Algebraic Aspects of Interactions of Massive Spinning Particles In Three Dimensions

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Abstract

The most general 2+1 dimensional spinning particle model is considered with the configuration space $\mathcal{M}^5 = R^{1,2} \times \mathcal{L}$ where $\mathcal{L}$ is a Lobachevsky plane being a space of the spinning modes. The action functional may involve all the possible first order Poincare invariants of $\mathcal{M}^5$ world lines, and the particular class of actions is specified thus the corresponding gauge algebra to be unbroken by inhomogeneous external fields. Nevertheless, the consistency problem reveals itself as a requirement of the global compatibility between first and second class constraints. These compatibility conditions, being unnoticed before in realistic second class theories, can be satisfied for a particle iff the gyromagnetic ratio takes the critical value $g = 2$. The quantization procedure is suggested for a particle in the generic background field by making use of a Darboux co-ordinates, being found by a perturbative expansion in the field multipoles and the general procedure is described for constructing of the respective transformation in any order.

1 Introduction

It is always a problem to construct consistent interactions for systems which involve higher spins. The problem appears in any dimension both at level of particles, fields and strings. For the massless case in low dimensions, the interacting spin fields allow for a uniform description at the level of equations of motion in the AdS space [1, 2]. Free massive field actions have long been known, it is not the case for interactions. When an action possesses invariance under higher-spin transformations, the reason of inconsistency is well known - the gauge algebra may be broken down when the interaction deforms theory. However, the peculiar feature of massive higher spin fields is that they may be formulated as pure second class constrained theories having no gauge invariance from the very beginning. The example of gauge noninvariant action was given in Ref. [3] for the massive higher spin fields. In this case, the consistency criteria become less obvious, however one should require at least Poincare invariance, renormalizability, unitarity and possibly somewhat yet unknown. Using the action of Ref. [3] the prescription was given in Refs. [4, 5] for imposing tree-level unitarity condition on the whole energy spectrum of the Compton scattering amplitudes of a single massive higher spin field. Some of coefficients in the action were fixed unambiguously, in particular, in a homogeneous e/m field gyromagnetic ratio takes the value $g = 2$ [4].

Another way to get consistent interacting field equations is to quantize an appropriate classical mechanical model of massive spinning particle which is constructed as a constrained Hamiltonian system with a finite number degrees of freedom. The advantage of the method is that it is simpler to achieve consistent couplings to backgrounds at the classical level than to examine then in the quantum field theory. The reason is that there is only a finite number of gauge symmetries (first class constraints) in the classical particle description and continuous infinity of field theoretical symmetries are generated from them through a quantization procedure. In Ref. [10] the systematic procedure was developed to construct the most general action functional (involving some external parameters) for a particle of an arbitrary spin. One may restrict parameter's values by appropriate conditions to the special case which admits consistent interactions with a general background. However, as further will be clarified, the consistency problem is not exhausted by noncontradictory deformation of the gauge algebra. It appears again as a requirement of global smoothness of equations of motion on the phase space. More precisely, when the interaction is switching on, the special surface appears where the Dirac brackets become singular, and to make equations of motion regular upon this surface we should impose some smoothness conditions restricting the choice of the free parameters in the action. These conditions, being applied to the case of the homogeneous e/m field, fix the gyromagnetic ratio as $g = 2$ which is the same value that has been deduced from the field theoretical approach [4].

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The note is organized as follows. We begin in section 2 with a brief view of the universal model of 3D particle, being the 3D analog of the model, allowing for the consistent interaction in D=4 [10]. In section 3 the problem of interaction is considered, the singular structure of the phase space is clarified, and the consistency conditions are obtained. In section 4 we turn to quantization of the model and in section 5 we conclude with some general remarks on the perturbative quantization procedure and the global consistency problem. The Appendix is attached with the some details of the geometric constructions.

