Bounds on masses of light composite Higgs bosons from Chiral Symmetry Rules

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Abstract

The effective quark models with quasilocal interaction, in strong coupling (tricritical) regime, are employed to construct the minimal extension of the Standard Model with two Higgs doublets, below the compositeness scale. The low energy lagrangean of Two-Higgs Doublet Standard Model is obtained in the large N_c and large-log approximation. The two-point correlators of scalar and pseudoscalar Higgs fields are calculated. The adjustment of their asymptotics at high energies allows to implement the chiral symmetry restoration in correspondence to the operator product expansion in QCD-like models. Some hints on the existence of new physical phenomena in the TeV energy region are made.

1 Definition of Two-Higgs Quasilocal Quark Model

The Two Higgs Doublet Model (2HDM) [1, 2] represents one of the possible extensions of the Standard Model (SM) which has two complex doublets of Higgs bosons instead of only one in SM. In the general 2HDM flavour-changing neutral currents (FCNC) [3] are too strong and the branching ratios of related processes are not acceptable by the phenomenology of electroweak decays [4]. One possibility to diminish FCNC is to couple the fermions only to a fixed combination of the two Higgs doublets and to its charge conjugated one, which is known as a Model I [2]. There is another possibility to couple the first Higgs doublet to down-type quarks while the second one to up-type quarks, which is known as a Model II [3].

The physical content of the Higgs sector includes a pair of CP-even neutral scalar Higgs bosons, H^0 and h^0, a CP-odd neutral pseudoscalar Higgs boson A and a pair of charged Higgs bosons $H^{\pm}$ if the CP parity is not violated. The mass spectrum of Models I and II has been extensively studied theoretically [1, 2, 5, 6] and bounded from the phenomenology of EW interactions at available high energies (for recent analyses, see [7, 8] and refs. therein).

If Higgs bosons are composite and their masses are created by a mechanism [9] of spontaneous chiral symmetry breaking [10]–[12], one should expect the chiral symmetry restoration (CSR) at high energies [13, 14] which provides the CSR constraints on phenomenological parameters of a Higgs model. These constraints on Higgs boson masses (and other parameters) may serve to select out the signatures of compositeness in the future experiments on Higgs particle observation.

Recently we have developed effective quark models including higher-dimensional operators made of fermion fields and their derivatives, which can be used [15] for the parameterization of unknown heavy particle dynamics beyond SM. The inclusion of higher dimensional operators in the fermionic lagrangean of SM opens the ways to built the two (and more) Higgs doublet extension of SM with composite Higgs bosons.

In our talk we explore particular Effective Quasilocal Quark Models (EQQM)[16] which accept relatively light Higgs boson states and conceivably inherit main properties of a underlying vector gauge theory [17, 18] giving rise to effective four-fermion operators at low energies. The latter theory may be thought of, for instance, as an extended QCD with additional heavy gluons (possibly manifesting extra spatial dimensions [18, 19] which induce the effective four-fermion interaction and enforce t-quarks to condensate). The effective potential of EQQM for boson fields and the Higgs boson mass spectrum is

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derived in sec. 2 where we restrict ourselves with the consideration of EQQM of type I. The lower bound for the h^0 boson mass is found to be $m_{h^0} \geq 120$ GeV for the t-quark mass $m_t \simeq 175$ GeV whereas the pseudoscalar A bosons can be as light as it is allowed by the experimental bounds.

The most important property of EQQM turns out to be the Chiral Symmetry Breaking (CSB) at low energies and, on the other hand, the Chiral Symmetry Restoration (CSR) at high energies. Here we deal with the two-point correlators of scalar and pseudoscalar quark densities [14, 20] which are saturated at low energies by Higgs-particle resonances of a definite parity. The difference between these correlators is decreasing rapidly in accordance to OPE of a vector-like gauge theory [21]. In the framework of either EQQM or a low-energy Higgs-field model it leads to the CSR constraints on some parameters of composite Higgs particles which are obtained in sec. 3.

