Chiral and Non-chiral Quark Models and their matching to QCD

A. A. Andrianov[a,b], V. A. Andrianov[a], D. Espriu[b] and R.Tarrach[b]

[a] Department of Theoretical Physics, St.Petersburg State University,

198904 St.Petersburg, Russia

[b] Departament d'ECM, Universitat de Barcelona

08028 Barcelona, Spain

Abstract

We discuss the low energy effective action of QCD in the quark sector. It has a qualitatively different structure depending on whether it is built above the CSB (chiral symmetry breaking) scale by means of perturbation theory or below the CSB scale when the major chiral symmetry breaking effect - the formation of light pseudoscalar mesons - is implemented manifestly. In the first case the models generalize the Nambu-Jona-Lasinio (NJL) one with CSB due to strong attractive 4-fermion forces in the scalar channel. In the second case the model represents an extension of the chiral quark model with a constituent quark mass and the non-linear realization of chiral symmetry. We show that if the lowest scalar meson state is sufficiently heavy approaching the mass of heavy pseudoscalar π' then QCD favours a low-energy effective theory which is largely dominated by the simplest chiral quark model, whereas higher dimensional operators (of the Nambu-Jona-Lasinio type) can be relatively small. On the contrary, a light scalar meson ($m_\sigma \simeq 500 MeV$) provides evidence in favour of the NJL mechanism.

1 Introduction

The Chiral Quark Model (CQM) [1] is built with colored quark fields $q_i^\alpha, \bar{q}_i^\alpha; i = 1, \ldots, N_c, \alpha = 1, 2, .., N_F$ interacting with the colorless chiral field $U(x)$, by means of the Yukawa-type operator

$$\mathcal{L}_{CQM} = \bar{q}\left(i\,\partial\!\!\!/ - M_0(UP_L + U^+P_R)\right)q + \frac{1}{4}f_0^2\mathrm{tr}\left(\partial_\mu U\partial^\mu U^+\right), \tag{1}$$

where $\partial\!\!\!/ \equiv \gamma^\mu\partial_\mu$ and the projectors $P_{L(R)} = (1 \pm \gamma_5)/2$ are used. M_0 is a chirally invariant constituent mass, which is assumed to acquire a non-zero value once chiral symmetry is spontaneously broken. For two light flavors $U(x) = \exp(i\pi(x)/F_0)$ is a $SU(2)$ matrix with generators $\pi \equiv \pi^a T^a$, $a = 1, 2, 3$. This massless lagrangean (1) is invariant under $SU_L(2) \bigotimes SU_R(2)$ symmetry transformations of chiral and quark fields. Hence the fields $\pi(x)$ correspond to the massless Goldstone bosons – pions.

The constant F_0 is the pion decay constant, whereas f_0 represents a bare pion decay constant, which is supposed to contain residual gluon contributions not accounted for by the constituent mass.

The second approach to interpolate the QCD effective action is provided by the Nambu-Jona-Lasinio model (NJL)[2]. It contains a chirally invariant four-fermion interaction. For massless quarks it reads

$$\mathcal{L}_{NJL} = i\bar{q}\,\partial\!\!\!/ q + \frac{g_0}{4N_c\Lambda^2}\left[(\bar{q}q)^2 - (\bar{q}\gamma_5\tau^a q)^2\right]. \tag{2}$$

This lagrangean is also invariant under $SU_L(2)\bigotimes SU_R(2)$ symmetry transformations of quark fields. It is however well known that for strong enough coupling g_0 dynamical breaking of chiral symmetry occurs in this model. This leads to the creation of a massless pion state, to the appearance of a dynamical quark mass Σ_0 (similar to M_0), and to the presence of a massive scalar meson state with the mass $m_\sigma \simeq 2\Sigma_0$, the σ-particle.

Both models provide, after integrating out the quark fields, basically the same set of low energy structural constants describing interactions between (pseudo) Goldstone bosons and therefore may serve as first approximations for interpolating the true QCD low-energy effective action.

Two ways have been proposed [3, 4] to improve the above models in order to get a better agreement with meson phenomenology at low and intermediate energies and to conform to the requirements of QCD.

