Approximate Analytical Solutions of RG Equations in MSSM

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Abstract

Simple approximate analytical formulae as solutions for RGE's for Yukawa couplings in MSSM are presented. An accuracy of few per cent is achieved for a wide range of boundary conditions. An application to solving analytically the equations for masses and soft SUSY breaking parameters in MSSM involving the so called Grassmannian expansion is given.

1 Introduction

The renormalization group equations (RGE) for the rigid couplings and soft parameters in SUSY gauge theories play a crucial role in applications. Actually, all predictions of the MSSM are based on solutions to these equations in leading and next-to-leading orders [1].

Typically, one has three gauge couplings, one or three Yukawa couplings (for the case of low or high $\tan\beta$, respectively and taking into account only the 3rd generation) and a set of soft couplings. In leading order, solutions to the RGE for the gauge couplings are simple, however, already for Yukawa couplings, they are known in an analytical form only for the low $\tan\beta$ case, where only the top coupling is left. In a recent paper [2], it has been shown that solutions to the RGE for the soft couplings follow from those for the rigid ones in a straightforward way. One takes the solution for the rigid coupling (gauge or Yukawa), substitute instead of the initial conditions their modified expressions [3]

$$\alpha_i \quad \Rightarrow \quad \tilde{\alpha}_i = \alpha_i(1 + M_i\eta + \bar{M}_i\bar{\eta} + 2M_i\bar{M}_i\eta\bar{\eta}), \tag{1}$$

$$Y_k \quad \Rightarrow \quad \tilde{Y}_k = Y_k(1 - A_k\eta - \bar{A}_k\bar{\eta} + A_k\bar{A}_k\eta\bar{\eta} + \Sigma_k\eta\bar{\eta}). \tag{2}$$

where $\eta = \theta^2, \bar{\eta} = \bar{\theta}^2$, θ and $\bar{\theta}$ are the Grassmannian parameters, and expand over these parameters. This gives the solution to the RGEs for the soft couplings. Hereafter the following notation is used:

$$\alpha_i \equiv \frac{g_i^2}{16\pi^2}, \quad Y_k \equiv \frac{y_k^2}{16\pi^2}, \quad \Sigma_k = \sum_{j=1}^{3} m^2_{j(k)}\,.$$

where g_i and y_k are the gauge and Yukawa couplings, respectively, and m_j^2 are the soft masses associated with each scalar field.

The procedure described above, however, assumes that one knows solutions to the RGE for the rigid couplings in the analytic form. At the same time, in many cases such solutions are unknown. Actual examples are the MSSM with high $\tan\beta$ and NMSSM. Below we propose simple analytical formulae which give an approximate solution to the RGE for Yukawa couplings in an arbitrary SUSY theory with the accuracy of a few per cent. Performing the Grassmannian expansion in these approximate solutions one can get those for the soft couplings in a straightforward way. As an illustration we consider the MSSM in high $\tan\beta$ regime. The approximate solutions obtained in this way possess infrared quasi-fixed points [4] which can be found analytically. They appear in the limit when the initial values of the Yukawa couplings are much larger than those for the gauge ones. Then, one can trace analytically how the initial conditions for the soft terms disappear from their solutions in the above mentioned limit.

2 Approximate analytical solutions in MSSM (high $\tan\beta$ case)

Consider the MSSM in the high $\tan\beta$ regime. One has to take into account three gauge and three Yukawa coupling (belonging to 3rd generation). The one-loop RG equations are

$$\dot{\alpha}_i \quad = \quad -b_i\alpha_i^2, \qquad b_i = (\frac{33}{5}, 1, -3), \quad i = 1, 2, 3.$$

$$
\begin{aligned}
\dot{Y}_t &= Y_t\left(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1 - 6Y_t - Y_b\right), \\
\dot{Y}_b &= Y_b\left(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{7}{15}\alpha_1 - Y_t - 6Y_b - Y_\tau\right), \\
\dot{Y}_\tau &= Y_\tau\left(3\alpha_2 + \frac{9}{5}\alpha_1 - 3Y_b - 4Y_\tau\right).
\end{aligned}
$$

with the initial conditions: $\alpha_i(0) = \alpha_0$, $Y_{t,b,\tau}(0) = Y_{(t,b,\tau)0}$ and $t = \ln(M_X^2/Q^2)$ (dots in l.h.s. of the equations above denote derivative on t).