In Sec. 4 we examine how CSR at high energies is realized in the 2HQQM of type I in the Nambu-Jona-Lasinio phase near tricritical point and show that it is compatible with existence of relatively light scalar, h^0, and pseudoscalar, A, Higgs bosons.

Let us remind the effective quark lagrangean of a EQQM which incorporates all higher-dimensional operators necessary for the description of Two-Higgs Doublet Standard Model in the low-energy limit. The two-flavor, third generation quark models with quasilocal interaction are considered in which the t- and b-quarks are involved in the DCSB.

We restrict ourselves by investigation of the Two–Higgs Quark Models of type I [16, 22] with Quasilocal interaction (2HQQM) with the following lagrangean:

$$\mathcal{L}_I = \bar{q}_L \not{D} q_L + \bar{t}_R \not{D} t_R + \bar{b}_R \not{D} b_R + \frac{1}{N_c \Lambda^2} \sum_{k,l=1}^{2} a_{kl} \left(J^T_{t,k} + g \widetilde{J}^T_{b,k} \right) i\tau_2 \left(g J_{b,l} - \widetilde{J}_{t,l} \right). \tag{1}$$

Here we have introduced the notations for doublets of fermion currents:

$$J_{t,k} \equiv \bar{t}_R f_{t,k}(\hat{s})\, q_L\,, \qquad J_{b,k} \equiv \bar{b}_R f_{b,k}(\hat{s})\, q_L\,, \qquad q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \tag{2}$$

and the tilde in $\widetilde{J}_{t,k}$ and $\widetilde{J}_{b,k}$ marks charge conjugated quark currents, rotated with Pauli matrix τ_2,

$$\tilde{J}_{t,k} = i\tau_2 J^{\star}_{t,k}\,, \qquad \tilde{J}_{b,k} = i\tau_2 J^{\star}_{b,k}\,. \tag{3}$$

The subscripts t, b indicate right components of t and b quarks in the currents, the index k enumerates the formfactors:

$$\begin{aligned} &f_{t,1}(\hat{s}) = 2 - 3\hat{s}\,; \qquad f_{t,2}(\hat{s}) = -\sqrt{3}\hat{s}\,; \qquad \hat{s} \equiv -\frac{\partial^2}{\Lambda^2}\,; \\ &f_{b,1}(\hat{s}) = 2 - 3\hat{s}\,, \qquad f_{b,2}(\hat{s}) = -\sqrt{3}\hat{s}\,; \end{aligned} \tag{4}$$

which are orthonormal on the interval $0 \leq s = \langle \hat{s} \rangle \leq 1$. In these notations coupling constants of the four-fermion interaction are represented by 2×2 matrix a_{kl} and contributed also from the Yukawa constant g. This coupling constant induces the quark mass ratio $m_b/m_t \sim g$.

2 Effective potential and mass spectrum of composite Higgs bosons

For sufficiently strong four-fermion coupling constants the lagrangean (1) reveals the phenomenon of dynamical chiral symmetry breaking due to generation of dynamical quark mass.

In order to describe it in details the lagrangean density (1) of the Model I must be rearranged by means of incorporation of auxiliary bosonic variables and by integrating out fermionic degrees of freedom [15]. Namely, we define two scalar $SU(2)_L$-isodoublets:

$$\Phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix}, \tag{5}$$

and their charge conjugates:

$$\widetilde{\Phi}_1 = i\tau_2\Phi_1^* = \begin{pmatrix} \phi_{12}^\star \\ -\phi_{11}^\star \end{pmatrix}, \qquad \widetilde{\Phi}_2 = \begin{pmatrix} \phi_{22}^\star \\ -\phi_{21}^\star \end{pmatrix}. \tag{6}$$