2 Extended chiral quark model

The low-energy effective lagrangian of ECQM $\mathcal{L}_{ECQM}$ is built of quark, chiral and external fields, to be invariant under left and right $SU(2)$ rotations. It is implemented by means of the QCD Dirac operator:

$$\hat{D} \equiv i\gamma_\mu(\partial_\mu + \bar{V}_\mu + \gamma_5\bar{A}_\mu) + i(\bar{S} + i\gamma_5\bar{P}), \tag{3}$$

where $\langle S\rangle = m_q$, matrix of current quark masses.

It is convenient to introduce the 'rotated', 'dressed' or 'constituent' quark fields

$$Q_L \equiv \xi q_L, \qquad Q_R \equiv \xi^\dagger q_R, \qquad \xi^2 \equiv U, \tag{4}$$

which transform nonlinearly under $SU_L(2)\bigotimes SU_R(2)$ but identically for left and right quark components.

Changing to the 'dressed' basis implies the following replacements in the external vector, axial, scalar and pseudoscalar sources

$$\begin{aligned}
\bar{V}_\mu \to v_\mu &= \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \partial_\mu\xi\xi^\dagger + \xi^\dagger\bar{V}_\mu\xi + \xi\bar{V}_\mu\xi^\dagger - \xi^\dagger\bar{A}_\mu\xi + \xi\bar{A}_\mu\xi^\dagger\right),\\
\bar{A}_\mu \to a_\mu &= \frac{1}{2}\left(-\xi^\dagger\partial_\mu\xi - \partial_\mu\xi\xi^\dagger - \xi^\dagger\bar{V}_\mu\xi + \xi\bar{V}_\mu\xi^\dagger + \xi^\dagger\bar{A}_\mu\xi + \xi\bar{A}_\mu\xi^\dagger\right),\\
\bar{\mathcal{M}} \to \mathcal{M} &= \xi^\dagger\bar{\mathcal{M}}\xi^\dagger.
\end{aligned} \tag{5}$$

In these variables the relevant part of ECQM action can be represented as,

$$\mathcal{L}_{ECQM} = \mathcal{L}_{ch} + \mathcal{L}_{\mathcal{M}} + \mathcal{L}_{vec}, \tag{6}$$

where $\mathcal{L}_{ch}$ accumulates the interaction of chiral fields and quarks in the chiral limit in the presence of vector and axial-vector external fields, $\mathcal{L}_{\mathcal{M}}$ extends the description for external scalar and pseudoscalar fields and, in particular, for massive quarks.Further on we simplify our analysis taking the chiral limit, $m_q = 0$ and therefore neglect $\mathcal{L}_{\mathcal{M}}$. As well we do not consider here the operators $\mathcal{L}_{vec}$ generating meson states in vector and axial-vector channels (see [5]).

In more detail

$$\mathcal{L}_{ch} = i\bar{Q}\left(\not{D} + M_0\right)Q - \frac{f_0^2}{4}\mathrm{tr}(a_\mu^2) + \frac{G_{S0}}{4N_c\Lambda^2}\left(\bar{Q}_L Q_R + \bar{Q}_R Q_L\right)^2 - \frac{G_{P1}}{4N_c\Lambda^2}\left(-\bar{Q}_L\vec{\tau}Q_R + \bar{Q}_R\vec{\tau}Q_L\right)^2, \quad (7)$$

where

$$Q \equiv Q_L + Q_R, \qquad \not{D} \equiv \not{\partial} + \not{v} - \gamma_5 g_A \not{a}, \quad (8)$$

with the 'bare' pion decay constant f_0 and the axial coupling $g_A \equiv 1 - \delta g_A$. In general, $G_{S0} \neq G_{P1}$.

To find the characteristics of composite boson states in colorless quark channels we introduce the auxiliary bosinic fields:

$$\mathcal{L}_{4-quark} = \bar{Q}\left[i\widetilde{\Sigma} - \gamma_5\widetilde{\Pi}\right]Q + N_c\Lambda^2\left[\frac{\widetilde{\Sigma}^2}{G_{S0}} + \frac{(\widetilde{\Pi}^a)^2}{G_{P1}}\right]. \quad (9)$$

Due to quark loops the auxiliary fields obtain kinetic terms and propagate, i.e. interpolate scalar and pseudoscalar resonance states. One fulfills the confinement requirement for a finite number of resonances if one retains only that part of the quark loop which contains the leading logarithm of the cutoff Λ. In this approach one coherently neglects both the threshold part of quark loop and (the infinite number of) heavier resonance poles. This is supported by the large-N_c approximation which associates all momentum dependence in the bosonized action solely with meson resonances.

