Event Orientation in e^+e^- Annihilation

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Abstract

We review the orientation of $e^+e^- \to q\bar{q}g$ events in terms of the polar and azimuthal angles of the event plane w.r.t. the electron beam direction. The asymmetry of the azimuthal-angle distribution is, like the left-right forward-backward polar-angle asymmetry, sensitive to parity-violating effects in three-jet events; these are presently being explored experimentally. We present these observables at $O(\alpha_s)$ in perturbative QCD and discuss their dependence on longitudinal beam polarisation and c.m. energy. A moments analysis in terms of the orientation angles allows a more detailed test of QCD by isolating the independent helicity cross-sections.

1 Introduction

In e^+e^- annihilation, events containing three distinct jets of hadrons were first observed at the PETRA storage ring in 1979 [1]. Such events were interpreted in terms of the fundamental process $e^+e^- \to q\bar{q}g$, providing direct evidence for the existence of the gluon, the vector boson of QCD [2]. Subsequent studies of the properties of such events [3] have confirmed this interpretation.

Many jet observables have been explored experimentally in e^+e^- annihilations, yielding information on QCD as well as on the electroweak theory. We here consider the orientation of the $q\bar{q}g$ plane or 'event plane' [4] in terms of the angles θ and χ, where θ is the polar angle of the quark direction with respect to the electron beam, and χ is the azimuthal orientation angle of the event plane with respect to the quark-electron plane, such that (Fig. 1):

$$\cos\chi = \frac{\mathbf{q}\times\mathbf{g}}{|\mathbf{q}\times\mathbf{g}|}\cdot\frac{\mathbf{q}\times\mathbf{e}^-}{|\mathbf{q}\times\mathbf{e}^-|}. \tag{1}$$

The polar angle can also be defined in two-jet events of the type $e^+e^- \to q\bar{q}$, in which case the distribution in θ is determined in the electroweak theory [5], and displays a c.m. energy-dependent forward-backward asymmetry which has been observed in many experiments [6, 7]. For e^+e^- annihilation at the Z^0 resonance, the polar-angle asymmetry is large only if one, or both, of the beams are longitudinally polarised, as at SLC/SLD [8]. The azimuthal angle χ is, of course, undefined in $q\bar{q}$ events, but in $q\bar{q}g$ events, it also displays an asymmetry which can be large at the Z^0 resonance in the case of highly polarised electrons. This azimuthal-angle distribution is currently being investigated experimentally [9].

2 The $e^+e^- \to q\bar{q}g$ Differential Cross-Section

Let $\mathbf{q}$, $\bar{\mathbf{q}}$, and $\mathbf{g}$ denote the quark, antiquark and gluon momenta respectively (see Fig. 1) and x, $\bar{x}$ and x_g be the scaled energies (for simplicity, we consider the limit of massless quarks):

$$|\mathbf{q}\,| = x\sqrt{s}, \qquad |\bar{\mathbf{q}}\,| = \bar{x}\sqrt{s}, \qquad |\mathbf{g}\,| = x_g\sqrt{s}, \tag{2}$$

with $x + \bar{x} + x_g = 2$. Allowing for longitudinal beam polarisation, the fully-differential three-jet cross-

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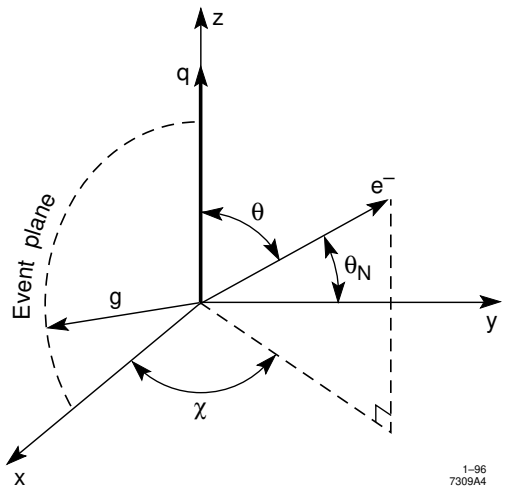

Figure 1: Definition of the angles θ and χ.

