# A polarized laser beam as “an anisotropic crystal” for high-energy photons and electrons

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**Abstract**

The elastic light-light scattering below the threshold of the $e^+e^-$ pair production leads to a variation in polarization of hard $\gamma$-quanta traversing (without loss in intensity) a region where the laser light is focused. We show that this effect allows for a full control of the $\gamma$-quantum polarization using the existing lasers. It can be observed in the experiment of the type of E-144 at SLAC and it should be taken into account in experiments at future $\gamma\gamma$ colliders.

We apply the same method to calculate the variation in the polarization of electrons traversing a laser beam. Equations are obtained which determine the variation of the electron density matrix. The change in the electron polarization depends not only on the Compton cross sections but on the real part of the forward Compton amplitudes as well. It should be taken into account, for example, at simulation of the $e \to \gamma$ conversion performed presently for $\gamma\gamma$ colliders.

## 1 Introduction

This report is based on the papers [1, 2]. Let us speak, for definiteness, about high-energy photons traversing a laser beam. The case of high-energy electrons traversing the laser beam will be discussed in section 4.

It is well known that a region with an electromagnetic field can be regarded as anisotropic medium (see, for example, [3] §129-130 and [4]). A possibility to consider a bunch of laser photons as a “crystal” is pointed out in ref. [5], but the definite estimates and calculations of this possibility are not given.

In the present paper we study in detail properties of such a “crystal” considering head-on collision of hard $\gamma$-quanta with the bunch of polarized laser photons. For the energy of $\gamma$-quanta below the threshold of the $e^+e^-$ pair production

$$\hbar\omega < \hbar\omega_{\rm th} = \frac{m_e^2c^4}{\hbar\omega_L} = \frac{260\ \text{GeV}}{(\hbar\omega_L/\text{eV})} \tag{1}$$

(here $\hbar\omega_L$ is the laser photon energy and $m_e$ is the electron mass) the main interaction is the elastic light-light scattering $\gamma\gamma_L \to \gamma\gamma_L$. Cross section of this process $\lesssim \alpha^2 r_e^2$ is approximately by 5 order of magnitude less then a typical cross section for the pair production $\sim \pi r_e^2$, where $\alpha = e^2/(\hbar c) = 1/137$ and $r_e = e^2/(m_e c^2)$ is the classical electron radius. Therefore, the laser bunch is practically transparent for such $\gamma$-quanta. On the other hand, the variation in polarization for the $\gamma$-quantum traversing the bunch is determined by the interference of the incoming wave and the wave scattered at zero angle. In other words, the variation **is not given by the cross section** (which is proportional to square of the light-light scattering amplitude of the order of $\alpha^4$), but by **the scattering amplitude itself** $\sim \alpha^2$. As a result, in this case the essential variation in the $\gamma$-quantum polarization can occur practically without loss in intensity of $\gamma$-quanta. This effect can be interesting for the following reasons:

(i) It can be used to control the polarization of hard $\gamma$-quanta without loss in their intensity. In particular, it is possible to transform the circular polarization into the linear one or the linear polarization into the circular one and to rotate the direction of the linear polarization.

(ii) The experimental observation of variation in the $\gamma$-quantum polarization will be an indirect observation of the process of elastic light-light scattering. The conditions close to those necessary for observation of this effect are realized now at SLAC E-144 experiment [6].

(iii) The discussed problem is also actual in connection with projects of $\gamma\gamma$ colliders[7] which under development now (see refs. [8, 9, 10]). In these projects it is suggested to obtain the required high-energy $\gamma$-quanta by backward Compton scattering of laser light on the electron beam of a linear collider. The planned almost the whole conversion $e \to \gamma$ will take place under condition that an electron travels in a laser “target” an optical thickness $t$ of the order of one:

$$t \sim \pi\, r_e^2\, n_L\, l_L \ \sim\ 1,$$

where $n_L$ is the concentration of the laser photons in the bunch and $l_L$ is its length. The $\gamma$-quantum obtained inside the bunch will travel further through the same bunch varying its polarization. We will show that this variation is determined approximately by the same parameter $t$ and it may be quite essential. Therefore, such a change in polarization should be, generally speaking, taken into account at simulations of the $e \to \gamma$ conversion process performed just now for such colliders.