The actual value of constituent mass $< \Sigma >= \Sigma_0$ in ECQM is described by the mass-gap equation

$$\frac{\Lambda^2}{G_{S0}}\left(\Sigma_0 - M_0\right) = -\frac{\Sigma_0^3}{4\pi^2}\ln\frac{\Lambda^2}{\Sigma_0^2} \equiv \Sigma_0^3 I_0. \quad (10)$$

Therefrom it is evident that the natural scale for the four-fermion interaction is given rather by Σ_0 than by Λ and it is useful to redefine the related coupling constants: $\bar{G}_C = G_C I_0 \frac{\Sigma_0^2}{\Lambda^2}$ for characterizing the weak coupling regime by $\bar{G}_C \ll 1$. The pion decay constant can be found by taking into account the bare pion kinetic term (7)

$$F_0^2 = f_0^2 + N_c\Sigma_0^2 I_0 g_A^2. \quad (11)$$

The pion mass is set by the quark condensate

$$C_q = \left(2c_0 + \frac{N_c}{4\pi^2}\Sigma_0^3\ln\frac{\Lambda^2}{\Sigma_0^2}\right) \equiv -B_0 F_0^2, \quad (12)$$

according to the Gell-Mann-Oakes-Renner formula, $m_\pi^2 = 2m_q B_0$ and the masses of the u, d quarks are taken equal for simplicity.

Respectively the heavy Π mass is found to be

$$m_{\Pi}^2 = \frac{2\Sigma_0^2}{\delta^2}(\frac{1}{\bar{G}_P} + 1), \quad \delta \equiv \frac{f_0}{F_0}. \tag{13}$$

The scalar meson mass is obtained in the form

$$m_{\sigma}^2 = 2\Sigma_0^2(\frac{1}{\bar{G}_S} + 3). \tag{14}$$

3 Non-chiral quasilocal quark models of NJL type

The quasilocal approach of [4, 6] represents a generalization of the NJL model where many-fermion vertices with derivatives are incorporated with the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the perturbative QCD effective action. Gluons themselves are not explicitly included. Rather the effect of low energy gluons is contained in a number of parameters.

For sufficiently strong couplings, the new operators promote the formation of additional scalar and pseudoscalar states. These models allow an extension of the linear σ model, with the pion being a broken symmetry partner of the lightest scalar meson just as in NJL model, and with excited pions and scalar particles coming in pairs. In particular, when only scalar and pseudoscalar color-singlet channels are examined one derive the minimal two-channel lagrangean in the separable form:

$$\mathcal{L}^{QQM} = \bar{q}i\hat{\partial}q + \frac{1}{4N_f N_c \Lambda^2}\sum_{k,l=1}^{2} a_{kl}\left[\bar{q}f_k(\hat{s})q\,\bar{q}f_l(\hat{s})q - \bar{q}f_k(\hat{s})\tau^a\gamma_5 q\,\bar{q}f_l(\hat{s})\tau^a\gamma_5 q\right], \tag{15}$$

where $\hat{\partial} \equiv \gamma^\mu\partial_\mu$, a_{kl} represents a symmetric matrix of real coupling constants and polynomial formfactors are chosen as follows:

$$f_1(\hat{s}) = 2 - 3\hat{s}\,; \qquad f_2(\hat{s}) = -\sqrt{3}\hat{s}\,; \qquad \hat{s} \equiv -\frac{\partial^2}{\Lambda^2}\,. \tag{16}$$

As this model interpolates the low-energy QCD action it is supplied with the cutoff $\Lambda \sim 1$ GeV which bounds virtual quark momenta in quark loops. We restrict ourselves with consideration of two-flavor case, thus τ_a denote Pauli matrices.

The evaluation of physical characteristics in the QQM of NJL type is similarly based on the introduction of auxiliary scalar and pseudoscalar fields:

$$\mathcal{L}_{4-quark} = \sum_{k=1}^{2} i\bar{q}\left(\sigma_k + i\gamma_5\pi_k^a\tau^a\right)f_k(\hat{s})q + N_f N_c\Lambda^2\sum_{k,l=1}^{2}\left(\sigma_k a_{kl}^{-1}\sigma_l + \pi_k^a a_{kl}^{-1}\pi_l^a\right). \tag{17}$$

Let us parametrize the matrix of coupling constants in a close vicinity of tricritical point:

$$8\pi^2 a_{kl}^{-1} = \delta_{kl} - \frac{\Delta_{kl}}{\Lambda^2}, \qquad |\Delta_{kl}| \ll \Lambda^2. \tag{18}$$

The last inequality turns out to be equivalent to require the dynamical mass to be essentially less than the cutoff.