If one keeps only top-Yukawa coupling then an exact analytical solution is known [5]

$$
\alpha_i(t) = \frac{\alpha_0}{1 + b_i\alpha_0 t}, \quad Y_t(t) = \frac{Y_0 E_t(t)}{1 + 6Y_0 F_t(t)}, \tag{3}
$$

where

$$
E_t(t) = \prod_i (1 + b_i\alpha_0 t)^{c_i/b_i}, \quad c_i = (\frac{13}{15}, 3, \frac{16}{3}), \quad F_t(t) = \int_0^t E_t(t')dt'. \tag{4}
$$

In the literature one can also meet attempts of analytical solutions for the case when τ-Yukawa is dropped [6, 7]. For the general case the exact analytical solution is absent or might be cumbersome.

2.1 The Choice of Approximate Solution

We look for solution in a simple form similar to (3). In choosing approximate solutions we follow the idea of Ref.[7] where an approximation for Y_t and Y_b ignoring Y_τ has been proposed in the form (updated to fit our notations)

$$
Y_{t,b}(t) = Y_{(t,b)0} E_t(t) / \left[1 + 6F_t(t)(Y_{t0} + Y_{b0})\right]^{1/6} \left[1 + 6F_t(t) Y_{(t,b)0}\right]^{5/6} \tag{5}
$$

Our suggestion is to go further and consider separate brackets for each propagator entering the Yukawa vertex. Then, one has the following expressions for the Yukawa couplings:

$$
\begin{aligned}
Y_t &= \frac{Y_{t0} E_t}{[1 + A(Y_{t0}F_t + Y_{b0}F_b)]^{\frac{1}{A}} [1 + 2BY_{t0}F_t]^{\frac{1}{B}} [1 + 3CY_{t0}F_t]^{\frac{1}{C}}}, \quad \frac{1}{A} + \frac{1}{B} + \frac{1}{C} = 1 \\
Y_b &= \frac{Y_{b0} E_b}{[1 + A(Y_{t0}F_t + Y_{b0}F_b)]^{\frac{1}{A}} [1 + 2BY_{b0}F_b]^{\frac{1}{B}} [1 + C(3Y_{b0}F_b + Y_{\tau 0}F_\tau)]^{\frac{1}{C}}}, \\
Y_\tau &= \frac{Y_{\tau 0} E_\tau}{[1 + A'Y_{\tau 0}F_\tau]^{\frac{1}{A'}} [1 + 2B'Y_{\tau 0}F_\tau]^{\frac{1}{B'}} [1 + C(3Y_{b0}F_b + Y_{\tau 0}F_\tau)]^{\frac{1}{C}}}, \quad \frac{1}{A'} + \frac{1}{B'} + \frac{1}{C} = 1
\end{aligned}
$$

where the brackets correspond to the Q, U, H_2, Q, D, H_1 and L, E, H_1 propagators, respectively. Here E_t and F_t are given by eqs. (4), and E_b and E_τ have the same form, but with $c_i^{(b)} = (7/15, 3, 16/3)$ and $c_i^{(\tau)} = (9/5, 3, 0)$, respectively. The brackets are organized so that they reproduce the contributions of particular diagrams to the corresponding anomalous dimensions. We introduced E and F–like functions for each Yukawa in order to preserve the idea and the way one gets the exact solution (3). The coefficients A, B, C, A' and B' are arbitrary and their precise values are not so important. Note that when Yukawa couplings Y_{i0} are small enough, one can make expansion in each bracket, and the dependence in these coefficients disappears. However, for large couplings, which are of interest for us because of the fixed points, we have some residual dependence. The requirement that the sum of exponents equals 1 follows from comparison with RGE. According to (5), one should expect the approximation to work well when the brackets are roughly equal to each other. Apparently, since $F_\tau < F_t \sim F_b$ and $Y_\tau \leq Y_b \leq Y_t$ one cannot completely satisfy this requirement. Our choice of the coefficients A, B, C, A' and B' is dictated mainly by simplicity. In the following we choose them as

$$
\begin{aligned}
B = A, C = 2/3A \quad &\rightarrow \quad A = 7/2,\ B = 7/2,\ C = 7/3, \\
B' = A'/2 \quad &\rightarrow \quad A' = 21/4,\ B' = 21/8.
\end{aligned}
$$

