Quark box in pionium decay to $\pi^0\pi^0$

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Abstract

The decay amplitude of pionium into two neutral pions via quark-box mechanism in a phenomenological quark model is considered. The main parameters of the model are the pion-quark couplings, which are fixed by decay of pion on photon pair. With constituent quark masses of 330 MeV for u- and d-quarks we estimate the pionium decay width to be 13eV and lifetime $0.05 \times 10^{-15} s$.

Modern experiments devoted to investigation propeties of hadron $\pi^+\pi^-$ atom (pionium) take place in CERN. It's aim is pionium lifetime measuring with 10% acuracy [1]. The first life time estimation was done in [2] $\tau = 2.9^{\infty}_{-2.1} \cdot 10^{-15} s$. Pionium – hadronic atom $(\pi^+\pi^-)$ – is the simple atom system. But in this system we must take into account both the electromagnetic and strong interactions. The pionium energy spectrum is determined by Coulomb potential, because the bore radius $r_B = 387 fm$ is much more then the effective range of strong interactions. But from the other hand the pionium decay is due to strong interaction. And the main contribution into decay is $\pi^+\pi^- \to \pi^0 + \pi^0$ recharge process. The corresponding decay rate is determined by well known approximate Deser formulae [3]:

$$\Gamma_0 = \frac{16\pi}{9}\sqrt{\frac{\Delta m_\pi}{m_\pi}}\sqrt{1-\frac{\Delta m_\pi}{2m_\pi}}(a_0-a_2)^2|\psi(0)|^2, \tag{1}$$

where $\Delta m_\pi = m_{\pi^+} - m_{\pi^0}$, $\psi(0)$ - wave function value at $r = 0$, a_0, a_2 - $\pi\pi$-scattering lengthes with isospin I=0,2. Theoretical predictions on S-wave $\pi\pi$-scattering lengthes were done in chiral perturbation theory: $a_0 = 0.217$, $a_2 = -0.041$ [4]. Corresponding experimental values are following: $a_0 = 0.26 \pm 0.05$, $a_2 = -0.028 \pm 0.012$ [5]. New experiment DIRAC will allow to determine the difference $(a_0 - a_2)$ with high accuracy. That's why the main theoretical investigation of pionium is connected with strong and electromagnetic corrections to (1). At the same time pionium atom may decay on two photons (but this contribution is rather small).

In chiral perturbation theory the quarks and gluons are integrated out, and hence they do not appear in effective Lagrangians. In this paper we evaluate the decay amplitude of pionium into two neutral pions via quark-box mechanism in a constituent quark model framework.

The pionium decay amplitude into π^0-pair may be presented as multiplication between amplitude $T(\vec{p}, P)$ of $\pi^+ + \pi^- \to \pi^0 + \pi^0$ and quasipotential wave function $\psi_M(\vec{p})$ [6]:

$$F = \int \frac{d^3p}{\sqrt{M}(2\pi)^3}\psi_M(\vec{p})T(\vec{p}, P), \tag{2}$$

where 4-momentums of pi-mesons p_1 and p_2 are expressed through totall 4-momentum $P = p_1 + p_2$ and relative 4-momentum $p = (p_1 - p_2)/2$, M- pionium mass.

We have suggested that the mechanism for decay may be given by the the quark-box diagrams. Only the u- and d-type quarks contribute since the π does not have any other content. To simplify the calculations we shall assume that the u- and d-quarks are degenerate in masses, i.e. $m_u = m_d = m$ and let $m = 330 MeV$ [7]. It is easy to generalize to unequal masses. The strong pion coupling constant $g_{\pi uu} = g_{\pi dd} = \frac{m}{f_\pi} \approx 3.57$, here $f_\pi = 92.4$ MeV [8].