After integrating out the quark fields one comes to the bosonic effective action $\mathcal{W}(\sigma_k, \pi_k^a)$ and, for constant meson variables, to the effective potential. The conditions on extremum of the effective potential, the mass-gap equations,

$$\begin{aligned}
\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2 &= 16\sigma_1^3 \ln\frac{\Lambda^2}{4\sigma_1^2} - \frac{159}{4}\sigma_1^3 - \frac{15\sqrt{3}}{4}\sigma_1^2\sigma_2 + \frac{9}{4}\sigma_1\sigma_2^2 + \frac{\sqrt{3}}{4}\sigma_2^3; \\
\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2 &= -\frac{5\sqrt{3}}{4}\sigma_1^3 + \frac{9}{4}\sigma_1^2\sigma_2 + \frac{3\sqrt{3}}{2}\sigma_1\sigma_2^2 + \frac{9}{4}(\sigma_2)^3,
\end{aligned} \tag{19}$$

allow to find the relations between the components of dynamical mass function and (reduced) coupling constants Δ_{kl}. In practice, one uses the v.e.v.'s of scalar fields as input parameters, in particular, $2\sigma_1 = m_{dyn} = 200 \div 300$ MeV, and determines the required Δ_{kl}. One recovers two isosinglet states in the scalar channel and two triplet states in the pseudoscalar one. The lightest multiplet consists of the massless pion, $m_\pi = 0$ and the NJL scalar meson, $m_\sigma = 4\sigma_1 = 2m_{dyn}$ in the leading-log approach (which status is the same as in ECQM, see above). The masses of heavier mesons depend essentially on the pattern of CSB in the vicinity of tricritical point. But in the leading-log approach they are approximately equal, $m_{\sigma'}^2 \sim m_{\pi'}^2 \sim -\frac{4}{3}\Delta_{22} = \mathcal{O}(\log\Lambda)$. The last estimation follows from the mass-gap eqs.(19). A more precise relation takes places:

$$m_{\pi'}^2 \simeq -\frac{4}{3}\Delta_{22} + \sigma_1^2 + \frac{2\sqrt{3}}{3}\sigma_1\sigma_2 + 3\sigma_2^2 \gg m_{\sigma'}^2 - m_{\pi'}^2 \simeq 2\sigma_1^2 + \frac{4\sqrt{3}}{3}\sigma_1\sigma_2 + 6\sigma_2^2 > 0. \tag{20}$$

4 Chiral symmetry restoration constraints

Let us exploit the constraints based on chiral symmetry restoration at QCD at high energies. We focus on two-point correlators of colorless quark currents

$$\Pi_C(p^2) = \int d^4x \exp(ipx)\langle T\,(\bar{q}\Gamma q(x)\bar{q}\Gamma q(0))\rangle, \tag{21}$$

where $c = S, P$ and respectively $\Gamma = 1, i\gamma_5$. In the chiral limit the scalar correlator and the pseudoscalar one coincide at all orders in perturbation theory and also at leading order in the non-perturbative O.P.E.[7, 3, 8]. As the above differences decrease rapidly with increasing momenta, one can expect that the lowest lying resonances included into ECQM successfully saturate the constraints from CSB restoration.

For ECQM and QQM, in the scalar channel, one obtains the following sum rules

$$\sum Z_\sigma^{(n)} = Z_\pi + Z_\Pi, \qquad Z_\pi = 4B_0^2F_0^2, \tag{22}$$

$$\sum Z_\sigma^{(n)} m_{n,\sigma}^2 - Z_\Pi m_\Pi^2 \simeq 24\pi\alpha_s C_q^2 \sim 0, \tag{23}$$

where $Z_\sigma^{(n)}, Z_\pi, Z_\Pi$ stand for the residues in resonance pole contributions in the scalar and pseudoscalar correlators. In the ECQM one reveals only one scalar meson, $n = 1$; $Z_\sigma^{(1)} \equiv Z_\sigma$ and in the QQM one

has two scalar mesons, $n = 1, 2$; $Z_\sigma^{(2)} \equiv Z'_\sigma$. The last relation is essentially saturated by heavy pion and heavy scalar parameters.