This gives approximate solutions as

$$Y_t \approx \frac{Y_{t0}E_t}{\left(1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)\right)^{2/7}(1+7Y_{t0}F_t)^{5/7}}, \tag{6}$$

$$Y_b \approx \frac{Y_{b0}E_b}{\left(1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)\right)^{2/7}(1+7Y_{b0}F_b)^{2/7}\left(1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)\right)^{3/7}} \tag{7}$$

$$Y_\tau \approx \frac{Y_{\tau 0}E_\tau}{\left(1+\frac{21}{4}Y_{\tau 0}F_\tau\right)^{4/7}\left(1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)\right)^{3/7}}. \tag{8}$$

To get the solutions for the soft terms it is enough to perform the substitution (1,2) and expand over η and $\bar{\eta}$ in the analytical solutions (6–8) for the Yukawa couplings. To get the solution for the Σ term one has to make expansion up to $\eta\bar{\eta}$. This can be done with the help of the following relations

$$\left.\frac{d\tilde{E}}{d\eta}\right|_\eta = M_0 t\frac{dE}{dt}, \quad \left.\frac{d\tilde{F}}{d\eta}\right|_\eta = M_0(tE-F) \quad \left.\frac{d^2\tilde{E}}{d\eta d\bar{\eta}}\right|_{\eta,\bar{\eta}} = M_0^2\frac{d}{dt}\left(t^2\frac{dE}{dt}\right), \quad \left.\frac{d^2\tilde{F}}{d\eta d\bar{\eta}}\right|_{\eta,\bar{\eta}} = M_0^2 t^2\frac{dE}{dt} \tag{9}$$

and leads to the formulae given in Appendix. We took $M_{i0} = m_{1/2}$ the common GUT value for gaugino's masses. Initial conditions were taken to be $A_i(0) = A_0$, $\Sigma_i(0) = \Sigma_0$ just for the sake of simplicity (case of universality). As one can see, the non-universality of boundary conditions for A's and Σ's can be easily taken into account by modifying relations (9). The corresponding solutions for the individual soft masses can be obtained by expressing them through Σ's in an exact way. We present the explicit expressions for the soft terms and masses in Appendix.

2.2 Numerical Analysis

We start by investigating the precision of approximate solutions for the Yukawa couplings. To estimate the accuracy, we introduce a relative error which is defined as

$$\varepsilon = \frac{Y_{approx} - Y_{numeric}}{Y_{numeric}}, \tag{10}$$

and corresponds to the M_Z scale ($t = 66$) at the end of the integration range. The accuracy for the solutions of soft terms is defined in the same way.

For the beginning, we take all three Yukawa couplings to be equal at the GUT scale and having their common value Y_0 in the range $(.01 \div 25)\alpha_0$. The upper limit is taken in order not to leave the perturbativity regime. We find that for $Y_0 \leq \alpha_0$ the approximation errors are less than 3% for all Y's. While for Y_t it remains smaller than 2% over the whole range of initial values at the GUT scale, for Y_b the error increases up to 4% and for Y_τ up to 14% (for large values of Y_0). It is worth mentioning that for small Y_0 (around $\alpha_0/2$ and below) the accuracy is very good (fractions of per cent or better).

Consider now $Y_{b0} = Y_{\tau 0} = Y_0 \leq 10\alpha_0$ and let the top Yukawa coupling vary within the limits $1 \leq Y_{t0}/Y_0 \leq 10$ in order to examine the applicability of our formulae. In this case the accuracy spoils a little bit with increase of the initial values. Namely, the error for Y_t increases up to 10%, and for Y_b and Y_τ up to 20%. However, if one keeps Y_0 in the range $(\frac{1}{100} \div \frac{1}{2})\alpha_0$ the accuracy for Y_t remains better than 3%, and for Y_b and Y_τ better than 10%. A particular case considered above seems to have the worst accuracy. This is not surprising since our approximate formulae are supposed to work best of all when all three Yukawa couplings are nearly equal. If we keep $Y_{\tau 0} \leq Y_{b0} \leq Y_{t0}$ and the relative ratios less than 5, we get an average error of less than 5% for Y_t, about 5% for Y_b and 10% for Y_τ. Further on, we narrow the range of initial values up to $(1/10 \div 10)\alpha_0$ because the errors (defined as in (10)) come to an asymptotic value for $Y_0 > 10\alpha_0$ and almost vanish for $Y_0 < \alpha_0/10$. The comparison of numerical and approximate solutions is shown in Fig.1 for three different sets of Y_0's.