Then the lagrangean (1) can be rewritten in the following way:

$$\mathcal{L}_I = L_{kin} + N_c\Lambda^2 \sum_{k,l=1}^{2} \Phi_k^\dagger (a^{-1})_{kl}\Phi_l + i\sum_{k=1}^{2}\left[\widetilde{\Phi}_k^\dagger J_{t,k} + g\Phi_k^\dagger J_{b,k}\right] + \text{h.c.} \tag{7}$$

The integrating out of fermionic degrees of freedom produces the effective action for Higgs bosons of which we shall keep only the kinetic term and the effective potential consisting of two- and four-particles operators. The omitted terms are supposedly small, being proportional to inverse powers of a large scale factor Λ and/or a large $\ln(\Lambda/v)$; $v \simeq 246$ GeV.

A systematic approximation can be developed in the vicinity of (poly)critical point,

$$8\pi^2 a_{kl}^{-1} \sim \delta_{kl} + \frac{\Delta_{kl}}{\Lambda^2}, \qquad |\Delta_{kl}| \ll \Lambda^2, \tag{8}$$

which signifies the cancellation of quadratic divergences [23].

In general there exist the regimes where at the minimum of the effective potential the ratio of v.e.v. of neutral Higgs fields is complex [23] and v.e.v. of charged components are also non-zero. Due to $U(2)$ symmetry of the effective potential one can always choose the parameterization of Higgs fields in the vicinity of a minimum with only one of v.e.v. being complex:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \theta \\ \phi_2 + i\rho \end{pmatrix}. \tag{9}$$

The appearance of a complex v.e.v. leads to CP violation [1, 24, 25] and a non-zero v.e.v. of charged fields breaks electric stability of the vacuum and supplies the photon with a mass. The latter phase is not realized in the Universe at zero temperatures as it follows from the severe bounds on both electric charge asymmetry and on the photon mass (see, e.g. [26]). We assume the electric charge stability of the vacuum, i.e. that only neutral components of both Higgs doublets may have nonzero v.e.v. [2, 5, 27]. It can be shown [28] that the relevant solutions with $\theta = \rho = 0$ are favorable if

$$\Delta_{22} \leq (1+g^4)\left[\phi_2^2 + \frac{1}{8}(\sqrt{3}\phi_1 + \phi_2)^2\right]. \tag{10}$$

This bound follows also from the absence of tachyons among composite bosons.

Taking the latter in mind one can find the minimum of the effective potential from the mass-gap equations [28]:

$$\begin{aligned}
\frac{8\pi^2}{N_c}\frac{\delta V_{eff}(\phi_k, \rho=\theta=0)}{\delta\phi_1} &= -\Delta_{11}\phi_1 - \Delta_{12}\phi_2 + 8\phi_1^3\ln\frac{\Lambda^2}{2\phi_1^2} + 8\phi_1^3 g^4 \ln\frac{\Lambda^2}{2g^2\phi_1^2} + \\
&\quad + \frac{1+g^4}{8}\left[-159\phi_1^3 - 15\sqrt{3}\phi_1^2\phi_2 + 9\phi_1\phi_2^2 + \sqrt{3}\phi_2^3\right] = 0, \\
\frac{8\pi^2}{N_c}\frac{\delta V_{eff}(\phi_k, \rho=\theta=0)}{\delta\phi_2} &= -\Delta_{12}\phi_1 - \Delta_{22}\phi_2 + \frac{1+g^4}{8}\Big[-5\sqrt{3}\phi_1^3 + 9\phi_1^2\phi_2 + \\
&\quad + 3\sqrt{3}\phi_1\phi_2^2 + 9\phi_2^3\Big] = 0.
\end{aligned} \tag{11}$$

One can choose ϕ_1, ϕ_2 as independent scales of 2HQQM. Then the mass-gap equations (11) allow to find the parameters Δ_{11}, Δ_{12} describing the deviation from critical coupling constants as functions of $\phi_1, \phi_2, \Delta_{22}$.