2 Universal 3D spinning particle model

Configuration space of 3D spinning particle [8, 9] can be chosen as $\mathcal{M}^5 = R^{1,2} \times \mathcal{L}$, where the Lobachevsky plane $\mathcal{L}$ carries the spinning degree of freedom [a]. Poincare generators

$$P_a = p_a \qquad J_a = \varepsilon_{abc} x^b p^c + j_a \ , \tag{2.1}$$

being functions on the extended phase space $T^*\mathcal{M}^5$, form the algebra with respect to the Poisson brackets

$$\begin{array}{rcl} \{P_a, P_b\} & = & 0 \\ \{P_a, J_b\} & = & \varepsilon_{abc}\eta^{cd} P_d \\ \{J_a, J_b\} & = & \varepsilon_{abc}\eta^{cd} J_d \end{array} \tag{2.2}$$

Level surface of Casimir functions $P^2 = -m^2$ and $PJ = ms$ fixes particle's spin and mass parameters in the space of values (P_a, J_a). Due to commutation relations (2.2), Hamiltonian counterparts for the Casimirs on the extended phase space should appear as first class constraints. Existence of two independent gauge symmetries reveals itself in the form of classical Zitterbewegung. When the effect is present the particle's world-surface is no more straight line but takes topology of cylinder [8]. The same phenomenon may happen not only in d=3, but in d=4 [10] and higher-dimensional case [13]. To reduce extra dimensions of the world-surface, one should require the gauge symmetries to be dependent on space-time part of the extended phase space

$$\left\{ \begin{array}{rcl} \delta_{1\varepsilon} x^a & = & 2\varepsilon p^a \\ \delta_{1\varepsilon} p^a & = & 0 \end{array} \right\} \sim \left\{ \begin{array}{rcl} \delta_{2\mu} x^a & = & \mu j^a \\ \delta_{2\mu} p^a & = & 0 \end{array} \right\} \tag{2.3}$$

Taking into account the Casimirs surfaces, the requirement results in $p_a = -m/s j_a$.

To construct the most general action functional [10], we need to subject it to the following natural conditions:

1) the Lagrangian does not contain higher derivatives

2) the action is invariant under reparametrization of particle's world-line

3) mass-shell $P^2 + m^2 = 0$ and spin-shell $PJ - ms = 0$ conditions should arise in the theory as constraints

There are five true Poincare invariants (for definition of n_a and other notations, see Appendix)

$$\begin{array}{rcl} \Gamma_1 & = & \dot{x}^2 \\ \Gamma_2 & = & (\dot{x}n)^2 \\ \Gamma_3 & = & (\dot{x}\dot{n}) \\ \Gamma_4 & = & \varepsilon_{abc}\dot{x}^a n^b \dot{n}^c \\ \Gamma_5 & = & \dot{n}^2 \end{array} \tag{2.4}$$

and one more object which transforms by a total derivative

$$\Gamma = i\frac{\dot{z}\bar{z} - \dot{\bar{z}}z}{1 - z\bar{z}} \tag{2.5}$$

The most general Poincaré- and reparametrization-invariant action reads

$$S = \int d\tau (L(\Gamma_i) + \beta\Gamma) \equiv \int d\tau \mathcal{L} \tag{2.6}$$

[a] For the details of inner space geometry and notations to be used in the text see the Appendix, more details of this geometry could be found in [8].

Imposing the above mentioned conditions on the action to be consistent with interactions, one gets the one-parametric family of the Lagrangians

$$\mathcal{L}_\gamma = m\gamma(\dot{x}n) + \frac{m}{s}\sqrt{(\gamma^2-1)(\dot{x}^2 + (\dot{x}n)^2 - \frac{2s}{m}\varepsilon_{abc}\dot{x}^a n^b \dot{n}^c + \frac{s^2}{m^2}\dot{n}^2)} + i\gamma s\frac{\dot{z}\bar{z} - \dot{\bar{z}}z}{1-z\bar{z}} \tag{2.7}$$

where $\gamma = \beta/s$. When the parameter $\gamma \neq 1$, the Lagrangian possesses two gauge symmetries dependent on the space-time part of $\mathcal{M}^5$. If $\gamma = 1$, these gauge symmetries become dependent on the whole configuration space, and they both reduce to reparametrization invariance only. In what follows we consider only the case $\gamma = 1$.