In the paper we take into account the lowest order in intensity of the laser electromagnetic field. With the growth of the intensity of laser flash it is necessary to take into account the effects of high orders in this intensity (see ref. [11] and literature therein) which we neglect in the present paper.

## 2 Equations for Stokes parameters of the $\gamma$-quantum

Let us consider the head-on collision of $\gamma$-quanta with the bunch of laser photons. We choose the $z$ axis along the momenta of $\gamma$-quanta. The polarization state of $\gamma$-quantum is described by Stokes parameters $\xi_{1,2,3}$, among them $\xi_2$ is the degree of the circular polarization (which is equal to the mean $\gamma$-quantum helicity) and $\sqrt{\xi_1^2+\xi_3^2}$ is the degree of the linear polarization. In the helicity basis ($\lambda,\ \lambda' = \pm 1$) the density matrix of $\gamma$-quantum has the form (see, for example, ref. [3] §8):

$$\rho^{\gamma}_{\lambda\lambda'} = \frac{1}{2}\begin{pmatrix} 1+\xi_2 & -\xi_3+i\xi_1 \\ -\xi_3-i\xi_1 & 1-\xi_2 \end{pmatrix}. \tag{2}$$

For the laser photon the corresponding matrix is described by the following parameters: the degree of the circular polarization $P_c$, the degree of the linear polarization $P_l$ and the direction of the linear polarization. Let us choose the $x$ axis along this direction [1], then

$$\rho^{L}_{\lambda\lambda'} = \frac{1}{2}\begin{pmatrix} 1+P_c & -P_l \\ -P_l & 1-P_c \end{pmatrix}. \tag{3}$$

We also use a compact expression describing the polarization of both photons

$$\rho_{\Lambda\Lambda'} = \rho^{\gamma}_{\lambda_1\lambda_1'}\ \rho^{L}_{\lambda_2\lambda_2'}\,. \tag{4}$$

Let us derive the equations for Stokes parameters $\xi_i$ of $\gamma$-quantum traversing a laser bunch. As is well known the variations in intensity and polarization of the wave passing a medium are due to its interference with the wave scattered at zero angle. Let the incoming wave has the form $A_\Lambda\,\mathrm{e}^{ikz}$. Here the amplitude $A_\Lambda$ describes the polarization state of the $\gamma$-quantum and the laser photon, the wave vector $k=\omega/c$ (the frequency of laser photon $\omega_L \ll \omega$). When the wave passes through a "target" layer of a thickness $dz$ the forward scattered wave is given by

$$f_{\Lambda\Lambda'}\,A_{\Lambda'}\,2n_L\,dz \int \frac{\mathrm{e}^{ikr}}{r}\,dx\,dy = \frac{2\pi\,i}{k}\,f_{\Lambda\Lambda'}\,A_{\Lambda'}\,2n_L\,dz\,\mathrm{e}^{ikz}\,.$$

Therefore, the amplitude variation is equal to

$$dA_\Lambda = \frac{2\pi\,i}{k}\,f_{\Lambda\Lambda'}\,A_{\Lambda'}\,2n_L\,dz\,, \tag{5}$$

where $f_{\Lambda\Lambda'}$ is the forward amplitude for the process of elastic scattering light by light. The factor 2 in front of $n_L$ is due to relative motion of the $\gamma$-quanta and the "target".

Now we can calculate the variation of density matrix $\rho_{\Lambda\Lambda'}$ from eq. (4) which is expressed via the product of $A_\Lambda$:

$$\rho_{\Lambda\Lambda'} = \frac{\langle A_\Lambda A^*_{\Lambda'}\rangle}{N}\,,\qquad N = \langle A_\Lambda A^*_\Lambda\rangle\,, \tag{6}$$

where $\langle ...\rangle$ denotes a statistical averaging. The quantity $N$ is proportional to the $\gamma$-quantum intensity $J$. When the wave passes through the layer of a thickness $dz$ its relative variation in intensity is equal to

$$\frac{dJ}{J} = \frac{dN}{N} = -\frac{4\pi}{k}\,\mathrm{Im}\,(f_{\Lambda\Lambda'}\,\rho_{\Lambda'\Lambda})\,2n_L\,dz = -\,\sigma_{\gamma\gamma}\,2n_L dz\,. \tag{7}$$

[1] We restrict ourselves to the case when the direction of the linear polarization is constant inside the laser flash. More general case corresponds only to a little more cumbersome expressions.