For the soft couplings, A, we take the initial values at the GUT scale to be $A_0 = (-2, -1, 0, 1, 2)m_{1/2}$ and leave Y_0's in the narrowed range as stated above. Then, we get an accuracy of $(3 \div 5)\%$ for A_t

and A_b. For A_τ the approximation is worse when A_0 is taken to be negative or smaller than $m_{1/2}$, but things go better for large initial values of A_0, and we get an accuracy of about 10%. Again it should be mentioned that this is an accuracy at the end point where A_τ itself is close to 0 and the accuracy defined as (10) gives merely an odd hint about precision. Along the curves the accuracy is much better. In Fig.1 we have plotted the behaviour of A_t, A_b and A_τ for three different initial values of A_0, namely $\{-m_{1/2}, 0, m_{1/2}\}$ and for one set of Y_0's. As for the Σ's, keeping the range of parameter space for Y_0 and A_0 as above, we get an accuracy of typically 2% for Σ_t (even better for almost equal Y_0's). For Σ_b the precision is around 4%. With Σ_τ we get into the same troubles as for A_τ. For it the approximation becomes good (about 10%) only for a large enough ratio of $m_0^2/m_{1/2}^2$. The approximation errors for A's and Σ's are linked with those for Y. If one considers only the sets of small initial values for Y_0 (less than $\alpha_0/2$), then Σ's are approximated with a precision better than 1%, regardless of the A_0 values. Precision for Σ increases with A_0, but this dependence is not so striking as the one on Y_0.

The approximate formulae for the soft masses may be derived from Σ's using eqs.(11-17). In this case the approximate solutions give an accuracy of about $1 \div 3\%$ for $m_{Q_3}^2$, $m_{U_3}^2$ and $3 \div 5\%$ for $m_{D_3}^2$. For the Higgs masses we get a good approximation (of about 5% on the average) for $m_{H_2}^2$, and a satisfactory one for $m_{H_1}^2$ (typically 10%). This accuracy is almost insensitive to the A_0 variation (we took it in the range $(-2 \div 2)m_{1/2}$) and on the ratio $m_0^2/m_{1/2}^2$ (taken to be $.5 \div 2$). The slepton masses are not approximated properly in an analogous way. This is mainly due to a poor approximation of Y_τ. As one can expect, a good analytical approximation for Yukawa couplings would give also a good precision for soft terms and masses.

As a concluding remark on numerical analysis, it should be mentioned that one has a rather good approximation for small (less than $\alpha_0/2$) initial values of the Yukawa couplings. For larger values of Y's one can still have a sufficiently good approximation only in the case of unification of the Yukawa couplings.

2.3 The Fixed Points

One can see that solutions (6-8) exhibit the quasi-fixed point behaviour as mentioned in Introduction, when the initial values $Y_{t0} = Y_{b0} = Y_{\tau 0} \geq \alpha_0$. In this case, one can drop 1 in the denominator and the resulting expressions become independent of the initial conditions

$$Y_t^{FP} \approx \frac{E_t}{\left(\frac{7}{2}(F_t+F_b)\right)^{\frac{2}{7}}(7F_t)^{\frac{5}{7}}},\ Y_b^{FP} \approx \frac{E_b}{\left(\frac{7}{2}(F_t+F_b)\right)^{\frac{2}{7}}(7F_b)^{\frac{2}{7}}\left(\frac{7}{3}(3F_b+F_\tau)\right)^{\frac{3}{7}}},\ Y_\tau^{FP} \approx \frac{E_\tau}{\left(\frac{21}{4}F_\tau\right)^{\frac{4}{7}}\left(\frac{7}{3}(3F_b+F_\tau)\right)^{\frac{3}{4}}}.$$

These expressions being expanded over the Grassmannian variables give the quasi-fixed points for the soft terms and masses. This is illustrated in Fig.1 We see that the IRQFP behaviour is sharply expressed for Y_t and Y_b (see Fig. 1), and our approximate solution describes the fixed point line well. The same takes place for the corresponding A's and Σ's. For Y_τ, A_τ and Σ_τ the accuracy is worse, however, the solution is still reliable. The soft mass terms there exhibit the same IRQFP behaviour, though some residual dependence on the initial conditions is left in full analogy with the exact solutions in the low $\tan\beta$ case. The approximate solutions allow one to calculate the IRQFP analytically.