section can then at the tree-level, and for massless quarks, be expressed as [10]:

$$\begin{aligned} 2\pi\frac{\mathrm{d}^4\sigma}{\mathrm{d}(\cos\theta)\mathrm{d}\chi\mathrm{d}x\mathrm{d}\bar{x}} &= \left[\frac{3}{8}(1+\cos^2\theta)\,\frac{\mathrm{d}^2\sigma_{\mathrm{U}}}{\mathrm{d}x\mathrm{d}\bar{x}} + \frac{3}{4}\sin^2\theta\,\frac{\mathrm{d}^2\sigma_{\mathrm{L}}}{\mathrm{d}x\mathrm{d}\bar{x}}\right. \\ &\quad \left. +\frac{3}{4}\sin^2\theta\cos 2\chi\,\frac{\mathrm{d}^2\sigma_{\mathrm{T}}}{\mathrm{d}x\mathrm{d}\bar{x}} + \frac{3}{2\sqrt{2}}\sin 2\theta\cos\chi\,\frac{\mathrm{d}^2\sigma_{\mathrm{I}}}{\mathrm{d}x\mathrm{d}\bar{x}}\right] h_f^{(1)}(s) \\ &\quad +\left[\frac{3}{4}\cos\theta\,\frac{\mathrm{d}^2\sigma_{\mathrm{P}}}{\mathrm{d}x\mathrm{d}\bar{x}} - \frac{3}{\sqrt{2}}\sin\theta\cos\chi\,\frac{\mathrm{d}^2\sigma_{\mathrm{A}}}{\mathrm{d}x\mathrm{d}\bar{x}}\right] h_f^{(2)}(s), \end{aligned} \tag{3}$$

where at lowest order in the electroweak theory the dependences on flavour and beam polarisation are very transparent, and given by the functions:

$$h_f^{(1)}(s) = Q_f^2\Xi - 2Q_f\mathrm{Re}\,f(s)(v\Xi - a\xi)v_f + |f(s)|^2[(v^2+a^2)\Xi - 2va\xi](v_f^2+a_f^2), \tag{4}$$

$$h_f^{(2)}(s) = -2Q_f\mathrm{Re}\,f(s)(a\Xi - v\xi)a_f + |f(s)|^2[-(v^2+a^2)\xi + 2va\Xi]2v_f a_f, \tag{5}$$

with

$$f(s) = \frac{1}{4\sin^2 2\theta_W}\,\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z^{\mathrm{tot}}}, \tag{6}$$

where Q_f is the charge of quark flavour f. Furthermore, v, a (v_f, a_f) are the vector and axial vector couplings of the Z^0 to the electron (quark of flavour f), respectively. The longitudinal beam polarisations enter through the coefficients

$$\Xi = 1 - P_-^{\parallel}P_+^{\parallel}, \qquad \xi = P_-^{\parallel} - P_+^{\parallel}, \tag{7}$$

where $P_-^{\parallel}$ and $P_+^{\parallel}$ are the longitudinal polarisations of the electron and positron beams.

Several important points concerning eq. (3) should be noted. The cross-section can be written as a sum of 6 terms, each of which may be factorised into three contributions: the first factor is a simple trigonometric function of the polar and azimuthal orientation angles θ and χ, and the second, $\mathrm{d}^2\sigma_i/\mathrm{d}x\mathrm{d}\bar{x}$ (i=U, L, T, I, P, A), is a function of the parton momentum fractions; these are determined by QCD and kinematics; the third factor, $h_f^{(1,2)}(s)$, is a function containing the dependence on the fermion electroweak couplings. Hence, in each term there is factorisation both between the dynamical contributions of the QCD and electroweak sectors of the Standard Model and between the orientation of the event plane and the relative orientation of the jets within the plane. One may exploit this property by defining moments in terms of $\cos\theta$ and $\cos\chi$ in order to isolate the different terms [4]. The σ_i are often referred to in the literature as *helicity cross-sections*, and the form of eq. (3), with six terms, each containing one of the

independent helicity cross-sections, has been shown [11] to be valid for massless partons up to $O(\alpha_s^2)$ in perturbative QCD.

When quark masses are introduced, each of the first four terms in eq. (3) (proportional to $h_f^{(1)}(s)$) gets an additional contribution proportional to $\tilde{h}_f^{(1)}(s)\,\mathrm{d}^2\tilde{\sigma}_i$, where

$$\tilde{h}_f^{(1)}(s) = |f(s)|^2[(v^2+a^2)\Xi - 2va\xi]a_f^2, \tag{8}$$

and where $\mathrm{d}^2\tilde{\sigma}_i$ is proportional to the square of the quark mass, m_f^2.