3 Interactions

The full description of the most general interacting Lagrangian and its constrained Hamiltonian analysis is described in Ref. [11]. In this section we examine the case of homogeneous e/m field:

$$L = (\dot{x}n)(m + e\gamma(Fn)) + is\frac{\dot{z}\bar{z} - \dot{\bar{z}}z}{1-z\bar{z}} - e\dot{x}^a(A_a + \mu F_a + \delta F_{ab}n^b) + \mathcal{O}(e^2)\ . \tag{3.1}$$

Here γ, μ and δ are arbitrary constants. When the field is homogeneous, the term $-e\dot{x}^a\mu F_a$ does not contribute to dynamics as it vanishes modulo a total time derivative. The complete set of the constraints on $\mathcal{M}^8 = T^*(R^{1,2}) \times \mathcal{L}$ reads

$$T_a = p_a - mn_a - e\gamma(Fn)n_a + e\delta F_{ab}n^b \tag{3.2}$$

Their brackets on $\mathcal{M}^8$ may be presented in the form

$$\{T_a, T_b\} = \varepsilon_{abc}((1 - \frac{me\gamma}{s})F^c - (\frac{m^2}{s} + \frac{2me\gamma}{s}(Fn))n^c + \frac{me\delta}{s}F^{ck}n_k) \equiv \varepsilon_{abc}N^c \tag{3.3}$$

The maximal rank of this matrix equals 2 and corresponding null-vectors are N_c. Thus, in a general position, there are two second class constraints and one first class among T_a. As is obvious, the rank decreases to zero, if the r.h.s of (3.3) vanishes $N_c = 0$. Locally, i.e. *on the surface* $N_c = 0$, all the constraints become the "first class". The Dirac brackets, being constructed of the second class constraints in the general position, become singular on this surface that gives rise to the discontinuity in the equations of motion. To study the dynamics near the singular surface, it is convenient to chose another constraint basis which is equivalent to the original constraints $T_a = 0$ (3.2), the equivalence immediately follows from the identity (6.5),

$$H = p^2 + m^2 + 2em\gamma(Fn) + e\delta F_{ab}p^a n^b \tag{3.4}$$

$$\theta = (p\xi) + ie\delta(F\xi) \qquad \bar{\theta} = (p\bar{\xi}) - ie\delta(F\bar{\xi}) \tag{3.5}$$

The algebra of these constraints reads

$$\{H, \theta\} = -2ie(pn)(F\xi)((1 - \frac{m\gamma}{s}) + \frac{i\delta}{s}(pn)) + (\sim \theta, \bar{\theta}) \tag{3.6}$$

$$\{H, \bar{\theta}\} = 2ie(pn)(F\bar{\xi})((1 - \frac{m\gamma}{s}) - \frac{i\delta}{s}(pn)) + (\sim \theta, \bar{\theta}) \tag{3.7}$$

$$\{\theta, \bar{\theta}\} = -\frac{i\zeta^2}{2s}((pn)^2 + s(Fn)) + (\sim \theta, \bar{\theta}) \equiv \Phi + (\sim \theta, \bar{\theta}) \tag{3.8}$$

$$\Phi \equiv -\frac{i\zeta^2}{2s}((pn)^2 + s(Fn)) \tag{3.9}$$

Denote constraints $(\theta, \bar{\theta}) = \theta_i$, $i = 1, 2$ and co-ordinates on $\mathcal{M}^6 = T^*(R^{1,2})$ as $(x_a, p^b) = \Gamma^A$. Then Dirac brackets take the form

$$\omega^{AB} = \{\Gamma^A, \Gamma^B\}_{DB} = \{\Gamma^A, \Gamma^B\} + \frac{1}{\Phi}\{\Gamma^A, \theta_i\}\varepsilon_{ij}\{\theta_j, \Gamma^B\} \tag{3.10}$$

To reduce the inner space, from $T_a = 0$ we exclude vector n_a [11] as

$$n_a = \frac{p_a}{(-p^2)^{1/2}} - e\delta\frac{F_{ab}p^b}{p^2} + \mathcal{O}(e^2) \tag{3.11}$$