The true minimum is derived from the positivity of the second variation of the effective action around a solution of the mass-gap equation,

$$\Phi_k = \begin{pmatrix} \pi_k^+ \\ \frac{1}{\sqrt{2}}(\phi_k + \sigma_k + i\pi_k) \end{pmatrix}, \qquad k = 1, 2. \tag{12}$$

This variation reads:

$$\frac{16\pi^2}{N_c} S^{(2)}_{eff} \equiv \frac{1}{2}\left(\sigma, (\hat{A}^\sigma p^2 + \hat{B}^\sigma)\sigma\right) + \left(\pi^-, (\hat{A}^{\pi^\pm} p^2 + \hat{B}^{\pi^\pm})\pi^+\right) + \frac{1}{2}\left(\pi, (\hat{A}^\pi p^2 + \hat{B}^\pi)\pi\right), \qquad (13)$$

where two symmetric matrices - for the kinetic term $\hat{A}^i = \left(A^i_{kl}\right)$, $\quad i = (\sigma, \pi^\pm, \pi)$ and for the constant, momentum independent part, $\hat{B}^i = \left(B^i_{kl}\right)$ - have been introduced in the CP conserving phase. Their explicit form is displayed in [28].

The mass spectrum of related bosonic states is determined by the solutions of the secular equations :

$$\det(\hat{A}^i p^2 + \hat{B}^i) = 0\,, \qquad (14)$$

at $-m^2 = p^2 < 0$ in both scalar and pseudoscalar channels.

For large $\ln\left(\Lambda^2/m_t^2\right) \gg 1$ both the solutions of (11) and the mass spectrum look differently in two regimes: $\Delta_{11} \sim m_t^2 \ln\left(\Lambda^2/m_t^2\right) \gg \Delta_{12} \sim \Delta_{22} \sim m_t^2$ or $\Delta_{11} \sim \Delta_{12} \sim \Delta_{22} \sim m_t^2 \ln\left(\Lambda^2/m_t^2\right) \gg m_t^2$.

In the first case the mass spectrum of physical scalar bosons H^0, h^0 reads:

$$m^2_{H^0} = 8\phi_1^2 = 4m_t^2 \simeq \frac{\Delta_{11}}{\ln\left(\Lambda^2/m_t^2\right)}\,; \quad m^2_{h^0} = \frac{1}{2}\left(8\phi_2^2 + (\sqrt{3}\phi_1 + \phi_2)^2\right) - \frac{4}{3}\Delta_{22} \equiv \frac{3}{2}\delta m^2 - \frac{4}{3}\Delta_{22}\,, \qquad (15)$$

and for physical pseudoscalar bosons $\pi, A, H^\pm$:

$$m_\pi^2 = 0\,; \qquad m_A^2 = m^2_{H^\pm} = \frac{1}{6}\left(8\phi_2^2 + (\sqrt{3}\phi_1 + \phi_2)^2\right) - \frac{4}{3}\Delta_{22} \equiv \frac{1}{2}\delta m^2 - \frac{4}{3}\Delta_{22}\,, \qquad (16)$$

where we have introduced the notation for the mass splitting between excited states:

$$m^2_{h^0} - m^2_A = \delta m^2\,. \qquad (17)$$

One can see that, in general, $m^2_{h^0} - m^2_A = \delta m^2 \neq m^2_{H^0} - m^2_\pi$ and the splitting between scalar and pseudoscalar states is not equidistant. But if the dynamical mass is a true constant, i.e. $\sqrt{3}\phi_1 + \phi_2 = 0$, then one recovers the remarkable relation:

$$m^2_{h^0} - m^2_A = m^2_{H^0} - m^2_\pi = 4m_t^2\,. \qquad (18)$$

As $m_t^2 = 2\phi_1^2$ the parameter ϕ_1 is fixed from the phenomenology and never takes zero values. Then for a given Δ_{22} the lowest masses of excited bosons arise at $\phi_1 = -3\sqrt{3}\phi_2$ and amount to:

$$\delta m^2_{min} = \frac{1}{9}m^2_{H^0}\,; \quad m^2_{h^0} = \frac{1}{6}m^2_{H^0} - \frac{4}{3}\Delta_{22} \geq \frac{1}{9}m^2_{H^0}\,; \quad m_A^2 = \frac{1}{18}m^2_{H^0} - \frac{4}{3}\Delta_{22} \geq 0\,. \qquad (19)$$