Here we introduce (in accordance with the optical theorem) the total cross section for the light-light scattering $\sigma_{\gamma\gamma}$. Analogously,

$$d\rho_{\Lambda\Lambda'} = d\frac{\langle A_\Lambda A^*_{\Lambda'}\rangle}{N} = \frac{2\pi\, i}{k}\,(f_{\Lambda\Lambda''}\rho_{\Lambda''\Lambda'} - f^*_{\Lambda'\Lambda''}\rho_{\Lambda\Lambda''})\, 2n_L dz - \rho_{\Lambda\Lambda'}\,\frac{dN}{N}\,. \tag{8}$$

Among the 16 helicity amplitude $f_{\Lambda\Lambda'} \equiv f_{\lambda_1\lambda_2\,\lambda'_1\lambda'_2}$ only six are not equal zero for the forward scattering, namely $f_{++++} = f_{----}\,,\quad f_{+-+-} = f_{-+-+}\,,\quad f_{++--} = f_{--++}\,.$ We will further use the following real dimensionless quantities $R$ and $I$ proportional correspondingly to the real and imaginary parts of the scattering amplitudes[2]:

$$(2\pi/k)\,(f_{++++} + f_{+-+-}) = \pi r_e^2\,(R_{np} + iI_{np})\,, \tag{9}$$

$$(2\pi/k)\,(f_{++++} - f_{+-+-}) = \pi r_e^2\,(R_c + iI_c)\,,\quad (2\pi/k)\, f_{++--} = \pi r_e^2\,(R_l + iI_l)\,. \tag{10}$$

By substituting eqs. (4), (9), (10) into eqs. (7), (8) we obtain the equations for Stokes parameters. To write down these equations it is convenient to introduce the quantity

$$dt(z) = 2\pi r_e^2\, n_L dz \tag{11}$$

which we will call **the reduced optical thickness** of the layer $dz$. Then

$$\begin{aligned} \frac{d\xi_1}{dt} &= (-R_c\xi_3 + I_c\xi_1\xi_2)\,P_c + (R_l\xi_2 + I_l\xi_1\xi_3)\,P_l\,, \\ \frac{d\xi_2}{dt} &= -I_c(1-\xi_2^2)\,P_c + (-R_l\xi_1 + I_l\xi_2\xi_3)\,P_l\,, \\ \frac{d\xi_3}{dt} &= (R_c\xi_1 + I_c\xi_2\xi_3)\,P_c - I_l\,(1-\xi_3^2)\,P_l\,. \end{aligned} \tag{12}$$

Integrating these equations one obtains Stokes parameters as functions of the reduced optical thickness $t$ – see sections 3 below. Then they are used to calculate the cross section $\sigma_{\gamma\gamma} = \pi r_e^2\,(I_{np} + \xi_2 P_c\, I_c + \xi_3 P_l\, I_l)$ and the intensity (7).

The forward scattering amplitudes (and, therefore, quantities $R$ and $I$) depend on the single variable $s/(m_e^2c^4) = 4\omega/\omega_{\rm th}$, where $s = 4\,\hbar\omega\,\hbar\omega_L$ is the square of the total energy of the $\gamma$-quantum and the laser photon in their centre-of-mass system. The corresponding formulae are given in ref. [1], functions $R$ are presented in fig. 1. Note that the extrema of $R_c$ and $R_l$ are at the threshold of the pair production:

$$R_c = 0.315,\quad R_l = -0.348 \quad \text{at } s = 4m_e^2c^4\,. \tag{13}$$

In the region below the threshold these functions decrease very rapidly with decreasing of $s$.

# 3 A laser bunch as a transparent anisotropic medium

A laser bunch becomes practically transparent for $\gamma$-quanta with the energy $\omega < \omega_{\rm th}$ (below the threshold of the pair production).

