation relations of the Poincaré algebra are fulfilled. One usually includes $\hat{U}$ in the mass square operator of the free two particle system in the additive way [9]: $P^2 = (p_1 + p_2)^2 \;\rightarrow\; \hat{M}_I^2 = P^2 + \hat{U}$. In the case of IF dynamics the Poincaré algebra is conserved if $\hat{U}$ commutes with the total angular momentum operator $\hat{\vec{J}} = (\hat{J}_1, \hat{J}_2, \hat{J}_3)$, with the operator of total 3-momentum $\hat{\vec{P}}$ and with the operator $\vec{\nabla}_P$. The complete set of commuting operators for the two-particle system with interaction contains now: $\hat{M}_I^2$, $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$. In the case of IF dynamics the operators $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ coincide with the appropriate operators of the two-particle system without interaction and one can construct the basis in which these three operators are diagonals. While working in this basis to obtain the wave function one needs to diagonalize $\hat{M}_I^2$. Eigenvalue problem for operator $\hat{M}_I^2$ has the form of nonrelativistic Schrödinger equation.

In RHD the Hilbert space of composite particle states is the tensor product of single particle Hilbert spaces: $\mathcal{H}_{q\bar{Q}} \equiv \mathcal{H}_q \otimes \mathcal{H}_{\bar{Q}}$ and the state vector in RHD is a superposition of two-particle states. As a basis in $\mathcal{H}_{q\bar{Q}}$ one can choose the following set of vectors:

$$
\begin{aligned}
|\,\vec{p}_1\,, m_1;\,\vec{p}_2\,, m_2 > &= |\,\vec{p}_1\,, m_1 > \otimes |\,\vec{p}_1\,, m_2\; >, \\
< \vec{p}, m\,|\,\vec{p}\,'\,, m' > &= 2p_0\,\delta(\vec{p} - \vec{p}\,')\,\delta_{mm'}\;, \qquad (1)
\end{aligned}
$$

Here $\vec{p}_1$, $\vec{p}_2$ are particle momenta, m_1 , m_2 are spin projections.

Since we consider a two-quark system as one composite system, then a natural basis is one with separated center-of-mass motion:

$$
|\,\vec{P},\; \sqrt{s},\; J,\; l,\; S,\; m_J\; >\,, \qquad (2)
$$

with $P_\mu = (p_1+p_2)_\mu$, $P^2_\mu = s$, $\sqrt{s}$ is the invariant mass of two-particle system , l — the angular momentum in the center-of-mass frame, S is the total spin, J is the total angular momentum, m_J is a projection of the total angular momentum.

The basis (2) is connected with (1) through the Clebsch – Gordan decomposition of the Poincaré group.

Three of four operators in complete set of commuting operators are diagonal in basis (2). The composite system wave function in basis (2) is following

$$< \vec{P}\,',\, \sqrt{s'},\, J',\, l',\, S',\, m'_J|\, p_c >= N_c \delta(\vec{P}\,' - \vec{p}_c)\delta_{JJ'}\delta_{m_J m'_J}\delta_{ll'}\delta_{SS'}\,\varphi^J_{lS}(k)\,. \qquad (3)$$

$k = \sqrt{(s^2 - 2s(M^2_{\bar{s}} + M^2_u) + \eta^2)/4s}\,$, $\quad \eta = M^2_q - M^2_{\bar{Q}}$. N_c, N_{C-G} are factors due to normalization. A concrete form of N_c and N_{C-G} will not be used. For the pseudoscalar mesons ($J = l = S$=0) $\varphi^J_{lS}(k) \to \varphi_c(k)$.

At study in the framework of our approach electromagnetic properties of $\pi-$ and $K-$ mesons the following integral representation is obtained for the electromagnetic form factor [25]:

$$F_c(Q^2) = \int d\sqrt{s}\; d\sqrt{s'}\;\; \varphi_c(k)\, g_0(s, Q^2, s')\, \varphi_c(k'). \qquad (4)$$

For $\varphi(k)$ one can use any phenomenological wave function, normalized using the relativistic density of states: $\varphi(k) = \sqrt{\sqrt{s}(1-\eta^2/s^2)}\,u(k)\,k$, $u(k)$ is nonrelativistic phenomenological wave function. Expression for the function $g_0(s, Q^2, s')$ can be found in the work [23] for example.

We have obtained the expression (4) for the form factors in the framework of the principally relativistic approach: the instant form of RHD. The form factors are expressed in terms of relativistic function $g_0(s,\, Q^2,\, s')$ and nonrelativistic wave functions $u(k)$. The behavior of form factor depends essentially on the model type of wave function. The RHD instant form enables one to obtain easily the nonrelativistic limit of Eq.(4). The relativistic effects are important: the difference between relativistic and nonrelativistic form factors for one and the same wave function is very large.