One can see that tensor ω^{AB} becomes singular on the surface

$$\Phi \equiv p^2 - s\frac{Fp}{(-p^2)^{1/2}} = 0 \tag{3.12}$$

Thus the phase space $\mathcal{M}^6$, being equipped with the bracket tensor $\omega^{AB} = \{\Gamma^A, \Gamma^B\}$, is not a smooth manifold anymore, it is decomposed into 3 parts: $\mathcal{M}^6 = \mathcal{M}^- \cup \Phi \cup \mathcal{M}^+$, where $\mathcal{M}^-$ and $\mathcal{M}^+$ are domains of analyticity of the bracket tensor. These domains are defined by the conditions $\Phi < 0$ and $\Phi > 0$ respectively. The equations of motion

$$\dot{\Gamma}^A = \{\Gamma^A, H\} + \frac{1}{\Phi}\{\Gamma^A, \theta_i\}\varepsilon_{ij}\{\theta_j, H\} \tag{3.13}$$

may suffer from a discontinuity at $\Phi = 0$. To make the equations of motion smooth, one should restrict dynamics on physical space with the following condition

$$\{H, \theta_i\} = \Pi_{ij}\theta_j + K_i H \tag{3.14}$$

where Π_{ij} and K_i are arbitrary functions. When the condition (3.14) is satisfied, the singular second term in (3.13) disappears, at least on shell. For the algebra (3.7-8) this means that parameters take their critical values $\delta = 0$ and $\gamma = s/m$. The equations of motion become smooth in the whole phase space $\mathcal{M}^6$ and take the well-known "Lorentz" form

$$\dot{x}^a = 2p^a + \mathcal{O}(F^2, \partial F, ...) \tag{3.15}$$

$$\dot{p}^a = -2eF^{ab}p_b + \mathcal{O}(F^2, \partial F, ...) \tag{3.16}$$

$$H = p^2 + m^2 + 2se\frac{(Fp)}{(-p^2)^{1/2}} + \mathcal{O}(F^2, \partial F, ...) = 0 \tag{3.17}$$

So far we have considered only the first order in the charge e, but, in the homogeneous field, the procedure may be closed in coupling constant. The equations (3.15-17) remain the same and Lagrangian takes the form

$$L = (\dot{x}n)\sqrt{m^2 + 2se(Fn)} + is\frac{\dot{z}\bar{z} - \dot{\bar{z}}z}{1 - z\bar{z}} - e\dot{x}^a A_a \tag{3.18}$$

If one calls numerical coefficient at the vertex (Fj) in Lagrangian (3.18) and Hamiltonian (3.17) as a gyromagnetic ratio [12], then one may conclude that the smoothing condition (3.14) fixes it as $\gamma = 2$.

Consider the consistency problem (which has been treated above in the framework of the constrained dynamics) from the viewpoint of the symplectic geometry. Symplectic view allows to formulate the consistency condition directly in the reduced phase space $\mathcal{M}^6$. The symplectic 2-form associated to Dirac brackets

$$\Omega = \frac{1}{2}(\omega^{-1})_{AB}d\Gamma^A \wedge d\Gamma^B \tag{3.19}$$

degenerates on the singular surface $\Phi = 0$: $\det\Omega = \Phi^2$. Equations of motion have the form

$$\Omega_{AB}\dot{\Gamma}^B|_{H=0} = \partial_A H|_{H=0} \tag{3.20}$$

and symplectic counterpart for the condition (3.13) reads as

$$Z^i_A \partial^A H|_{\Phi=H=0} = 0 \tag{3.21}$$

where Z^i_A are $[i]$ linear independent *local* null-vectors of the two-form (3.19):

$$\Omega_{AB}Z^B_i|_{\Phi=0} = 0 \tag{3.22}$$

The vectors Z^i_A are defined by the two-form Ω regardless of the Hamiltonian H. Since Z^i_A are fixed, relations (3.21) should be treated as restrictions to the admissible form of the Hamiltonian. It is the restriction which fixes the gyromagnetic ratio in the case of the particle in external e/m field.

