Einstein-Podolsky-Rosen Paradox and Bell Inequality

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Abstract

We present a solution of the problems of the Einstein-Podolsky-Rosen paradox and the Bell inequality within the framework of a composite quantum model with memory.

Perhaps the most disputed situation arises in quantum mechanics at joint consideration of two problems: the Einstein-Podolsky-Rosen [1] paradox and the Bell inequality [2].

The explanation of the EPR paradox looks quite natural in the models with hidden parameters. Its explanation in the orthodox interpretation of quantum mechanics seems to me not so convincing. On the other hand, the Bell inequalities are easily proved in the models with hidden parameters. The experiment does not confirm these inequalities, and it agrees with the orthodox interpretation.

I would like to discuss how these two problems are solved within the framework of the model proposed by me [3, 4, 5, 6]. In this model it is supposed that the quantum object consists of a local nucleus, carrier of dynamic properties and a carrier of memory — a shell extended in space, which is a wave information field. The behaviour of a concrete quantum object depends on the information which is stored in its memory. In particular, the state of the shell determines the result of individual experiment. Each state of a shell is unique.

At first we shall discuss which place could be occupied by the quantum-mechanical wave function in the model proposed. It is clear that the wave function should be tightly connected with the information field, but they cannot coincide in so far as the latter is unique, while the wave function describes the whole ensemble of quantum objects.

In this connection we shall introduce at first a concept of a global pure ensemble. In the proposed model it is supposed that the behaviour of a concrete quantum object is determined by the information stored in its information field. In order to use this information, at first we should reveal it. For this purpose a series of measurements should be performed. At each measurement the information field is subjected to an influence of a measuring device, which changes an initial structure of the field. Fulfilling a sequence of measurements, we expose the information, connected not with one state of the field, but to a series of various states. Therefore, only the part of the information can be detected for each state of the field .

We shall designate by an abbreviation MIES (maximal information exposed simultaneously) the information, stored in the information field, which can be detected as a result of a measurement of magnitudes which correspond to the mutually commuting elements of algebra of observables. Strictly speaking, the condition of simultaneity is not compulsory, but such abbreviation agrees well with the concept accepted in quantum mechanics — a set of simultaneously measurable observables.

We shall name as a global pure ensemble a set of all quantum systems where the same MIES is stored in the memory. We shall consider such ensemble as the ensemble described by a uniform wave function. The global pure ensemble is physically not realizable. In order to avoid possible complications we shall accept the hypothesis, that any sufficiently large random sample of the global pure ensemble can serve as its rather good (in statistical sense) representative. This sample will be refered to as a pure ensemble and will be characterized by the same wave function.

Mathematically, we can determine the wave function as an equivalence class of information fields of quantum objects which store the same MIES in their memory. It is natural to assume that the additive and wave properties of the wave function are the consequences of corresponding properties of the information field. At the same time, the information field is not obliged to be an element of any Hilbert space as opposed to the wave function.

For the same quantum object the set of simultaneously measurable observables can be chosen in various modes. Respectively, different variants of the MIES can be extracted from the memory of the same quantum object. Therefore, the quantum object with fixed state of the information field can be referred to various pure ensembles.

Thus, the change of measuring devices which are used for extraction of the information from the memory of the quantum object results in a modification of its wave function. This modification of the wave function is not operated by any equation of motion. It has all indications of a collapse. If the information field of a quantum object does not change at such collapse, one can define this phenomenon as a passive or subjective collapse.

The joint consideration of the EPR paradox and the Bell inequalities is facilitated by the fact that the same experiment is suitable for their illustration. Bohm [7] has proposed a very spectacular experiment for this purpose. It looks as follows (see Figure 1).

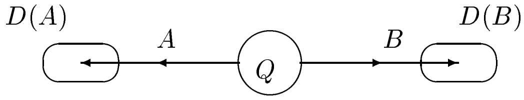

Figure 1.

A quantum object Q (particle with spin 0 in the elementary variant of the experiment) decays into two objects A and B (particles with spins 1/2). The objects A and B scatter at large distance and hit the measuring devices D (A) and D (B), respectively. The object A has a set of observables A_a (double projection of spin onto the direction a), which are labeled by the index a. The observables, corresponding to different indices, are not simultaneously measurable. Each of the observables can take two values ± 1. In a concrete measurement the device D (A) measures an observable A_a. For the object B everything is similar. In the devices D (A) and D (B) the measurements are independent.

We shall at first consider the Bell inequality. We shall assume that a quantum object Q has a hidden parameter λ. In each individual event the parameter λ accepts a definite value. The distribution of events according to the parameter λ is characterized by a measure $\mu(\lambda)$ with usual properties

$$\mu(\lambda) \geq 0, \qquad \int d\mu(\lambda) = 1.$$

All magnitudes, connected with an individual event, depend on the parameter λ. In particular, the values of observables A_a and B_b, obtained in an individual experiment, are functions $A_a(\lambda)$, $B_b(\lambda)$ of the parameter λ. For an individual event the correlation of observables A_a and B_b is characterized by the magnitude of $A_a(\lambda)\,B_b(\lambda)$. The average value of this magnitude is referred to as the correlation function $E(a,b)$,

$$E(a,b) = \int d\mu(\lambda)\,A_a(\lambda)\,B_b(\lambda).$$

Giving various values to the indices a and b and taking into account that

$$A_a(\lambda) = \pm 1, \quad B_b(\lambda) = \pm 1, \tag{1}$$

we shall obtain the inequality

$$|E(a,b) - E(a,b')| + |E(a',b) + E(a',b')| \leq \tag{2}$$

$$\leq \int d\mu(\lambda)\,[|A_a(\lambda)|\,|B_b(\lambda) - B_{b'}(\lambda)| + |A_{a'}(\lambda)|\,|B_b(\lambda) + B_{b'}(\lambda)|] =$$

$$= \int d\mu(\lambda)\,[|B_b(\lambda) - B_{b'}(\lambda)| + |B_b(\lambda) + B_{b'}(\lambda)|].$$

