Two Photon Pionium Decay

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Abstract

We calculate the pionium decay width on two photons on the basis of quasipotential method. The tree level annihilation contribution in scalar electrodynamics and quark loop contribution on the basis of quasipotential quark model are taken into account. Numerical value of the pionium decay width is 0.94 meV.

Last time experiments devoted to the investigation of hadronic $\pi^+\pi^-$ atom (pionium) properties took place at CERN. The aim is pionium life time measurement with 10% accuracy [1]. The first life time estimation was done in [2]: $\tau = 2.9^{\infty}_{-2.1} \cdot 10^{-15}$ s. Hadronic atom $(\pi^+\pi^-)$ is the simple atom system, which gives us an opportunity to study both electromagnetic and strong interactions. The pionium energy spectrum is determined by Coulomb potential, because the Bohr radius $r_B = 387$ fm is much larger than the range of strong interactions. At the same time the pionium decay is due to strong interaction. The main contribution to the decay width is determined by the process of $\pi^+\pi^- \to \pi^0 + \pi^0$ annihilation. The corresponding decay rate is described by well known Deser formula [3]:

$$\Gamma_0 = \frac{16\pi}{9}\sqrt{\frac{\Delta m_\pi}{m_\pi}}\sqrt{1-\frac{\Delta m_\pi}{2m_\pi}}(a_0-a_2)^2|\psi(0)|^2, \tag{1}$$

where $\Delta m_\pi = m_{\pi^+} - m_{\pi^0}$, $\psi(0)$ is the value of pionium wave function at the origin, a_0, a_2 are $\pi\pi$ scattering lengths with isospin I=0,2. Theoretical predictions on S-wave $\pi\pi$ scattering lengths were done in chiral perturbation theory: $a_0 = 0.217$, $a_2 = -0.041$ [4]. Corresponding experimental values are the following: $a_0 = 0.26 \pm 0.05$, $a_2 = -0.028 \pm 0.012$ [5]. Increasing of precision in the measurement of pionium date rate Γ, which is planed in new experiment DIRAC, will allow to determine the difference $(a_0 - a_2)$ with higher accuracy. That's why the main theoretical investigation of pionium was connected with the calculation of strong and electromagnetic corrections to the expression (1) [6-12]. At the same time pionium atom $(J^{PC} = 0^{++})$ may decay on two photons (rare decay). The interest to this decay is connected with the possibility to obtain new information about such fundamental properties of pion as electric (α) and magnetic (β) polarizabilities [13, 14, 15]. The process $\gamma+\gamma \to \pi^+ + \pi^-$ was studied earlier in the effective chiral theory of pion-nucleon interaction [16]. The annihilation of pions $\pi^+ + \pi^- \to \gamma + \gamma$ and Compton pion effect were studied on the basis of Nambu-Iona-Lasinio model [17, 18]. In this work we have calculated the pionium decay rate on two photons in the framework of quasipotential quark model [19, 20, 21].

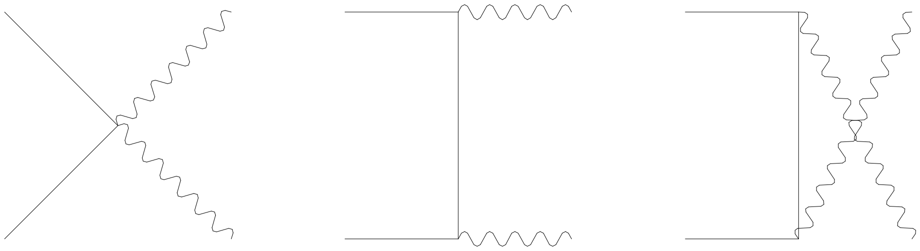
The decay amplitude of pionium on two photons can be presented as the product of an $\pi^+ + \pi^- \to \gamma + \gamma$ annihilation amplitude $M(\vec{p}, P)$ times a quasipotential wave function $\psi_M(\vec{p})$ [22]:

$$F = \int \frac{d^3p}{\sqrt{M}(2\pi)^3}\psi_M(\vec{p})M(\vec{p}, P), \tag{2}$$

where four momenta of π -mesons p_1 and p_2 are expressed through total four momentum $P = p_1 + p_2$ and relative four momentum $p = (p_1 - p_2)/2$, M is the pionium mass. In scalar quantum electrodynamics the main contribution to the amplitude F is determined by Feynman diagrams of fig.1 (a, b, c). Corresponding amplitudes of $\pi^+\pi^-$ annihilation take the following form:

$$M_0^{(a)} = 2e^2 g^{\mu\nu}\epsilon^\mu(k_1)\epsilon^\nu(k_2), \tag{3}$$

$$M_0^{(b)} + M_0^{(c)} = -e^2\left[\frac{(2p_1-k_1)^\mu(p_1-p_2-k_1)^\nu}{(p_1-k_1)^2 - m_\pi^2 + i\epsilon} + \frac{(2p_1-k_2)^\mu(p_1-p_2-k_2)^\nu}{(p_1-k_2)^2 - m_\pi^2 + i\epsilon}\right]\epsilon^\mu(k_1)\epsilon^\nu(k_2), \tag{4}$$



(a) (b) (c)

Figure 1: Two photon pionium annihilation in scalar quantum electrodynamics

where k_1, k_2 are the four momenta of photons, $\epsilon^\mu(k_1)$, $\epsilon^\nu(k_2)$ are the polarization vectors.

The sum of amplitudes (3), (4) contains tensor structure $(g^{\mu\nu} - p_1^\mu p_2^\nu / p_1 k_1 - p_1^\nu p_2^\mu / p_2 k_1)$ and is gauge invariant. Doing calculation of a squared modulus of amplitude F we can consider $\epsilon^\mu(k_1) p_1^\mu = \epsilon^\nu(k_2) p_1^\nu = 0$, what corresponds to transverse polarization of photons and to zero momentum of $\pi^\pm$ mesons relative motion. Two photon wave function s_{ik} for even state with J=0 is symmetric tensor [23].