One can see that the fixed points for the soft terms naturally follow from the Grassmannian expansion of our approximate solutions and inherit their stability properties, as has been shown in Ref.[8]. The behaviour of Σ's essentially repeats that of the Yukawa couplings in agreement with Ref.[9].

3 Discussion

We hope to convince the reader that the approximate solutions presented above reproduce the behaviour of the Yukawa couplings with good precision in the whole integration region. Relative accuracy is typically a few per cent and is worse only at the end of the integration region mainly due to the smallness of the quantities themselves. Moreover, we have shown how the approximate solutions for the soft terms and masses follow from those for the rigid couplings. This demonstrates how the Grassmannian expansion, advocated in Ref.[2], works in the case of approximate solutions as well.

Since the form of our approximate solutions has been "guessed" ad hoc starting from some reasonable arguments, there is no direct way to improve them. However, one can imagine more constructive derivation of those solutions which would allow one to make corrections. Needless to say that it is enough to construct a solution for the rigid terms. Solutions for the soft terms will follow automatically.

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Appendix

We present here approximate expressions for the soft couplings and masses corresponding to eqs.(6-8).

$$A_t \approx A_0\left(1-\frac{Y_{t0}F_t+Y_{b0}F_b}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}-\frac{5Y_{t0}F_t}{1+7Y_{t0}F_t}\right)-m_{1/2}\left(\frac{t}{E_t}\frac{dE_t}{dt}-\frac{Y_{t0}(tE_t-F_t)+Y_{b0}(tE_b-F_b)}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}-\frac{5Y_{t0}(tE_t-F_t)}{1+7Y_{t0}F_t}\right)$$

$$A_b \approx A_0\left(1-\frac{Y_{t0}F_t+Y_{b0}F_b}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}-\frac{2Y_{b0}F_b}{1+7Y_{b0}F_b}-\frac{3Y_{b0}F_b+Y_{\tau 0}F_\tau}{1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)}\right)$$
$$-\ m_{1/2}\left(\frac{t}{E_b}\frac{dE_b}{dt}-\frac{Y_{t0}(tE_t-F_t)+Y_{b0}(tE_b-F_b)}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}-\frac{2Y_{b0}(tE_b-F_b)}{1+7Y_{b0}F_b}-\frac{3Y_{b0}(tE_b-F_b)+Y_{\tau 0}(tE_\tau-F_\tau)}{1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)}\right)$$

$$A_\tau \approx A_0\left(1-\frac{3Y_{\tau 0}F_\tau}{1+\frac{21}{4}Y_{\tau 0}F_\tau}-\frac{3Y_{b0}F_b+Y_{\tau 0}F_\tau}{1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)}\right)-m_{1/2}\left(\frac{t}{E_\tau}\frac{dE_\tau}{dt}-\frac{3Y_{\tau 0}(tE_\tau-F_\tau)}{1+\frac{21}{4}Y_{\tau 0}F_\tau}-\frac{3Y_{b0}(tE_b-F_b)+Y_{\tau 0}(tE_\tau-F_\tau)}{1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)}\right)$$

$$\Sigma_t \approx \Sigma_0\left(1-\frac{Y_{t0}F_t+Y_{b0}F_b}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}-\frac{5Y_{t0}F_t}{1+7Y_{t0}F_t}\right)$$
$$-\ \frac{A_0^2Y_{t0}F_t-2A_0Y_{t0}m_{1/2}(tE_t-F_t)+Y_{t0}m_{1/2}^2t^2\frac{dE_t}{dt}+A_0^2Y_{b0}F_b-2A_0Y_{b0}m_{1/2}(tE_b-F_b)+Y_{b0}m_{1/2}^2t^2\frac{dE_b}{dt}}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}$$
$$+\ \frac{\frac{7}{2}\left(-A_0Y_{t0}F_t+Y_{t0}m_{1/2}(tE_t-F_t)-A_0Y_{b0}F_b+Y_{b0}m_{1/2}(tE_b-F_b)\right)^2}{\left(1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)\right)^2}+m_{1/2}^2\frac{d}{dt}\left(\frac{t^2}{E_t}\frac{dE_t}{dt}\right)$$
$$-\ \frac{5\left(A_0^2Y_{t0}F_t-2A_0Y_{t0}m_{1/2}(tE_t-F_t)+Y_{t0}m_{1/2}^2t^2\frac{dE_t}{dt}\right)}{1+7Y_{t0}F_t}+\frac{35\left(-A_0Y_{t0}F_t+Y_{t0}m_{1/2}(tE_t-F_t)\right)^2}{(1+7Y_{t0}F_t)^2}$$