The fit of these CSR rules shows [3, 5] that in the ECQM they can be satisfied in the weak coupling regime of QCD as the 4-quark condensate correction is nearly irrelevant, being amount to 2% of main contribution from the heavy pion. We take $F_0 = 90$ MeV, $m_\pi^2 = 140$ MeV. We adopt [9] $\hat{m}_q(1\text{ GeV}) \simeq 6$ MeV, $B_0(1\text{ GeV}) \simeq 1.5$ GeV, and engage the phenomenological value for the heavy pion mass $m_\Pi \simeq 1.3$ GeV [10]. An optimal fit brings $m_\sigma \simeq 1$ GeV, $g_A = 0.6$ and $\Sigma_0 \simeq 200$ MeV. Therefrom one derives that the bare pion coupling $f_0 \simeq 62$ MeV and for the rest of the parameters we find: $\bar{G}_S \simeq 0.11$, $\bar{G}_P \simeq 0.13$. We see that indeed the four fermion coupling constants $\bar{G}_S$ and $\bar{G}_P$ are slightly different and their values $\ll 1$ signifying the weak coupling regime.

In turn, in the QQM one finds that the residues in poles are of different order of magnitude:

$$Z_\sigma \sim Z_\pi = \mathcal{O}\left(\frac{Z_0}{\ln\frac{\Lambda^2}{4\sigma_1^2}}\right) \ll Z_{\sigma'} \sim Z_{\pi'} = \mathcal{O}\left(Z_0\right) . \tag{24}$$

Then the leading asymptotics (22) represents the generalized σ-model relation and is automatically fulfilled.

As to the second constraint the possibility to satisfy it depends on the value of the QCD coupling constants α_s. Eventually it can be written as follows:

$$\begin{aligned} m_{\sigma'}^2 - m_{\pi'}^2 &\simeq \frac{\Delta_{SP}}{Z_0}; \\ \sigma_1^2 + \frac{2\sqrt{3}}{3}\sigma_1\sigma_2 + 3\sigma_2^2 &\simeq \frac{3N_c\alpha_s}{8\pi}(\sigma_1 - \sqrt{3}\sigma_2)^2. \end{aligned} \tag{25}$$

As the left part is always positive there exists a lower bound for $\alpha_s \geq \frac{8\pi}{9N_c}$ providing solutions of the constraint. The lowest value of $\alpha_s \simeq 0.9$ is given by $\sigma_1 = -\sqrt{3}\sigma_2$ and for these v.e.v.'s one obtains the following splitting between the σ'- and π'-meson masses:

$$m_{\sigma'}^2 - m_{\pi'}^2 \simeq \frac{8}{3}\sigma_1^2 = \frac{1}{6}m_\sigma^2; \tag{26}$$

i.e. for $m_\sigma = 500 \div 600 MeV$ these masses practically coincide, $m_{\sigma'} \simeq m_{\pi'} = 1300 MeV$ and such a σ'-meson may be identified [10] with $f_0(1300)$. The above value of α_s lies in the region of rather strong coupling where next-to-leading corrections to the anomalous dimension of four-quark operator are not negligible, $\sim \frac{\alpha_s}{\pi} \sim 0.3$ and should be systematically taken into account to obtain a reasonable precision.

5 Conslusions

We have shown that both the Extended Chiral Quark Model and the Quasilocal Quark Models truncating low-energy QCD effective action can serve to describe the physics of heavy meson resonances. The

matching to nonperturbative QCD based on the chiral symmetry restoration at high energies improves the predictability of both models. In the ECQM an optimal fit supports the existence of rather heavy scalar quarkonium with the mass of order 1 Gev. Meanwhile QQM extends the NJL model and inevitably contains a rather light scalar meson which is however not excluded by the particle phenomenology [10] though this light meson is most probably a 2-pion bound state [11].

Some disadvantage of QQM as well as of the original NJL model is that they presuppose the large, critical values of four-quark coupling constants which is difficult to justify with perturbative calculations in QCD. As well the CSR matching has to be performed at a scale where the QCD coupling constant is rather large and the perturbation theory is unreliable. The Extended Chiral Quark Models seem to be free of these shortcomings. However the final choice between them may be done by the fit of a larger variety of meson characteristics which is in progress.

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