If the laser photons are linearly polarized ($P_l \neq 0,\ P_c = 0$) the solution of eqs. (12) has the form

$$\xi_1 = \xi_1^0\cos\varphi_l + \xi_2^0\sin\varphi_l\,,\quad \xi_2 = -\xi_1^0\sin\varphi_l + \xi_2^0\cos\varphi_l\,,\quad \xi_3 = \xi_3^0\,, \tag{14}$$

with phase $\varphi_l = P_l\, R_l\, t$ and $\xi_i^0$ as the initial Stokes parameters. It is seen from this solution that in this case the laser bunch is an anisotropic medium with different refraction indices $\tilde n_x$ and $\tilde n_y$ along the $x$ and $y$ axes $\tilde n_x - \tilde n_y = (c/\omega)\, 2\pi r_e^2\, n_L\, P_l\, R_l$. Such a medium transforms the circular polarization of $\gamma$-quanta into the linear one and vice versa.

[2]As usual, the real parts of amplitudes are connected with refraction indices and the imaginary parts are connected with the corresponding cross sections, among them $R_{np}$ and $I_{np}$ correspond to the nonpolarized beams and $R_c$, $I_c$ and $R_l$, $I_l$ correspond to asymmetries for the circular and linear polarizations, respectively – see ref. [1] for detail.

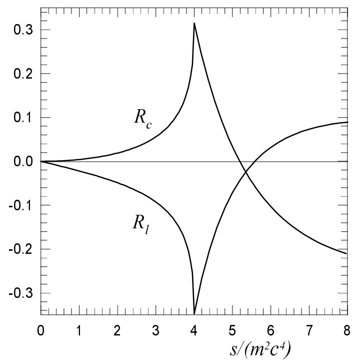

Fig. 1

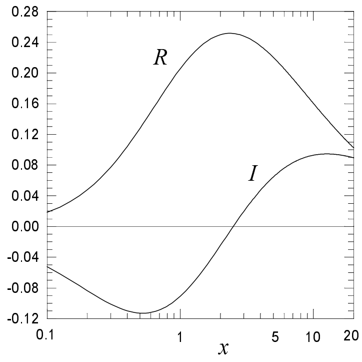

Fig. 2

**Fig. 1.** The real parts of the scattering amplitudes for the light-light scattering at zero angle (see eqs. (10) in dependence on the parameter $s/(m_e^2 c^4) = 4\omega/\omega_{\rm th}$.
**Fig. 2.** The real ($R$) and imaginary ($I$) parts of Compton scattering amplitudes at zero angle as function of the parameter $x = 4E_e\hbar\omega_L/(m^2c^4)$.

If the laser photons are circularly polarized ($P_c \neq 0,\ P_l = 0$), the solution of eqs. (12) has another form

$$\xi_1 = \xi_1^0 \cos\varphi_{\rm c} - \xi_3^0 \sin\varphi_{\rm c}\,,\ \ \xi_2 = \xi_2^0\,,\ \ \xi_3 = \xi_1^0 \sin\varphi_{\rm c} + \xi_3^0 \cos\varphi_{\rm c}\,, \tag{15}$$

with phase $\varphi_{\rm c} = P_c\, R_c\, t$. From this solution it is seen that in this case the laser bunch is an gyrotropic medium with different refraction indices for the right and left polarized $\gamma$-quanta. Such a medium rotates the direction of the $\gamma$-quantum linear polarization on the angle $(-\,\varphi_{\rm c}/2)$.

Above the threshold of the $e^+e^-$ pair production ($\omega > \omega_{\rm th}$ ) the variation in the $\gamma$-quantum polarization is accompanied by a reduction in their intensity in accordance with eq. (7). In this case the total $\gamma$-quantum degree of polarization are not conserved (for detail see ref. [1]).

# 4 The variation in polarization of high-energy electrons traversing a laser beam

Let an electron beam traverses a region where the laser light is focused. It is well known that the energies of these electrons as well as their polarizations are varied due to Compton scattering. The knowledge of this polarization is required, for example, if one considers the $e \to \gamma$ conversion with a conversion coefficient that is not small. In this case one has to take into consideration multiple electron scattering by laser photons. The polarization of the scattered electrons in dependence on the spin states of the colliding electrons and photons is given in [12].