$$\Sigma_b \approx \Sigma_0\left(1-\frac{Y_{t0}F_t+Y_{b0}F_b}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}-\frac{2Y_{b0}F_b}{1+7Y_{b0}F_b}-\frac{3Y_{b0}F_b+Y_{\tau 0}F_\tau}{1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)}\right)$$
$$-\ \frac{A_0^2Y_{t0}F_t-2A_0Y_{t0}m_{1/2}(tE_t-F_t)+Y_{t0}m_{1/2}^2t^2\frac{dE_t}{dt}+A_0^2Y_{b0}F_b-2A_0Y_{b0}m_{1/2}(tE_b-F_b)+Y_{b0}m_{1/2}^2t^2\frac{dE_b}{dt}}{1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)}$$
$$-\ \frac{2}{1+7Y_{b0}F_b}\left(A_0^2Y_{b0}F_b-2A_0Y_{b0}m_{1/2}(tE_b-F_b)+Y_{b0}m_{1/2}^2t^2\frac{dE_b}{dt}\right)+m_{1/2}^2\frac{d}{dt}\left(\frac{t^2}{E_b}\frac{dE_b}{dt}\right)$$
$$-\ \frac{3A_0^2Y_{b0}F_b+A_0^2Y_{\tau 0}F_\tau-6A_0Y_{b0}m_{1/2}(tE_b-F_b)-2A_0Y_{\tau 0}m_{1/2}(tE_\tau-F_\tau)+3Y_{b0}m_{1/2}^2t^2\frac{dE_b}{dt}+Y_{\tau 0}m_{1/2}^2t^2\frac{dE_\tau}{dt}}{1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)}$$
$$+\ \frac{\frac{7}{2}\left(-A_0Y_{t0}F_t+Y_{t0}m_{1/2}(tE_t-F_t)-A_0Y_{b0}F_b+Y_{b0}m_{1/2}(tE_b-F_b)\right)^2}{\left(1+\frac{7}{2}(Y_{t0}F_t+Y_{b0}F_b)\right)^2}+\frac{14\left(Y_{b0}m_{1/2}(tE_b-F_b)-A_0Y_{b0}F_b\right)^2}{(1+7Y_{b0}F_b)^2}$$
$$+\ \frac{\frac{7}{3}\left(-3A_0Y_{b0}F_b-A_0Y_{\tau 0}F_\tau+3Y_{b0}m_{1/2}(tE_b-F_b)+Y_{\tau 0}m_{1/2}(tE_\tau-F_\tau)\right)^2}{\left(1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)\right)^2}$$

$$\Sigma_\tau \approx \Sigma_0\left(1-\frac{3Y_{\tau 0}F_\tau}{1+\frac{21}{4}Y_{\tau 0}F_\tau}-\frac{3Y_{b0}F_b+Y_{\tau 0}F_\tau}{1+\frac{7}{3}(3Y_{b0}F_b+Y_{\tau 0}F_\tau)}\right)-\frac{3\left(A_0^2Y_{\tau 0}F_\tau-2A_0Y_{\tau 0}m_{1/2}(tE_\tau-F_\tau)+Y_{\tau 0}m_{1/2}^2t^2\frac{dE_\tau}{dt}\right)}{1+\frac{21}{4}Y_{\tau 0}F_\tau}$$

$$- \frac{3A_0^2 Y_{b0} F_b + A_0^2 Y_{\tau 0} F_\tau - 6A_0 Y_{b0} m_{1/2}(tE_b - F_b) + 3Y_{b0} m_{1/2}^2 t^2 \frac{dE_b}{dt} - 2A_0 Y_{\tau 0} m_{1/2}(tE_\tau - F_\tau) + Y_{\tau 0} m_{1/2}^2 t^2 \frac{dE_\tau}{dt}}{1 + \frac{7}{3}(3Y_{b0} F_b + Y_{\tau 0} F_\tau)}$$