At $O(\alpha_s)$ in perturbative QCD [10, 12]:

$$\frac{\mathrm{d}^2\sigma_i}{\mathrm{d}x\mathrm{d}\bar{x}} = \frac{\tilde{\sigma}}{(1-x)(1-\bar{x})}\,F_i, \tag{9}$$

where

$$\tilde{\sigma} = \frac{4\pi\alpha^2\alpha_s(s)}{3\pi s}. \tag{10}$$

For the coordinate system of Fig. 1 the F_{U}, F_{L}, F_{T}, F_{I}, F_{A} and F_{P} are given in [10], entirely in terms of 'internal' angles of the event. Since the quark and antiquark tend to have a small acollinearity angle, F_{L}, F_{T}, F_{I} and F_{A} will typically be small compared with F_{U} and F_{P}.

The results presented here are at $O(\alpha_s)$. Corresponding results at $O(\alpha_s^2)$, for massless quarks, could in principle be derived from the helicity cross-section expressions given in refs. [11, 13]. For massive quarks the helicity cross-sections have recently been calculated at $O(\alpha_s^2)$ in ref. [14]; in this case there are three additional cross-sections, corresponding to three new angular dependences:

$$\sin 2\theta \sin\chi, \qquad \sin^2\theta \sin 2\chi, \qquad \text{and} \qquad \sin\theta\sin\chi. \tag{11}$$

These terms are generated by absorptive parts in the scattering amplitude; they vanish in the massless limit even at the one-loop order. Thus, they are quite small. The last term, $\sin\theta\sin\chi$ can also be written as $\pm\cos\omega$, where ω is the angle between the normal to the event plane and the beam direction.

3 Polar- and Azimuthal-Angle Distributions

We now discuss the singly-differential cross-sections in terms of $\cos\theta$ or χ. Consider integrating eq. (3) first over x and $\bar{x}$, with the integration domain given by some standard jet resolution criterion y_c [15] and using the notation:

$$\hat{\sigma}_i \equiv \int_{y_c} \mathrm{d}x \int \mathrm{d}\bar{x} \frac{\mathrm{d}^2\sigma_i}{\mathrm{d}x\mathrm{d}\bar{x}}. \tag{12}$$

Integrating over χ we then obtain:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta)} = \left(\frac{3}{8}(1+\cos^2\theta)\,\hat{\sigma}_{\mathrm{U}} + \frac{3}{4}\sin^2\theta\,\hat{\sigma}_{\mathrm{L}}\right) h_f^{(1)}(s) + \frac{3}{4}\cos\theta\,\hat{\sigma}_{\mathrm{P}}\,h_f^{(2)}(s), \tag{13}$$

where the term containing $\hat{\sigma}_{\mathrm{P}}$ represents the well-known quark forward-backward asymmetry resulting from parity violation in the weak interaction, but for the three-jet case. Similarly, by integrating over $\cos\theta$ we obtain:

$$2\pi\,\frac{\mathrm{d}\sigma}{\mathrm{d}\chi} = \left(\hat{\sigma}_{\mathrm{U}} + \hat{\sigma}_{\mathrm{L}} + \cos 2\chi\,\hat{\sigma}_{\mathrm{T}}\right) h_f^{(1)}(s) - \frac{3\pi}{2\sqrt{2}}\cos\chi\,\hat{\sigma}_{\mathrm{A}}\,h_f^{(2)}(s), \tag{14}$$

where the term containing $\hat{\sigma}_{\mathrm{A}}$ represents an azimuthal, parity-odd asymmetry analogous to the last term in eq. (13) but owing its existence to the radiation of the gluon.