Our approach describes well the data on electromagnetic form factors of $\pi-, K-$ mesons [25].

Let us discuss in some detail the description of the semileptonic decays of pseudoscalar mesons in our approach. A hadron part of the invariant amplitude of these processes is represented in the following form:

$$\langle \vec{p}_c | J^\mu | \vec{p}\,'_{c'} \rangle = Q^\mu F_-(Q^2) + \tilde{P}^\mu F_+(Q^2)\,. \qquad (5)$$

Here $Q^\mu = (p_c - p'_{c'})^\mu$, $\tilde{P}^\mu = (p_c + p'_{c'})^\mu$, $Q^2 = -t = (p_c - p'_{c'})^2$.

By analogy with electromagnetic case (4) the following integral representation can be obtained for the form factors $F_-(Q^2)$, $F_+(Q^2)$ in the impulse approximation:

$$F_\pm(Q^2) = \int\; d\sqrt{s}\, d\sqrt{s'} \varphi_c(k(s))\, G^{(0)}_\pm(s, Q^2, s')\, \varphi_{c'}(k(s')) \qquad (6)$$

$\varphi_{c'}$, φ_c are meson wave functions of initial and final states. $G^{(0)}_\pm(s, Q^2, s')$ is so called a free two particle form factor which describes the weak decays of a free two–quark system. In the model of pointlike quarks explicit expressions for $G^{(0)}_\pm$ are following:

$$G^{(0)}_+(s,t,s') = \frac{\sqrt{s}(s+s'-t)\Theta(s,t,s')}{4\sqrt{s'}[\lambda(s,t,s')]^{\frac{3}{2}}\sqrt{\lambda(s',M_3^2,M_2^2)\lambda(s,M_3^2,M_1^2)}}\times$$
$$\{[f_3\lambda(s,t,s')(s-s'-t) - 2f_1 s(t(s+s'-t) + \eta(s-s'+t))]\cos\alpha +$$
$$+ f_6 M_2 \xi(s,t,s') s' t \sin\alpha\}$$
$$G^{(0)}_-(s,t,s') = \frac{\sqrt{s}(s+s'-t)\Theta(s,t,s')}{4\sqrt{s'}[\lambda(s,t,s')]^{\frac{3}{2}}\sqrt{\lambda(s',M_3^2,M_2^2)\lambda(s,M_3^2,M_1^2)}}\times \qquad (7)$$
$$\times\{[f_1((s+s'-t)^2(s-s') - 2s\eta(3s+s'-t))-$$
$$-f_3\lambda(s,t,s')(s+s'-t)]\cos\alpha + f_6 M_2\xi(s,t,s')s'(s-s')\sin\alpha\},$$

$$\eta = M_2^2 - M_1^2,$$

$$\Theta(s,t,s') = \theta(s-s_1) - \theta(s-s_2)$$

θ is step function, s_1 s_2 is determined by kinematics:

$$s_{1,2} = s' + t - \frac{1}{2M_2^2}(s'+\eta)(t+\eta) \mp .$$

$$\mp\frac{1}{2M_2^2}\sqrt{\lambda(s',M_1^2,M_2^2)\lambda(t,M_1^2,M_2^2)},$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab+bc+ac)$$

$$\xi(s,t,s') = \sqrt{-M_1\lambda(s,s',t) - ss't - s\eta(s-s'-t) - s\eta^2}.$$

$$\alpha = \omega_1 + \omega_2 + \omega_3,$$

ω_i are parameters of the Wigner rotation.

$$\omega_1 = \frac{\xi(s,t,s')}{M_3(s+s'-t+2\sqrt{ss'}) + \sqrt{ss'}(\sqrt{s}+\sqrt{s'}) - \eta\sqrt{s}},$$

$$\omega_2 = \frac{\xi(s,t,s')}{M_3(s+s'-t+2\sqrt{ss'}) + \sqrt{s}(s-t+\sqrt{ss'}) + \eta\sqrt{s}},$$

$$\omega_3 = \frac{\xi(s,t,s')}{M_1 s' + M_2(s-t) + \sqrt{s'}[(M_1+M_2)^2 - t] + \eta(M_2+M_1)},$$

Quarks form factors for pointlike quarks are following:

$$f(t) = \left[(M_1+M_2)^2 - t\right]^{-1/2}, \quad f_1 = (M_2+M_1)\,f(t)\,,$$

$$f_3 = -(M_2-M_1)\,f(t)\,, \quad f_6 = -\frac{4\,M_2}{M_1^2}\,f(t)\,.$$

M_i are quark masses.