4 Quantization

There exist a good many different methods of quantization of a given constrained system. Nevertheless, it is always a problematic task to quantize when phase space possesses nonlinear Poisson brackets. Usually, the quantization procedure is understood in the sense of deformation of Plank constant of the classical brackets to quantum commutators whose first order term is proportional to the classical one. This approach is commonly regarded as practically ineffective for the case of general manifold, because there is no representation theory for quantum brackets. The quantization problem is strongly simplified, if one finds the canonical realization (Darboux co-ordinates) for the initial classical Poisson brackets. This formal idea takes sensible form when phase space of interacting spinning particle is considered. The interacting theory contains coupling constant e, and knowing Darboux realization for free particle sector one may look for Darboux transformation of initial "interacting" co-ordinates in the form of perturbative expansion in coupling constant e. The attempt to construct the transformation in the first order in e was originally taken in Ref. [7] in the framework of a minimal anyon model. Below, we develop the general procedure for constructing of the respective transformation to the Darboux coordinates in any order in charge.

The structure of the proposed perturbation approach is most manifest in terms of symplectic geometry. In Darboux co-ordinates (x, p), 2-form of free case $\Omega_0 = \Omega(e = 0)$ takes the canonical form

$$\Omega_0 = \Omega(e = 0) \to \Omega = dp_a \wedge dx^a \tag{4.1}$$

To find canonical representation for the interacting 2-form $\Omega = \Omega(e)$ we proceed as follows: realize the initial variables Γ in the interacting theory as perturbation expansions in e of the initial variables of the free theory Γ_0 and then, making change of variables $\Gamma_0 = \Gamma_0(x, p)$, bring $\Omega = \Omega(e)$ to the canonical form:

$$\Omega_0 = \Omega(e) \stackrel{\Gamma=\Gamma(\Gamma_0)}{\longrightarrow} \Omega_0 = \Omega(e = 0) \stackrel{\Gamma_0=\Gamma_0(x,p)}{\longrightarrow} dp_a \wedge dx^a \tag{4.2}$$

In more practical terms the mentioned procedure means the following: starting with a pair of 2-forms, Ω_0 for the free case and Ω for the interacting one

$$\Omega = \Omega_{AB}(\Gamma; e) d\Gamma^A \wedge d\Gamma^B = \sum_{n=0}^{\infty} e^n \Omega^n_{AB}(\Gamma) d\Gamma^A \wedge d\Gamma^B \tag{4.3}$$

$$\Omega_0 = \Omega^0_{AB}(\Gamma_0) d\Gamma^A_0 \wedge d\Gamma^B_0 \tag{4.4}$$

(note that $\Omega^0(\Gamma_0)$ in (4.4) and first term in decomposition (4.3) are the same) one looks for the solution $\Gamma(\Gamma_0)$ of the equation

$$\Omega_{AB}(\Gamma(\Gamma_0)) d\Gamma^A(\Gamma_0) \wedge d\Gamma^B(\Gamma_0) = \Omega^0_{AB}(\Gamma_0) d\Gamma^A_0 \wedge d\Gamma^B_0 \tag{4.5}$$

in the form of the power series in the parameter of the charge e:

$$\Gamma^A = \Gamma^A(\Gamma_0) = \sum_{n=0}^{\infty} e^n \Gamma^A_n(\Gamma_0) \tag{4.6}$$

Substitution of (4.6) to (4.5) leads to

$$\Omega^0_{AC} \partial_B \Gamma^C_n + \Omega^0_{CB} \partial_A \Gamma^C_n + \partial_C \Omega^0_{AB} \Gamma^C_n = K_n(\Gamma_1, ..., \Gamma_{n-1}) \tag{4.7}$$

The last equation may be presented in the form

$$\mathcal{L}_{\Gamma_n} \Omega^0 = K_n(\Gamma_1, ..., \Gamma_{n-1}) \tag{4.8}$$

or in a manifest cohomological form

$$d \circ (\Omega^0 \lrcorner \Gamma_n) = K_n \tag{4.9}$$

where the operator d stands for an exterior derivation. Then the local compatibility condition for (4.9) reads as

$$d \circ K_n = 0 \tag{4.10}$$

The condition holds by virtue of Darboux theorem. General solution to the equation (4.9) has the form

$$\Gamma_n = (\Omega_0^{-1})^{AB}(\int_0^1 d\tau\tau\Gamma_0^C K^n_{BC}(\Gamma_0\tau) + \partial_B\Phi_n) \tag{4.11}$$

Arbitrary functions Φ_n, being involved in the general solution, do not contribute the respective two-form, hence Φ_n can be omitted.