For the case of longitudinally-polarised electrons and unpolarised positrons the dependences of these singly-differential distributions on the beam polarisation and c.m. energy are illustrated in Figs. 2(a,b) and 3(a,b) respectively. We present the σ_i at $O(\alpha_s)$ as in Ref. [10]. Fig. 2a shows the distribution in $\cos\theta$ at $\sqrt{s} = M_Z$ for down-type quarks, $y_c = 0.02$ and electron longitudinal polarisation $p = +1$, 0 and -1. (We refer to positive (negative) polarisation as right- (left-) handed respectively.) The recent

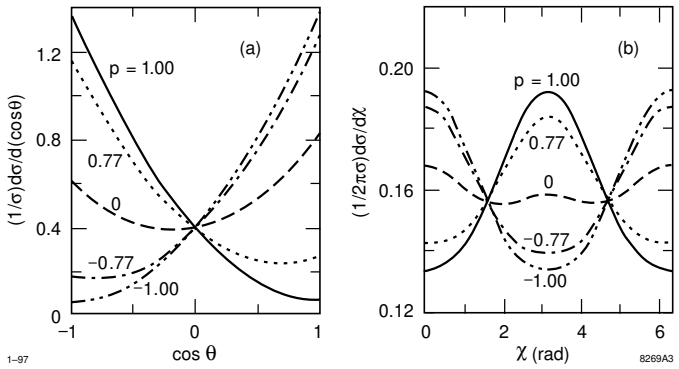

Figure 2: Angular orientation of the event plane for down-type quarks and $y_c = 0.02$. Distribution of (a) $\cos\theta$ and (b) χ at $\sqrt{s} = M_Z$, for 5 values of p.

SLC/SLD case of $p = \pm 0.77$ [16] is also indicated. The quark polar-angle forward-backward asymmetry is large for high beam polarisation, and its sign changes with the sign of the polarisation. The less familiar azimuthal-angle distribution is shown in Fig. 2b for the same cases as in Fig. 2a; the distribution is symmetric about $\chi = \pi$. The phase change of the χ distribution when the beam polarisation sign is changed is a reflection of the sign reversal of the forward-backward asymmetry in $\cos\theta$. Qualitatively similar results are obtained for up-type quarks, and for other values of y_c.

We illustrate the energy dependence of the $\cos\theta$- and χ-distributions in Figs. 3a and 3b, respectively, for down-type quarks at fixed electron polarisation $p = +1$, with results at $\sqrt{s} = 35$, 60, 91 and 200 GeV, corresponding to e^+e^- annihilation at the PETRA, TRISTAN, SLC/LEP and LEP2 collider energies. The variation with energy is due to the varying relative contribution of γ and Z^0 exchange in the e^+e^- annihilation process. Results are also shown for a possible high-energy collider operating with polarised electrons at $\sqrt{s} = 500$ GeV and 2 TeV. If such a facility could be operated at lower energies, where, apart from $\sqrt{s} = 91$ GeV (SLC), polarised beams were not previously available, measurements in the same experiment of the distributions shown in Figs. 3(a,b) would provide a significant consistency check of the Standard Model.

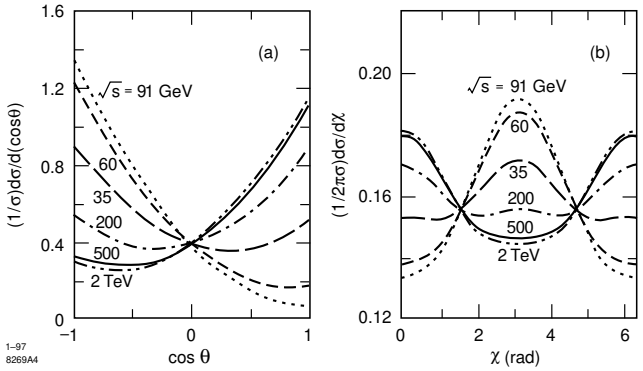

Figure 3: Angular orientation of the event plane for down-type quarks and $y_c = 0.02$. Distribution of (a) $\cos\theta$ and (b) χ for $p = +1$ at 6 values of $\sqrt{s}$.

4 Polar- and Azimuthal-Angle Asymmetries

By analogy with the *left-right forward-backward asymmetry* of the polar-angle distribution:

$$\tilde{A}_{\rm FB}(|p|)|_{\cos\theta} \equiv \frac{\int_0^1 \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta - \int_{-1}^0 \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta - \left(\int_0^1 \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta - \int_{-1}^0 \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta\right)}{\int_0^1 \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta + \int_{-1}^0 \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta + \int_0^1 \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta + \int_{-1}^0 \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\cos\theta}\,{\rm d}\cos\theta}, \qquad (15)$$

it is natural to define a corresponding asymmetry of the azimuthal-angle distribution [4]:

$$\tilde{A}(|p|)|_\chi \;\equiv\; \frac{\int_{\frac{\pi}{2}}^{\pi} \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\chi}\,{\rm d}\chi - \int_0^{\frac{\pi}{2}} \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\chi}\,{\rm d}\chi - \left(\int_{\frac{\pi}{2}}^{\pi} \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\chi}\,{\rm d}\chi - \int_0^{\frac{\pi}{2}} \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\chi}\,{\rm d}\chi\right)}{\int_{\frac{\pi}{2}}^{\pi} \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\chi}\,{\rm d}\chi + \int_0^{\frac{\pi}{2}} \frac{{\rm d}\sigma^{\rm L}}{{\rm d}\chi}\,{\rm d}\chi + \int_{\frac{\pi}{2}}^{\pi} \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\chi}\,{\rm d}\chi + \int_0^{\frac{\pi}{2}} \frac{{\rm d}\sigma^{\rm R}}{{\rm d}\chi}\,{\rm d}\chi}, \qquad (16)$$

where $\sigma^{\rm L,R} = \sigma(\mp|p|)$ is the $e^+e^- \to q\bar{q}g$ cross-section for a left- (L) or right- (R) handed electron beam of polarisation magnitude $|p|$.

For the case of e^+e^- annihilation at the Z^0 resonance using electrons of longitudinal polarisation p and unpolarised positrons, as at SLC, eqs. (4) and (5) reduce to the simple forms

$$\begin{aligned} h_f^{(1)}(M_Z^2) &= |f(M_Z^2)|^2[(v^2+a^2) - 2vap](v_f^2+a_f^2), \\ h_f^{(2)}(M_Z^2) &= |f(M_Z^2)|^2[-(v^2+a^2)p + 2va]2v_f a_f, \end{aligned} \qquad (17)$$

and hence

$$\tilde{A}_{\rm FB}(|p|)|_{\cos\theta} = \frac{3}{4}\,|p|\,\frac{\hat{\sigma}_{\rm P}}{\hat{\sigma}_{\rm U}+\hat{\sigma}_{\rm L}}\,A_f, \qquad \tilde{A}(|p|)|_\chi = \frac{3}{\sqrt{2}}\,|p|\,\frac{\hat{\sigma}_{\rm A}}{\hat{\sigma}_{\rm U}+\hat{\sigma}_{\rm L}}\,A_f, \qquad (18)$$

where we use the common notation $A_f \equiv 2v_f a_f/(v_f^2+a_f^2)$. Whereas both asymmetries are directly proportional to the beam polarisation $|p|$ and the electroweak coupling A_f, the $\cos\theta$ asymmetry is proportional to the helicity cross-section $\hat{\sigma}_{\rm P}$, and the χ asymmetry to the helicity cross-section $\hat{\sigma}_{\rm A}$. Since the electroweak factor A_f is predicted to a high degree of accuracy by the Standard Model, and, in the case of b and c quarks, has also been measured using predominantly $q\bar{q}$ final states at SLC and LEP [7], measurement of these asymmetries (18) in $q\bar{q}g$ events at SLC/SLD would allow one to probe $\hat{\sigma}_{\rm P}$ and $\hat{\sigma}_{\rm A}$. Preliminary results in this direction were reported recently [9]. Furthermore, the ratio of the asymmetries is independent of both polarisation and electroweak couplings and depends only on the ratio of $\hat{\sigma}_{\rm P}$ and $\hat{\sigma}_{\rm A}$:

$$\frac{\tilde{A}_{\rm FB}(|p|)|_{\cos\theta}}{\tilde{A}(|p|)|_\chi} = \frac{\sqrt{2}}{4}\,\frac{\hat{\sigma}_{\rm P}}{\hat{\sigma}_{\rm A}}. \qquad (19)$$

As a consequence of there being, up to $O(\alpha_s^2)$ in massless perturbative QCD, only the six independent helicity cross-sections given in eq. (3), the relations (18) and (19) are valid up to the same order.

We show in Fig. 4a, at $O(\alpha_s)$ the ratios $\hat{\sigma}_{\rm P}/(\hat{\sigma}_{\rm U}+\hat{\sigma}_{\rm L})$ and $\hat{\sigma}_{\rm A}/(\hat{\sigma}_{\rm U}+\hat{\sigma}_{\rm L})$ and their dependence on y_c; the dependence is weak. For completeness we also show $\hat{\sigma}_{\rm T}/(\hat{\sigma}_{\rm U}+\hat{\sigma}_{\rm L})$ and $\hat{\sigma}_{\rm I}/(\hat{\sigma}_{\rm U}+\hat{\sigma}_{\rm L})$. It would be worthwhile to investigate the size of higher-order perturbative QCD contributions by evaluating these ratios at $O(\alpha_s^2)$; this should be possible using the matrix elements described in Ref. [17] or the results of [14].