However, when the electrons pass the laser beam, the polarization varies also for those electrons which conserve their energies and directions of motion. In this section we discuss the polarization of those undeflected electrons. It can be done using just the same method as presented above for $\gamma$-quanta.

Indeed, the polarization state of an electron is described by the polarization vector $\boldsymbol{\zeta}$. The electron density matrix in the helicity basis ($\lambda,\ \lambda' = \pm\frac{1}{2}$) can be obtained from the $\gamma$-quantum density matrix (2) by substitutions

$$\xi_1 \to -\zeta_y,\ \ \xi_2 \to \zeta_z,\ \ \xi_3 \to -\zeta_x.$$

After that, the previous calculations is valid for electron case also with some simplifications since in this case $I_l = R_l = 0$. Because the Compton cross section does not have threshold, the laser bunch is not a transparent medium for electrons.

The function $I = I_c$ and $R = R_c$ are presented in fig. 2 (the exact expressions for these functions are given in ref. [2]).

Equations for the components of the electron polarization vector can be obtained from eqs. (12) and their solutions are given in ref. [2]. When an electron passes a laser bunch, its polarization vector changes absolute value and its transverse components are rotated. This phenomenon is similar to the well-known effect of the rotation of the neutron spin in the polarized target.

# 5 Discussion

**1.** As a result, we have shown that below the threshold of the $e^+e^-$ pair production the laser bunch is similar to a transparent anisotropic medium. Let us illustrate the magnitude of the discussed effects using as an example the parameters of the laser bunch given in ref. [8] (they are close to the parameters which are realized in the experiment E-144 at SLAC [6]): $\hbar\omega_L = 1.18$ eV, the energy of the laser flash is 1 J, the laser bunch length is 1.8 ps, and the peak intensity is about $10^{18}$ W/cm$^2$. The reduced optical thickness (11) for this flash is equal to $t = 1.4$. As is seen from fig. 1 phases $\varphi_l = P_l\, R_l\, t$ and $\varphi_c = P_c\, R_c\, t$ which determine the magnitude of the effect can reach values $\approx 0.3t \sim 1$. According to eqs. (14) and (15) it means that the variation in the $\gamma$-quantum polarization may be very large. It is also seen from fig. 1 that the effect depends strongly on the energy and it becomes very small at $\omega \ll \omega_{\rm th}$.

**2.** The linear polarization of $\gamma$-quanta is needed for a number of interesting experiments. For example, in ref. [13] it is stressed that the best way to determine the $CP$ value of the neutral Higgs boson with intermediate mass is to use $\gamma\gamma$ collisions with parallel or perpendicular linear polarizations. However, the $\gamma\gamma$ luminosity for such polarization is considerable less then the luminosity for the circular one. Using the additional laser bunches to transform the circular polarization into the linear one could obtain a luminosity of the $\gamma\gamma$ collision with linear polarizations close to that for the optimal conditions.

**3.** The variation in polarization for unscattered electrons should be taken into account in simulations of the conversion process. It can be realized in the following way. The electron state is defined by the current values of its energy $\varepsilon$, the direction of its momentum $z$ and its mean polarization vector $\boldsymbol{\zeta}$. The probability to scatter on the path $dz$ is equal to $dw = \sigma(\varepsilon, \zeta_z)\, 2\, n_L\, dz$, where $\sigma(\varepsilon, \zeta_z)$ is the total cross section of the Compton scattering process. Then, as usual, one can simulate whether the scattering takes place on this path $dz$ comparing $dw$ with the random number in the interval (0;1).

If the scattering takes place, one has to simulate the polar and azimuthal angles using the differential cross section. It allows to calculate the energy of the scattered $\gamma$-quantum and its Stokes parameters. A new value of the electron polarization vector $\boldsymbol{\zeta}'$ can also be calculated using the formulae of ref. [12].

If scattering does not take place, nevertheless, one has to change the electron polarization vector in accordance with equations which are obtained in this paper.

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