We have calculated the following values of semileptonic decays of K – meson: $K^- \to \pi^0 + \mu^- + \bar{\nu}_\mu$.

$$F_+(Q^2) = F_+(0)\{1 + \frac{\lambda_+}{m_\pi^2}Q^2 + \ldots\}\,, \tag{8}$$

$$F_0(Q^2) = F_0(0)\{1 + \frac{\lambda_0}{m_\pi^2}Q^2 + \ldots\}\,, \tag{9}$$

$$F_0(Q^2) = F_+(Q^2) - \frac{Q^2}{M_K^2 - m_\pi^2}F_-(Q^2)\,.$$

Here M_K and m_π are masses of the kaon and pion, respectively [27]. The calculations were made with wave functions of different models: the harmonic oscillator model [13], the dipole–type wave function [15] for n =1,2, the model with a linear confinement [26]. Our calculations have no free parameters. All parameters were fixed in the case of electromagnetic properties [25]. Results are represented in the table below. The experimental values are following: λ_+=0.033±0.008, λ_0= 0.004±0.007 [27]. Recent experiment [28] gives another result for λ_0: λ_0= 0.062±0.024. The results of our calculations describes well the data on λ_+ in all models. λ_0 is strong different for different experiments (see, e.g. [28]), therefore the average value [27] is questionable. Our results for λ_0 coincides with the experiment data [28].

model	$F_+(0)$	λ_+	λ_0
[13]	0.93	0.028	0.082
[15] n=2	0.94	0.026	0.073
[15] n=3	0.93	0.026	0.074
[26]	0.93	0.031	0.075

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References

[1] N. Isgur, C.H. Llewellyn Smith, Nucl. Phys. B 317 (1989) 526.
[2] O.C. Jakob, L.S. Kisslinger, Phys.Lett.B 243 (1990) 323.
[3] H. Ito, W.W. Buck, F. Gross, Phys.Lett.B 248 (1990) 28.
[4] F. Gross, Relativistic quantum mechanics and field theory, John Wiley & Sons Inc.,1993.
[5] G.E. Brown, A.D. Jackson, The Nucleon–Nucleon Interaction, North–Holland, Amsterdam, 1976.
[6] A.A. Logunov, A.N. Tavkhelidze, Nuovo Cim. 29 (1963) 380.
[7] R. Blankenbecler, R. Sugar, Phys.Rev. 142 (1966) 1051.
[8] J.W. Van Orden, N. Devine, F. Gross, Phys. Rev. Lett. 75 (1995) 4369.
[9] B.D. Keister, W.N. Polyzou, Advances in Nucl. Phys. 20 (1991) 225.
[10] P.A.M. Dirac, Rev.Mod.Phys. 21 (1949) 392.
[11] M.V. Terent'ev, Yad.Fiz.(USSR) 24 (1976) 207.
[12] V.B. Berestetskii, M.V. Terent'ev, Yad.Fiz.(USSR) 24 (1976) 1044; Yad.Fiz.(USSR) 25 (1977) 653.
[13] P.L. Chung, F.Coester, W.N. Polyzou, Phys.Lett. B 205 (1988) 545.
[14] M.M. Giannini, L. Kondratyuk, P. Saracco, Few–Body Systems 17 (1994) 21.
[15] F. Cardarelli, I.L. Grach, I.M. Narodetskii, G. Salmeé, S. Simula, Phys.Lett.B 332 (1994) 1.
[16] S. Simula, Phys.Lett.B 373 (1996) 193.
[17] M. Fuda, Ann.Phys.197 (1990) 265; 1991 Phys.Rev.D 44 (1991) 1880.
[18] B.D. Keister, Phis.Rev.D. 49 (1994) 1500.
[19] F. Lev, E-print hep-ph/9505373, 1995.
[20] S.N. Sokolov, A.N. Shatnii, Teor. Mat. Fiz. (USSR) 37 (1978) 291.
[21] A.F. Krutov, V.E. Troitsky, 1993 J. Phys. G: Nucl. Part. Phys. 19 (1993) L127.
[22] E.V. Balandina, A.F. Krutov, V.E. Troitsky, *Few-Body Problems in Physics, AIP Conference Proceedings* 334, ed. Franz Gross, p.887, 1994.
[23] E.V. Balandina, A.F. Krutov, V.E. Troitsky, 1995 Theor. Mat. Fiz.(Russia) 103 (1995) 41. (Engl. transl. Theor. Math. Phys. 103 (1995) 381.)
[24] E.V. Balandina, A.F. Krutov, V.E. Troitsky, *Proceedings of IXth Workshop on QFTHEP*, p.255. ed. B.B.Levtchenko, Moscow State University, 1995.
[25] E.V. Balandina, A.F. Krutov, V.E. Troitsky, J.Phys.G: Nucl.Phys. 22 (1996) 1585.
[26] H. Tezuka, J. Phys.A: Math.Gen. 24 (1991) 5267.
[27] Particle Data Group, Phys.Rev.D 45 (1992) Part II.
[28] V.M. Artemov et al. Preprint JINR P1–95–330, 1995.