It would be interesting to confront the theoretical predictions with experimental measurements, taking into account mass effects and higher-order QCD effects [14]. Significant deviations of the data from the predictions for the asymmetries, eqs. (18), would indicate anomalous parity-violating contributions to the process $e^+e^- \to q\bar{q}g$. The ratio of asymmetries, eq. (19), is at lowest order independent of the electroweak coupling factor A_f, and would help to unravel the dynamical origin of any such effect.

5 Inclusive Cross-Sections

All of the preceding discussions have been based on the assumption that the parton-type originator of jets is known, *i.e.* that in 3-jet events one can identify which jet originated from the quark, antiquark

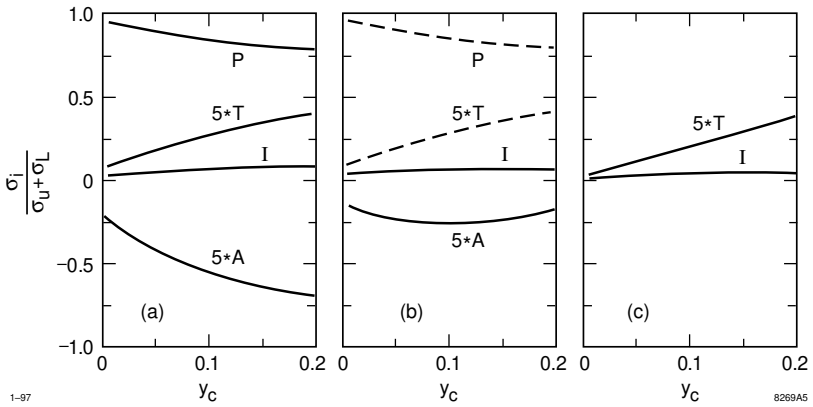

Figure 4: Helicity cross-section ratios (see text) as functions of y_c; (a) exclusive, (b) semi-inclusive and (c) fully-inclusive cases. For the sake of clarity, the ratios are multiplied by a factor of 5 where indicated.

and gluon. The definition of $\cos\theta$ requires that the quark jet be known, whereas the definition of χ requires that both the quark jet and a second jet origin be identified. It is difficult from an experimental point-of-view to make such exclusive identification for jets of hadrons measured in a detector. Quark and antiquark jets have been identified in predominantly 2-jet events in e^+e^- annihilation (see *e.g.* [8]). Currently, one has reached a single-hemisphere efficiency for b or $\bar{\text{b}}$ quarks of the order of 70%, and a double-tagging efficiency of about 50%. Identification of both quark and antiquark jets in $e^+e^- \to q\bar{q}g$ events is *a priori* more difficult due to the greater hadronic activity.

It is therefore useful to consider more inclusive quantities. Two possibilities are: (1) *Semi-inclusive*: the quark jet is assumed to be identified, and the *least energetic* jet in the event is taken to be the gluon and is used to define the angle χ (eq. (1)). While this assumption will be wrong part of the time, it will be wrong by a calculable probability, and hence this can in principle be fully corrected for. (2) *Fully-inclusive*: the jets are labelled only in terms of their energies, $x_3 \leq x_2 \leq x_1$; the polar angle θ is then defined by the angle of the fastest jet w.r.t. the electron beam direction and the azimuthal angle χ can be defined analogously to eq. (1) as:

$$\cos\chi = \frac{\mathbf{1}\times\mathbf{3}}{|\mathbf{1}\times\mathbf{3}|}\cdot\frac{\mathbf{1}\times\mathbf{e}^-}{|\mathbf{1}\times\mathbf{e}^-|} \tag{20}$$