Consider smooth structure of the mapping (4.6). On the surface $\Phi = 0$, the 2-form has null-vectors, therefore the Darboux co-ordinates on the map $\mathcal{M}^-$ cannot be smoothly continued through $\Phi = 0$ to $\mathcal{M}^+$. Consider the Jacobi matrix $J^A{}_B = \partial\Gamma^A/\partial\Gamma_0^B$. From the equation (4.5) one may get

$$\det J = \sqrt{\frac{\det\Omega^0}{\det\Omega}}, \qquad \det\Omega^0 = 1, \qquad \det\Omega = \Phi^2 \tag{4.12}$$

As is seen, the Jacobian J has a discontinuity on the singular surface. Thus, the perturbative quantization, being based on the expansion (4.6) of the Darboux coordinates, is able to reflect the dynamics in single part (of the phase manifold) alone of the two ones, $\mathcal{M}^+$ or $\mathcal{M}^-$.

5 Concluding remarks

In this paper the new understanding of the "consistency" is proposed for a massive spinning particle's dynamics in the background field. To have a consistent dynamics, one should satisfy the following conditions:

1) To provide a consistent deformation of gauge algebra, in the conventional sense.

2) To make particle's dynamics smooth upon the singular surface

The last condition results for the gyromagnetic ratio to be $g = 2$ for a particle. There is some point of issue in view of the quantization problem. Existence of the Dirac brackets possessing a *local* singularity appears to be unnoticed before in a realistic constrained system and because of this fact all the quantization prescriptions demand given Poisson tensor to be smooth on a whole phase manifold. It is unclear now how one should proceed to take the proper account of the singular surface in a quantization scheme. We drop out the singularity problem in the framework of perturbative approach to quantization, considering it as a first step towards the globally defined interacting quantum theory. The classical interaction is seen to be well defined globally iff $g = 2$. Note that in higher dimensions the interaction is well defined locally for an arbitrary spin massive particle, without any restriction on the giromagnetic ratio [14], although the global analysis, if it were performed for $d > 2 + 1$ along the lines of this note, would probably result in the same critical value for g in any dimension.

6 Appendix

We use realization of Lobachevsky plane as the unit open disk in a complex plane $\mathcal{L} \cong \{z \in C^1, z\bar{z} < 1\}$. The proper orthochronous Lorentz group $SO^\uparrow(1,2) \cong SU(1,1)/Z_2$ acts on $\mathcal{L}$ by fractional linear transformations

$$N: \quad z \to z' = (az - b)/(\bar{a} - \bar{b}z) \qquad N \in SU(1,1) \tag{6.1}$$

or in infinitesimal form

$$\delta z = -i\omega_a\xi^a \qquad \delta\bar{z} = i\omega_a\bar{\xi}^a \tag{6.2}$$

where ω_a - small real parameters and ξ_a is defined as follows

$$\xi_a = -1/2(2z, 1 + z^2, i(z^2 - 1)) \tag{6.3}$$

Inner space may be parametrized covariantly by unit timelike vector representing stereographic map of pseudosphere on Lobachevsky plane

$$n_a = (\frac{1 + z\bar{z}}{1 - z\bar{z}}, \frac{z + \bar{z}}{1 - z\bar{z}}, i\frac{z - \bar{z}}{1 - z\bar{z}}) \tag{6.4}$$

The following identity [9] holds for arbitrary 3-vector s_a

$$s_a = 2\frac{(s\xi)}{\zeta^2}\xi_a + 2\frac{(s\bar{\xi})}{\zeta^2}\bar{\xi}_a - (sn)n_a, \qquad \zeta = 1 - z\bar{z} \tag{6.5}$$

This identity allows to expand any 3d vector into fixed values (6.3), (6.4), that is directly and indirectly used in this note.

Acknowledgments

We are thankful to I.A.Bandos, I.A.Batalin, I.V.Gorbunov, M.Henneaux, A.A.Sharapov, K.M.Shekhter, M.A.Vasiliev for useful discussion related to various issues concerning the consistency problem for interactions.

This work is partially supported by the grant RFBR-98-02-16261, SLL is partially supported by the grant RFBR-99-01-00980

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