It follows from these inequalities that the second pseudoscalar Higgs boson can in principle be very light even massless whereas the second scalar Higgs particle can be lighter than the first one (of Nambu-Jona-Lasinio type) but still enough heavy.

For instance, if we treat these relations at the tree level then: $m_{H^0} = 2m_t \simeq 350$ GeV but the lightest mass for the second Higgs boson $m^2_{h^0} = \frac{1}{9}m^2_{H^0} + m^2_A \geq (130\mathrm{GeV})^2$ for $m_A \geq 50$ GeV (see [29]).

In the case when $m_t^2 \ln\left(\Lambda^2/m_t^2\right) \sim \Delta_{11} \sim \Delta_{12} \sim \Delta_{22} \gg m_t^2$ the mass spectra can be easily derived from eqs. (15), (16) retaining the last term, $m^2_{h^0} \simeq m^2_A \simeq -\frac{4}{3}\Delta_{22}$ and their splitting is again described by (17). Thus for such parameters Δ_{ik} the second multiplet of Higgs boson is indeed much heavier that the first one $m^2_{H^0} \ll m^2_{h^0} \simeq m^2_A \sim m_t^2 \ln\left(\Lambda^2/m_t^2\right) \sim (1\mathrm{TeV})^2$. Hence this scenario is practically equivalent to a One-Higgs SM for accessible energies.

Thus we have outlined the mass spectrum in EQQM generated due to CSB and established the range of four-fermion coupling constants which promotes the formation of relatively light Higgs bosons. On the other hand CSB occurs at low energies and one should expect the Chiral Symmetry Restoration for correlators of quark currents when in-, out-coming 4-momenta are sufficiently high.

3 Chiral symmetry restoration in vector-like gauge models

Let us examine the sensitivity to CSB of the two-point correlators of scalar quark densities, $\Pi_\sigma(p^2)$ and of pseudoscalar quark densities, $\Pi_\pi(p^2)$. In the large N_c approach [30, 31] and for the vanishing current quark mass, they consist of a sum of meson poles, with different positions and residues for scalar and pseudoscalar channels, and of a perturbative contribution, the same in both channels.

On the other hand, the high-energy asymptotics of the correlators for non-Abelian gauge theories are given by perturbation theory and OPE taking into account the asymptotic freedom of gauge interaction. As well the nonperturbative generation of gauge field and quark condensates [21] is assumed to determine subleading power-like corrections to perturbative asymptotics which, in particular, parameterize CSB.

Let us denote as g_s the coupling constant of a gauge interaction generating four-fermion forces and eventually, composite bosons. Then the conventional coupling constants are $\bar{\alpha}_s \equiv g_s/4\pi$. In the chiral limit ($m_q = 0$) CSB is unambiguously described by the difference between the scalar and pseudoscalar correlators which have the following behavior [21] at large p^2 (for one-flavor quarks) motivated by OPE (in Euclidean notations):

$$\left(\Pi_\pi(p^2) - \Pi_\sigma(p^2)\right)_{p^2\to\infty} \simeq -\frac{12\pi\bar{\alpha}_s\left(\langle\bar{q}q\rangle\right)^2}{p^4} \equiv \frac{\Delta_{SP}}{p^4} \tag{20}$$

where the vacuum dominance hypothesis [21] (in the large-N_c limit) has been exploited to estimate four-quark condensate contribution in terms of bilinear quark condensates $< \bar{q}q >$. It is also assumed that $\bar{\alpha}_s(\Lambda_{CSB}) \ll 1$.