For the semi-inclusive case we show at $O(\alpha_s)$ the ratios $\hat{\sigma}_T/(\hat{\sigma}_U+\hat{\sigma}_L)$, $\hat{\sigma}_I/(\hat{\sigma}_U+\hat{\sigma}_L)$, $\hat{\sigma}_P/(\hat{\sigma}_U+\hat{\sigma}_L)$ and $\hat{\sigma}_A/(\hat{\sigma}_U+\hat{\sigma}_L)$ in Fig. 4b. Whereas $\hat{\sigma}_P$ and $\hat{\sigma}_T$ are unchanged relative to the exclusive case, $\hat{\sigma}_I$ and $\hat{\sigma}_A$, which multiply terms proportional to $\cos\chi$ in eq. (3), are smaller in magnitude because of the sometimes incorrect gluon-jet identification. Though this implies that the parity-violating asymmetry $\tilde{A}(|p|)|_\chi$ in eq. (18) is smaller by a (y_c-dependent) factor of order 2, it will in fact be easier to access experimentally because the semi-inclusive case requires only one of the quark- and antiquark-jets to be identified explicitly.

In the fully-inclusive case the terms σ_A and σ_P, which are odd under interchange of quark and antiquark jets, cancel out; writing the cross-section in terms of thrust [18] one obtains at $O(\alpha_s)$:

$$2\pi\frac{d^3\sigma}{d(\cos\theta)d\chi dT} = \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma_U}{dT}+\frac{3}{4}\sin^2\theta\frac{d\sigma_L}{dT} \tag{21}$$

$$+\frac{3}{4}\sin^2\theta\cos 2\chi\frac{d\sigma_T}{dT}+\frac{3}{2\sqrt{2}}\sin 2\theta\cos\chi\frac{d\sigma_I}{dT}, \tag{22}$$

where expressions for $d\sigma_i/dT$ can be found in ref. [10]. Using the notation

$$\tilde{\sigma}_i \equiv \int_{y_c}\frac{d\sigma_i}{dT}\,dT, \qquad i = \text{U, L, T, I,} \tag{23}$$

we show at $O(\alpha_s)$ the ratios $\tilde{\sigma}_T/(\tilde{\sigma}_U+\tilde{\sigma}_L)$ and $\tilde{\sigma}_I/(\tilde{\sigma}_U+\tilde{\sigma}_L)$ in Fig. 4c. Their magnitudes and dependences on y_c differ relative to the exclusive and semi-inclusive cases due to the redefinition of θ and χ. Distributions of $\cos\theta$ and χ in this case have already been measured and found to be in agreement with $O(\alpha_s)$ QCD calculations [19, 20].

Another fully-inclusive observable is the polar-angle ω of the normal to the event plane with respect to the beam direction. The differential cross-section $d\sigma/d(\cos\omega)$ has been calculated at $O(\alpha_s^2)$ in massless perturbative QCD [21], and has been measured at $\sqrt{s} \simeq 35$ GeV [19] and $\sqrt{s} = 91$ GeV [20]. The effects of final-state interactions can induce a term linear in $\cos\omega$ whose sign and magnitude depend on the electron beam polarisation [22]; experimental limits on such a term have been set using hadronic Z^0 decays [23].

6 Conclusions

We have presented the orientation of $e^+e^- \to q\bar{q}g$ events in terms of the polar- (θ) and azimuthal- (χ) angle distributions. These distributions have been given at $O(\alpha_s)$ in perturbative QCD for massless quarks and their dependence on longitudinal electron-beam polarisation and centre-of-mass energy has been illustrated. The more complicated $O(\alpha_s^2)$ results are available for massless quarks [11, 13] and coming soon also for massive quarks [14]. We have considered the left-right forward-backward asymmetry of the $\cos\theta$ distribution and have presented a corresponding asymmetry of the χ distribution. Parity-violating 3-jet observables of this kind represent a new search-ground for anomalous contributions and are presently being explored experimentally [9].

For the case of e^+e^- annihilation at the Z^0 resonance using longitudinally-polarised electrons, the $\cos\theta$ asymmetry is proportional to the QCD helicity cross-section $\hat{\sigma}_P$, and the χ asymmetry to the helicity cross-section $\hat{\sigma}_A$; these are now being measured using the highly-polarised electron beam at SLC/SLD. To lowest electroweak order the ratio of these asymmetries is independent of electroweak couplings and the beam polarisation. These results are valid up to $O(\alpha_s^2)$ in QCD perturbation theory for massless quarks. At $O(\alpha_s)$ the dependence of $\hat{\sigma}_P$ and $\hat{\sigma}_A$ on the jet resolution parameter y_c is found to be weak. Higher-order perturbative QCD contributions, as well as quark mass effects, should be included before making a detailed comparison of these predictions with data. Even the extraction of $\hat{\sigma}_U$, $\hat{\sigma}_L$, $\hat{\sigma}_T$ and $\hat{\sigma}_I$, which does not require quark and antiquark jet identification, represents a detailed test of QCD, beyond what has so far been studied.