$$+ \frac{\frac{63}{4}\left(-A_0 Y_{\tau 0} F_\tau + Y_{\tau 0} m_{1/2}(tE_\tau - F_\tau)\right)^2}{\left(1 + \frac{21}{4} Y_{\tau 0} F_\tau\right)^2} + m_{1/2}^2 \frac{d}{dt}\left(\frac{t^2}{E_\tau}\frac{dE_\tau}{dt}\right)$$

$$+ \frac{\frac{7}{3}\left(-3A_0 Y_{b0} F_b - A_0 Y_{\tau 0} F_\tau + 3Y_{b0} m_{1/2}(tE_b - F_b) + Y_{\tau 0} m_{1/2}(tE_\tau - F_\tau)\right)^2}{\left(1 + \frac{7}{3}(3Y_{b0} F_b + Y_{\tau 0} F_\tau)\right)^2}$$

To find the individual soft masses one can formally perform integration of the RG equations and express the masses through Σ's solving a system of linear algebraic equations. This gives

$$m_{Q_3}^2 = \frac{13}{61} m_0^2 + m_{1/2}^2\left(\frac{64}{61} f_3 + \frac{87}{122} f_2 - \frac{11}{122} f_1\right) + \frac{1}{122}\left(17\Sigma_t + 20\Sigma_b - 5\Sigma_\tau\right) \quad (11)$$

$$m_{U_3}^2 = \frac{7}{61} m_0^2 + m_{1/2}^2\left(\frac{72}{61} f_3 - \frac{54}{61} f_2 + \frac{72}{305} f_1\right) + \frac{1}{122}\left(42\Sigma_t - 8\Sigma_b + 2\Sigma_\tau\right) \quad (12)$$

$$m_{D_3}^2 = \frac{19}{61} m_0^2 + m_{1/2}^2\left(\frac{56}{61} f_3 - \frac{42}{61} f_2 + \frac{56}{305} f_1\right) + \frac{1}{122}\left(-8\Sigma_t + 48\Sigma_b - 12\Sigma_\tau\right) \quad (13)$$

$$m_{H_1}^2 = -\frac{32}{61} m_0^2 + m_{1/2}^2\left(-\frac{120}{61} f_3 - \frac{3}{122} f_2 - \frac{57}{610} f_1\right) + \frac{1}{122}\left(-9\Sigma_t + 54\Sigma_b + 17\Sigma_\tau\right) \quad (14)$$

$$m_{H_2}^2 = -\frac{20}{61} m_0^2 + m_{1/2}^2\left(-\frac{136}{61} f_3 + \frac{21}{122} f_2 - \frac{89}{610} f_1\right) + \frac{1}{122}\left(63\Sigma_t - 12\Sigma_b + 3\Sigma_\tau\right) \quad (15)$$

$$m_{L_3}^2 = \frac{31}{61} m_0^2 + m_{1/2}^2\left(\frac{40}{61} f_3 + \frac{123}{122} f_2 - \frac{103}{610} f_1\right) + \frac{1}{122}\left(3\Sigma_t - 18\Sigma_b + 35\Sigma_\tau\right) \quad (16)$$

$$m_{E_3}^2 = \frac{1}{61} m_0^2 + m_{1/2}^2\left(\frac{80}{61} f_3 - \frac{60}{61} f_2 + \frac{16}{61} f_1\right) + \frac{1}{122}\left(6\Sigma_t - 36\Sigma_b + 70\Sigma_\tau\right) \quad (17)$$

$$M_i = \frac{m_{1/2}}{1 + b_i \alpha_0 t}, \quad f_i = \frac{1}{b_i}\left(1 - \frac{1}{(1 + b_i \alpha_0 t)^2}\right)$$