The decay amplitude structure is given by a straightforward evaluation of the Feynman diagrams. First we define the quantities U_i (i=1,...4) as in [7] by

$$T_i = 3g^4_{\pi qq}\int \frac{d^4Q}{(2\pi)^4}U_i \tag{3}$$

and

$$U_1 = 2\frac{Tr[(\hat{Q}+m)\gamma_5(\hat{Q}+\hat{p}_1+m)\gamma_5(\hat{Q}+\hat{p}_1+\hat{p}_2+m)\gamma_5(\hat{Q}+\hat{p}_1+\hat{p}_2+\hat{p}_3+m)\gamma_5]}{[Q^2-m^2][(Q+p_1)^2-m^2][(Q+p_1+p_2)^2-m^2][(Q+p_1+p_2+p_3)^2-m^2]},$$

$$U_2 = \frac{1}{2}U_1(p_1 \leftrightarrow p_4), \quad U_3 = U_1(p_3 \leftrightarrow p_4), \quad U_4 = U_2(p_3 \leftrightarrow p_4).$$

And totall decay amplitude

$$T = \sum_{i=1}^{4} T_i. \tag{4}$$

where $-p_3$, $-p_4$ - 4-momentums of final π^0 mesons.

After Feynman parametrization, trace and analitical evaluation integrals over momentum, we have obtained

$$T = \frac{3im^4}{16\pi^2 f_\pi^4}(-96L_0 - 48N_0 + 4L_2 + 2N_2 + 4L_1 + 2N_1), \tag{5}$$

where L_j, N_j – integrals over Feynman parameters.

$$L_0 = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} \ln L dz dy dx, \quad N_0 = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} \ln N dz dy dx,$$

$$L = x^2 + z^2 + w^2 - 2xz - x - z, \quad N = x^2 + z^2 + w^2 + 2xz - x - z, \quad w = \frac{m}{M_{\pi^\pm}},$$

$$L_2 = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx \frac{L_2(x,y,z)}{L^2}, \quad N_2 = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx \frac{N_2(x,y,z)}{N^2},$$

$$L_2(x,y,z) = 4x^4 - 16x^3z + 24x^2z^2 - 8x^2w^2 - 4x^2 - 16xz^3 + 16xzw^2 + 8xz + 4z^4 - 8z^2w^2 - 4z^2 + w^4 - 4w^2,$$

$$N_2(x,y,z) = 4x^4 + 16x^3z - 8x^3 + 24x^2z^2 - 24x^2z - 8x^2w^2 + 4x^2 + 16xz^3 -$$
$$-24xz^2 - 16xzw^2 + 8xz + 8xw^2 + 4z^4 - 8z^3 - 8z^2w^2 + 4z^2 + 8zw^2 + w^4 + 4w^2,$$

$$L_1 = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx \frac{-4 - 24x^2 + 48xz - 24z^2 + 16w^2}{L},$$

$$N_1 = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx \frac{4 - 24x^2 - 48xz + 24x - 24z^2 + 24z + 16w^2}{N}.$$

Some of the integrals in T are divergent. We handled this by the dimensional regularization. We have got in $\bar{M}S$ renormalization sceme for divergent part $\frac{i}{16\pi^2}\left(\frac{2}{\varepsilon} - \gamma_E - \ln\frac{M^2_{\pi^\pm}}{4\pi\mu^2}\right) \cdot 24$.

Numerical evaluation for T,F, decay width Γ and lifetime τ, $\Gamma = \frac{1}{64\pi M^2_{\pi^\pm}}|T|^2|\psi(0)|^2\sqrt{\frac{2\Delta M}{M_{\pi^\pm}}}, \tau = \Gamma^{-1}$ with additional significances $\Delta M = M_{\pi^\pm} - M_{\pi^0} = 139.57 - 134.98 = 4.59(MeV)$. and initial state wave function $\psi(0) = \frac{1}{\sqrt{4\pi}}\sqrt{\frac{a^3}{2}}$ have gave following results: $\Gamma \approx 13eV, \tau \approx 0.05 \times 10^{-15}s$. Note that our result for lifetime is less than known above written. It seems interesting and have to be check, ofcourse.

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