# Quantum effects in interaction of electromagnetic field and liquid-crystal films

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**Abstract**

As is known [1, 2], the biological membranes are cooperative systems. The properties of these systems are defined by peculiarities of their construction and peculiarities of their dynamical behaviour. The cooperative behaviour arises in processes with a local excitation transfer caused by physical actions on separate components of the membranes. Natural and artificial membranes can be in two basic states: in the form of a 2-D crystall (gel) or as a liquid cristall. In the paper a spectrum of fluctuations of a unordered one-dimension chain is analysed. That allows one to consider a number of special features of the electromagnetic field interaction with the membranes.

The equation of fluctuations of a partially unordered chain has the form

$$M_i \omega^2 U_i = K_i(U_{i+1} - U_i) - K_{i-1}(U_i - U_{i-1}). \tag{1}$$

We shall define the phase $\phi_i$ as follows

$$ctg\phi_i = \frac{U_i}{U_{i-1}}. \tag{2}$$

Then from (1) and (2) one can obtain the recurrent relation:

$$ctg\phi_{i+1} = 1 + \frac{K_{i-1}}{K_i} - \frac{M_i}{K_i}\omega^2 - \frac{K_{i-1}}{K_i} tg\phi_i. \tag{3}$$

After differentiation of $\phi_i$ in (3) with respect to $\omega^2$, we obtain

$$\frac{d\phi_{i+1}}{d(\omega^2)} = \frac{M_i}{K_i} sin^2\phi_{i+1} + \frac{K_{i-1}}{K_i} \frac{sin^2\phi_{i+1}}{cos^2\phi_i} \frac{d\phi_i}{d(\omega^2)}.$$

For all $i > 1$ one have $\frac{d\phi_i}{d(\omega^2)} > 0$, and therefore a number of states can be calculated in accordance with the formula [3]:

$$N(\omega_1^2, \omega_2^2) = \lim_{N\to\infty} [\frac{1}{\pi N a}(\phi_{N+1}(\omega_2^2) - \phi_{N+1}(\omega_1^2)],$$

where $g$ is a chain constant . First we shall consider an isotopical partially nonodered chain, when all force constants are equal: $K_i = K$. Then one can write (1) in the matrix form :

$$\begin{pmatrix} U_i \\ U_{i+1} \end{pmatrix} = T_i \begin{pmatrix} U_{i-1} \\ U_i \end{pmatrix}, \tag{4}$$

where

$$T_i = \begin{pmatrix} 0 & 1 \\ -1 & 2 - M_i \frac{\omega^2}{K} \end{pmatrix}.$$

Now let redefine the phase in order that it linearly fluctuates on districts occupied by the most light molecules. The matrix $T^0$ which describes the transition through the knot occupied by the lightest molecules has the form

$$T^0 = \begin{pmatrix} 0 & 1 \\ -1 & 2 - M^0 \frac{\omega^2}{K} \end{pmatrix}.$$

Because of $detT^0 = 1$ the frequencies satisfying the condition $Tr(T^0) \leq 2$ correspond to the wave propagation in the ideal chain with $M_i = M^0$. In this case the eigenvalues of the matrix $T^0$ are equal $e^{\pm iK}$,

$$2\cos K = -Tr(T^0),$$

with $0 < K < \pi$ and the corresponding eigenvectors $\vec{t_1}$ and $\vec{t_2} = \vec{t_1^*}$ may be normalizated in order that

$$t_1^{(1)} = -i,$$

$$t_1^{(2)} = R + iJ,$$

where

$$R = \frac{sinK}{T_{12}^0},$$

$$J = \frac{T_{11}^0 + cosK}{T_{12}^0}.$$

With the use of the matrix

$$S = \begin{pmatrix} -i & i \\ R+iJ & R-iJ \end{pmatrix}$$

the transformations of similarity lead $T^0$ to a diagonal form

$$S^{-1}T^0S = \begin{pmatrix} -e^{-iK} & 0 \\ 0 & e^{iK} \end{pmatrix}.$$

We shall introduce a new phase $\chi$ by means of the relation

$$S^{-1}\begin{pmatrix} sin\phi \\ cos\phi \end{pmatrix} = A\begin{pmatrix} e^{-i\chi} \\ e^{i\chi} \end{pmatrix},$$