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References

[1] TASSO Collab., R. Brandelik *et al.*, Phys. Lett. **86B** (1979) 243;
Mark J Collab., D.P. Barber *et al.*, Phys. Rev. Lett. **43** (1979) 830;
PLUTO Collab., Ch. Berger *et al.*, Phys. Lett. **86B** (1979) 418;
JADE Collab., W. Bartel *et al.*, Phys. Lett. **91B** (1980) 142.

[2] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. **47B** (1973) 365;
D.J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343;
H.D. Politzer, Phys. Rev. Lett. **30** (1973) 1346;
S. Weinberg, Phys. Rev. Lett. **31** (1973) 494.

[3] For a review see S.L. Wu, Phys. Rep. **107** (1984) 59.

[4] P.N. Burrows and P. Osland, Phys. Lett. **B384** (1997) 249.

[5] See, *e.g.*, G. Altarelli *et al.*, in Physics at LEP, CERN 86-02 (1986), eds. J. Ellis and R. Peccei, p. 1.

[6] See, *e.g.*, T. Kamae, in Proc. XXIV International Conference on High Energy Physics, Munich, August 4–10, 1988 (eds. R. Kotthaus and J.H. Kühn, Springer Verlag, 1989) p. 156.

[7] A. Blondel, in Proc. XXVIII International Conference on High Energy Physics, Warsaw, Poland, July 25-31 1996, World Scientific (1997).

[8] SLD Collab., K. Abe *et al.*, Phys. Rev. Lett. **74** (1995) 2890; *ibid* **74** (1995) 2895; *ibid* **75** (1995) 3609.

[9] SLD Collab, K. Abe *et al.*, SLAC-PUB-7570, Jun 1997. Contributed to 18th International Symposium on Lepton - Photon Interactions (LP 97), Hamburg, Germany, 28 Jul - 1 Aug 1997 and to International Europhysics Conference on High-Energy Physics (HEP 97), Jerusalem, Israel, 19-26 Aug 1997.

[10] H.A. Olsen, P. Osland, I. Øverbø, Nucl. Phys. **B171** (1980) 209.

[11] J.G. Körner, G.A. Schuler, Z. Phys. **C26** (1985) 559.

[12] E. Laermann, K.H. Streng, P.M. Zerwas, Z. Phys. **C3** (1980) 289; Erratum: Z. Phys. **C52** (1991) 352.

[13] G.A. Schuler and J.G. Körner, Nucl. Phys. **B325** (1989) 557.

[14] W. Bernreuther, A. Brandenburg and P. Uwer, Phys. Rev. Lett. **79** (1997) 189; A. Brandenburg, P. Uwer, PITHA-97-29, hep-ph/9708350; G. Rodrigo, A. Santamaria, M. Bilenkii, Phys. Rev. Lett. **79** (1997) 193.

[15] See, *e.g.* S. Bethke *et al.*, Nucl. Phys. **B370** (1992) 310.

[16] SLD Collab., K. Abe *et al.*, Phys. Rev. Lett. **78** (1997) 2075.

[17] S. Catani, M. Seymour, Nucl. Phys. **B485** (1997) 291.

[18] E. Farhi, Phys. Rev. Lett. **39** (1977) 1587.

[19] TASSO Collab., W. Braunschweig *et al.*, Z. Phys. **C47** (1990) 181.

[20] L3 Collab., B. Adeva *et al*, Phys. Lett. **B263** (1991) 551;
DELPHI Collab., P. Abreu *et al*, Phys. Lett. **B274** (1992) 498;
SLD Collab., K. Abe *et al.*, Phys. Rev. **D55** (1997) 2533.

[21] J.G. Körner, G.A. Schuler, F. Barreiro, Phys. Lett. **B188** (1987) 272.

[22] A. Brandenburg, L. Dixon, Y. Shadmi, Phys. Rev. **D53** (1996) 1264.

[23] SLD Collab., K. Abe *et al.*, Phys. Rev. Lett. **75** (1995) 4173.