In the right-hand side of the formula (2), due to the equalities (1), one of the expressions

$$|B_b(\lambda) - B_{b'}(\lambda)|, \qquad |B_b(\lambda) + B_{b'}(\lambda)| \tag{3}$$

is equal to zero, and other is equal to two for each value of λ. From here the Bell inequality follows

$$|E(a,b) - E(a,b')| + |E(a',b) + E(a',b')| \leq 2. \tag{4}$$

The correlation function $E(a,b)$ is easily calculated within the framework of the standard quantum mechanics. In particular, when A and B are particles with spins 1/2

$$E(a,b) = -\cos\theta_{ab}, \qquad \theta_{ab} \quad \text{is an angle between a and b.} \tag{5}$$

It is easy to verify that there are directions a, b, a', b', for which the formulas (4) and (5) contradict each other.

However there are the conditions, when the inconsistency is absent. In derivation of the inequality (4) we tacitly supposed that the expressions (3) exist for each λ. But this supposition is erroneous if the parameter λ takes different values in each individual event. Really, the observables $B_b(\lambda), \quad B_{b'}(\lambda)$ cannot have a definite value in one experiment, as they are not simultaneously measurable. On the other hand, the values of the parameter λ in $B_b(\lambda), \quad B_{b'}(\lambda)$ should be different in various experiments, as these values cannot repeat.

Just such situation is realized in the model proposed by me. In this model the behaviour of an individual quantum object is determined by the information, which is stored in its information field. Formally this information can be considered as a hidden parameter. However each value of this parameter is unique by virtue of a unrecurrence of states of the information field. Thus, the given proof of the Bell inequality is not correct in the model proposed by me.

We shall consider now the EPR paradox. We shall carry out argumentation on the basis of the same experiment. In brief, the paradox consists in the following. Let the device D (B) to be located much further from the birthplace of the particles A and B than the device D (A). Let the device D (A) measure a projection of spin onto the axis Z for the particle A, and this value appears equal to $S_z(A)$. After decay of the object Q the particles A and B were in a singlet state at once. Therefore, the device D (B) will find out a value

$$S_z(B) = -S_z(A)$$

for the particle B with probability equal to unity. Within the framework of the standard quantum mechanics this means that the particle B is in a state with the definite value of projection of spin $(-S_z(A))$ on the axis Z.

We shall now assume, that changed the setting and have decided to measure the projection of spin on the axis X for the particle A. Then, repeating the previous reasonings, we shall conclude that the particle B will appear in the state with definite value of projection $S_x(B) = -S_x(A)$. Thus, measuring the projection $S_z(A)$ or the projection $S_x(A)$ for the particle A, we place the remote particle B in one or another state. This means that the instant transmission of information at a large distance exists.

Bohr proposed the followig an explanation of this paradox. We should not consider a dynamic system with correlation (pair of the particles A and B) as consisting of two separated and independent particles, but should treat the measurement on one particle as a measurement on the whole system. Einstein sharply objected to this reasoning since to his opinion (and to my too) it contradicts the localization.

The paradox is absent in the model proposed here. The quantum object Q decays into two particles. The information fields of these particles keep correlation even after their separation from each other, as these particles have a common origin. Because of the correlation, we can receive some information about the structure of information field of the particle B producing a measurement on the particle A. It is a typical example of an indirect measurement, in which the structure of the information field of the particle B does not change, as the device D (A) has no long-range action.

Measuring the projection $S_z(A)$ for the particle A, we immediately expose MIES for the particle A and indirectly (by virtue of the correlation) we do MIES for the particle B. This indirect information is $S_z(B) = -S_z(A)$. If we measure the projection $S_x(A)$ by the device D (A), we can assign the particle B to pure ensemble, corresponding to the definite value $S_x(B)$. The transition in the device D (A) from one kind of measurement $(S_z(A))$ to another type $(S_x(A))$ is equivalent to the change of MIES which we indirectly extract from the information field of the particle B without changing this field. It is an example of a passive or subjective collapse.

We shall consider one more variant of the experiment. Let the devices D(A) and D(B) be at the same distance from the quantum object Q, and we simultaneously fulfill the measurements of $S_z(A)$ and $S_x(B)$. As a result, we simultaneously obtain for the particle B the values of two observables ($S_x(B)$ and $S_z(B) = -S_z(A)$), which are not simultaneously measurable in the usual terminology of quantum mechanics.

Such combination of direct and indirect measurements allows to receive a larger information than MIES. Therefore, strictly speaking, the MIES is not a maximal information. However this information has a specific character — it corresponds to a physical state of a quantum object in a definite time interval in the past. In the example considered by us this time interval is limited by the moment t_0 (moment of the decay of the object Q or birth of the particle B) and the moment t_1 (moment of the measurement $S_x(B)$ in the device D(B)). The indirect information, obtained by the device D(A), ceases to correspond to the state of the particle B after the moment t_1. Therefore, such information is useless for predictions of a behaviour of the particle B in future.

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