This rapidly decreasing asymptotics is a consequence of the chiral invariance of the lagrangean of a vector gauge theory with massless fermions. We stress that all perturbative contributions and nonperturbative terms induced solely by gluons or by resummation of perturbative series are completely canceled between two correlators as they do not break the chiral symmetry.

On the other hand, due to CSR, the saturation in the difference of scalar and pseudoscalar correlators may be successfully delivered by a finite number of low-lying resonances [14, 20]. Thus CSR at high energies may be thought of as a possible constraint on the EQQM to be a manifestation of compositeness in the Higgs model [23, 32]. We demand that, at the compositeness scale Λ_{CSB}, the relation (20) were approximately fulfilled and make the two-resonance ansatz for scalar and pseudoscalar correlators at intermediate energies supported by the Two-channel EQQM:

$$\Pi_\pi(p) - \Pi_\sigma(p) = \frac{Z_\pi}{p^2} + \frac{Z_{\pi'}}{p^2+m_{\pi'}^2} - \frac{Z_\sigma}{p^2+m_\sigma^2} - \frac{Z_{\sigma'}}{p^2+m_{\sigma'}^2}. \tag{21}$$

¿From the requirement of asymptotic CSR (20) it follows that:

$$Z_\sigma + Z_{\sigma'} = Z_\pi + Z_{\pi'}\,; \qquad Z_\sigma m_\sigma^2 + Z_{\sigma'} m_{\sigma'}^2 = Z_{\pi'} m_{\pi'}^2 + \Delta_{SP}\,. \tag{22}$$

Let us apply the above CSR sum rules to the 2HQQM (1). For this goal we have to obtain the appropriate two-point correlators for scalar and pseudoscalar composite Higgs fields.

4 CSR sum rules in 2HQQM

Let us introduce in 2HQQM the external sources which couple to the scalar and pseudoscalar quark densities. With their help one can derive required quark correlators. As these densities are doublets in our model, eq. (2), the relevant sources $X_1 = (\chi_{1j}), X_2 = (\chi_{2j})$ are also taken as doublets. The sources are complex: $\chi_{kj} = S_{kj} + iP_{kj}$, generating both scalar and pseudoscalar densities. The structure of the corresponding operators in the quark lagrangean is designed similar to the Yukawa operators in (7):

$$\mathcal{L}_I(S,P) = i\sum_{k=1}^{2}\left[\widetilde{X}_k^\dagger J_{t,k} + gX_k^\dagger J_{b,k}\right] + \text{h.c.} \tag{23}$$

We remind that in 2HQQM under consideration: $1 \gg g$ and take for a simplicity $g = 0$. Then the effect of external sources can be separated by shifting the Higgs fields in the quark part of the lagrangean (7):

$$\phi_{kj} \longrightarrow \bar{\phi}_{kj} + \chi_{kj}\,. \tag{24}$$

The dynamical boson fields arise as fluctuations around the solutions of the mass-gap equation (11):

$$\bar{\phi}_{k1} = \phi_{k1} - \chi_{k1}; \quad \bar{\phi}_{k2} = \phi_{k2} - \chi_{k2} + \langle\bar{\phi}_{k2}\rangle\,. \tag{25}$$

As we study the CP conserving phase with non-trivial v.e.v. only for scalar fields the physical parameters of neutral and charged pseudoscalar bosons are the same. Therefore we can consider neutral components only.