where $\phi$ is defined in (2):

$$\begin{pmatrix} U_{i-1} \\ U_i \end{pmatrix} = I_i \begin{pmatrix} sin\phi_i \\ cos\phi_i \end{pmatrix},$$

$$I_i > 0$$

and $A$ is the c-number. Then we have

$$-ctg\phi = Rctg\chi + J. \tag{5}$$

It follows from (4) and (5) that phases $\chi$ up to and after transition through the knot occupied by the lightest molecules ($T_i = T^0$) satisfy the equality:

$$ctg\chi_{i+1} = ctg(\chi_i + K), \tag{6}$$

that explains the aim of the introduction of the new phase. By differentiation of (5) one obtain

$$\left.\frac{d\chi_i}{d(\omega^2)}\right|_{[\frac{\chi_i}{\pi}]=0} = -sinK\left.\frac{d\phi_i}{d(\omega^2)}\right|_{[\frac{\phi_i}{\pi}]=0} < 0.$$

Therefore in case of fastening the end of the chain ( that corresponds to the boundary condition $[\frac{\phi_{N+1}}{\pi}] = 0$) the number of states in interval $(\omega_1^2, \omega_2^2)$, per unit of the length can be calculated with the use of the phase $\chi$:

$$N(\omega_1^2, \omega_2^2) = \lim_{N\to\infty}(Na)^{-1}[\frac{\chi_{N+1}(\omega_1^2) - \chi_{N+1}(\omega_2^2)}{\pi}]$$

The matrix $T_i$ of transition through an arbitraty knot has the form:

$$T_i = e^{(-\tilde{a}_i Q_{12})}T_0 \tag{7}$$

where

$$(Q_{12})_{ij} = \delta_{i2}\delta_{j1}\tilde{a}_i = \frac{M_i - M^0}{K}\omega^2 > 0.$$

It follows from (4) and (7) that in a general case the recurrent relation (5) will admit the following form

$$ctg\chi_{i+1} = ctg(\chi_i + K) + \frac{\tilde{a}_i}{R}.$$

With use of the condition $K(\omega_m^2) = 0$ and

$$\chi_{N+1}(\omega_m^2) = \chi_1(\omega_m^2) = O(N),$$

(which is resulted from (6)) one obtain the following formula for the number of states $N(\omega^2, \omega_m^2)$ in the surroundings of the fluctuative boundary

$$N(\omega^2, \omega_m^2) = \lim_{N\to\infty} (Na)^{-1}[\frac{1}{\pi}\chi_{N+1}(\omega^2)]. \tag{8}$$

We shall notice, that after the replacement $v_i = K_i(U_{i+1} - U_i)$, the equation analogous (1) may be received

$$-\frac{M_i\omega^2}{K}U_i = U_{i+1} - 2U_i + U_{i-1}. \tag{9}$$

A complete analogy of (8) with the well known solution of the Shrödinger equation [3, 4] for the random type potential

$$U(x) = \sum_i K_i\delta(x - i),$$

permits one to apply results obtained with account of quantum effects to the problem considered above with the use of the parameter

$$K = arccos(\frac{M^0\omega^2}{2K} - 1),$$

in (1) or

$$K = arccos(\frac{M\omega^2}{2\,min\{K_i\}} - 1),$$

in (9) as a quasi-momentum. In this case the true fluctuation boundary of the fluctuation spectrum corresponds to the maximum significance of the frequency $\omega_{max} = 2(\frac{K_{max}}{M_{min}})^{\frac{1}{2}}$.

# References


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