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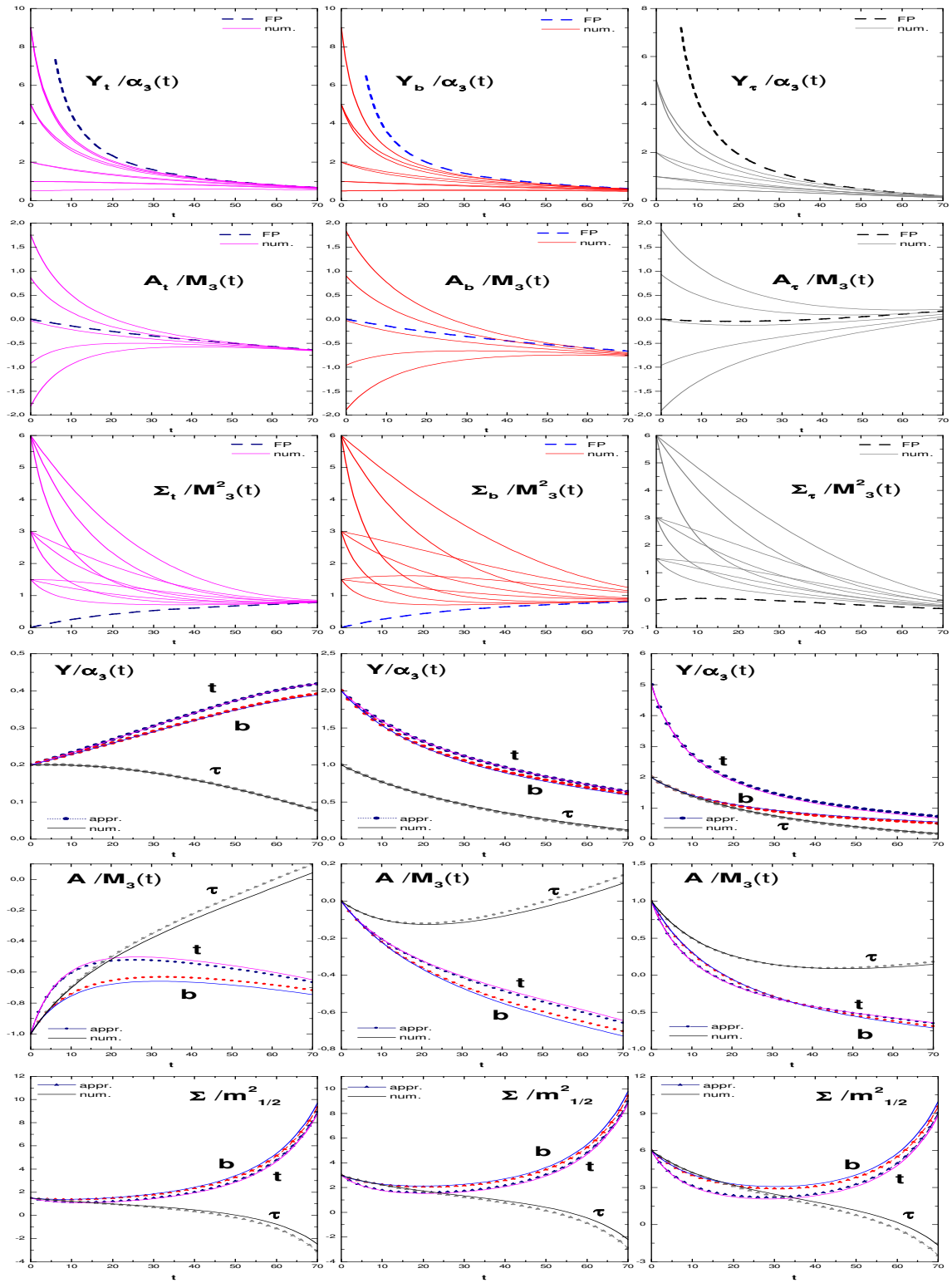

Figure 1: *On the top*: the IRQFP behaviour for Yukawa couplings, soft terms $A_{t,b,\tau}$ and Σ's. Numerical solutions for all A's are done for $Y_{t0} = 5\alpha_0$,$Y_{b0} = Y_{\tau 0} = 2\alpha_0$. For Σ we took $A_0 = 0$, three different sets of Y_0's and m_0^2. Dotted lines correspond to IRQFP given by analytical solutions. *On the bottom*: Comparison of approximate and numerical solutions for Y, A and Σ. Evolution of the soft terms A and Σ has been plotted for $Y_{t0} = 5\alpha_0\,, Y_{b0} = Y_{\tau 0} = 2\alpha_0$. Dotted lines correspond to analytical approximation, solid lines to numerical solution.