The resulting effective action for generating of two-point correlators is given by:

$$\begin{aligned} S^{(2)} &\simeq \frac{N_c\Lambda^2}{16\pi^2}\sum_{k,l=1}^{2}\left(S_k\Gamma^{(\sigma)}_{kl}S_l + P_k\Gamma^{(\pi)}_{kl}P_l\right) \\ \Gamma^{(\sigma)}_{kl} &= \delta_{kl} - 2\Lambda^2\left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl}\,; \qquad \Gamma^{(\pi)}_{kl} = \delta_{kl} - 2\Lambda^2\left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl}\,. \end{aligned} \tag{26}$$

When taking the approximate expressions [28] for matrices $\hat{A}, \hat{B}$ one derives the inverse propagators. In particular, the strictly local quark density can be presented as a superposition of two currents:

$$\bar{t}t = \frac{1}{2}\left(\bar{t}f_1t - \sqrt{3}\bar{t}f_2t\right). \tag{27}$$

The CSR restoration sum rules (22) for it can be found [28] from the difference: $\Pi^{aa}_{\pi}(p^2) - \Pi_{\sigma}(p^2)$ where the residues in poles happen to satisfy the relations:

$$Z_A = -Z_\pi + \frac{N_c\Lambda^4}{4\pi^2}\,; \qquad Z_{h^0} = -Z_{H^0} + \frac{N_c\Lambda^4}{4\pi^2}\,. \tag{28}$$

It can be shown that the residues in poles are of different order of magnitude:

$$Z_{H^0} \sim Z^\pi \sim \frac{1}{\ln\frac{\Lambda^2}{m_t^2}} \ll Z_{h^0} \sim Z_A\,, \tag{29}$$

The first CSR rule represents the generalized σ-model relation:

$$Z_{H^0} + Z_{h^0} = Z_\pi + Z_A = \frac{N_c\Lambda^4}{4\pi^2}\,, \tag{30}$$

which is fulfilled, in fact, at a high precision.

The second constraint in (22) is satisfied for an appropriate 4-fermion condensate contribution Δ_{SP}. Thus:

$$Z_{H^0}m^2_{H^0} + Z_{h^0}m^2_{h^0} = Z_A m^2_A + \Delta_{SP}\,. \tag{31}$$

Taking into account (29) and the sum rule (30) one concludes that:

$$Z_{h^0} \simeq Z_A \simeq \frac{N_c\Lambda^4}{4\pi^2}; \qquad \frac{N_c\Lambda^4}{4\pi^2}\left(m^2_{h^0} - m^2_A\right) \simeq \Delta_{SP}\,. \tag{32}$$

In order to estimate a maximal scale of compositeness $\Lambda \equiv \Lambda_C$ due to vector-like forces we assume that presumably the typical scale of gauge interactions is of order $\mu \sim m_t \ll \Lambda_C$ and one can apply effectively the one-loop approximation to calculate the strong interaction constant $\bar{\alpha}_s$ (see, [29]). As it follows from (19), the minimal value of mass splitting corresponds to $m^2_{H^0}/9$. Respectively the value of Δ_{SP} is described by the relation (20) and in the 2HQQM the condensates are derived from in (28). Let us combine these estimates and relations to derive the upper bound on the compositeness scale Λ_C:

$$\frac{N_c\Lambda_C^4 m_t^2}{9\pi^2} = \min(\Delta_{SP}) = 12\pi\bar{\alpha}_s\frac{N_c^2\Lambda_C^4 m_t^2}{16\pi^4}\,; \tag{33}$$

hence:

$$\bar{\alpha}_s N_c \simeq \frac{12\pi}{11\left(1 - \frac{2N_f}{N_c}\right)\ln\frac{\Lambda_C^2}{m_t^2}} > \frac{4\pi}{27} \quad \text{or} \quad \Lambda_C \leq 10^4\text{GeV}\,. \tag{34}$$

Thus we have proved that CSR at high energies is realized in the 2HQQM of type I in the Nambu-Jona-Lasinio phase near tricritical point. Moreover these models in the NJL phase can be regarded as effective models originating from a underlying vector-gauge theory with an electroweak compositeness scale of order 1 TeV or more (it may be also a fundamental, quantum gravity or string scale [18, 19] when additional spatial dimensions of inverse size > 10 Mev exist).

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