Pions are composite quark systems. So the amplitude of process $\pi^+ + \pi^- \to \gamma + \gamma$ is determined with the accuracy $O(\alpha)$ by some quark diagrams, shown on fig.2. Amplitude, corresponding to the first diagram has the following form:

$$M_1^{(1)} = 3g_\pi^2(Q_u^2+Q_d^2)\int \frac{-id^4k}{(2\pi)^4} Sp\{\gamma_5(\hat{k}-\hat{p}_1+m)\gamma_5(\hat{k}-\hat{P}+m)\gamma^\nu(\hat{k}-\hat{k}_1+m)\gamma^\mu(\hat{k}+m)\}\epsilon^\mu(k_1)\epsilon^\nu(k_2)\times \tag{5}$$

$$\times\{(k^2-m^2)[(k-P)^2-m^2][(k-p_1)^2-m^2][(k-k_1)^2-m^2]\}^{-1},$$

where g_π is the strong pion coupling constant: $L = ig_\pi \bar{q}\vec{\tau}\vec{\pi}\gamma_5 q$ [18], m is the quark mass, $P = p_1 + p_2 = k_1 + k_2$. For momentum integral evaluating in (5) we use Feynman parameterization:

$$\frac{1}{abcd} = 6\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} \frac{dz}{[ax+by+cz+d(1-x-y-z)]^4}. \tag{6}$$

After shift of integration momentum $k \to k + Px + p_1 y + k_1 z$ we can transform the denominator of the first amplitude to the form $1/(k^2-D)^4$, where $D = m_\pi^2\{\eta^2 - [4x(1-x-y-z) + y(1-y-2z)]\}$, $\eta = m/m_\pi$. Expression (5) contains the gauge invariant structures of two kinds: $(g^{\mu\nu} k_1 k_2 - k_1^\nu k_2^\mu)$ $(k_1 p_1 k_2^\mu p_1^\nu + k_2 p_1 k_1^\nu p_1^\mu - k_1 k_2 p_1^\nu p_1^\mu - k_1 p_1 k_2 p_1 g^{\mu\nu})$. Neglecting by relative momentum of pions, forming pionium, we can represent the amplitude $M_1^{(1)}$ in the following manner:

$$M_1^{(1)} = e^2[A_1\left(g^{\mu\nu}\frac{k_1 k_2}{m_\pi^2} - \frac{k_1^n u k_2^\mu}{m_\pi^2}\right) + B_1 g^{\mu\nu}]\epsilon^\mu(k_1)\epsilon^\nu(k_2), \tag{7}$$

$$A_1 = \frac{5g_\pi^2}{6\pi^2}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \left[\frac{f_1(x,y,z)}{D} + \frac{f_2(x,y,z}{D^2}\right], f_1(x,y,z) = 1+8xy+8xz-2x-3y-3z, \tag{8}$$

$$f_2(x,y,z) = 4x^2y^2 + 12x^2yz - 4x^2y + 8x^2z^2 - 4x^2z + 2xy^3 + 6xy^2z - 6xy^2 + 4xyz^2 -$$

$$-14xyz + 2xy\eta^2 + 4xy - 8xz^2 + 2xz\eta^2 + 4xz - y^3 - 3y^2z + 2y^2 - 2yz^2 + 2yz - y\eta^2 - y - z\eta^2,$$

$$B_1 = \frac{5g_\pi^2}{6\pi^2}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \left[\frac{f_3(x,y,z)}{D} + \frac{f_4(x,y,z)}{D^2} - 24\ln D\right], f_3(x,y,z) = 4xz - 6xy - 3x + \tag{9}$$

$$+5y^2+10yz-y-z+3\eta^2, f_4(x,y,z) = -4x^2y^2-8x^2yz+2x^2y-4x^2z+4xy^2z+3xy^2+8xyz^2+4xyz-4xy+$$

$$+4xz^2+4xz\eta^2-3x\eta^2+y^4+4y^3z-y^3+4y^2z^2-3y^2z+2y^2\eta^2-2y^2-2yz^2+4yz\eta^2-y\eta^2+\eta^4+2y-z\eta^2.$$

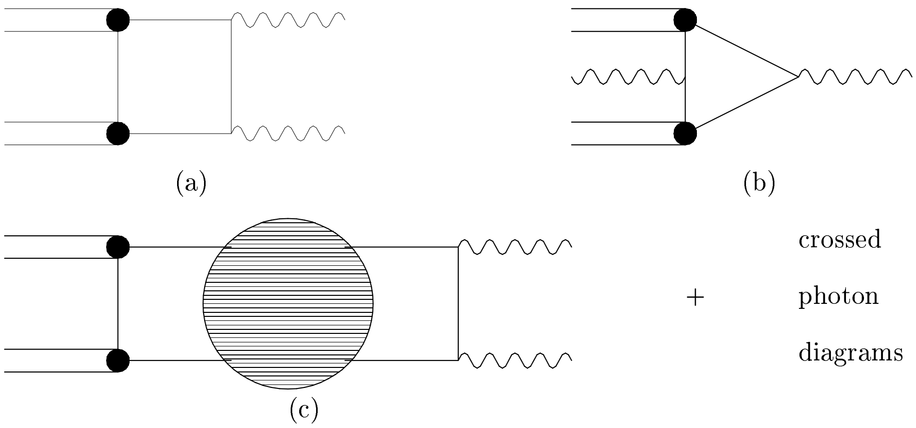


Figure 2: Quark annihilation diagrams for the process $\pi^+ + \pi^- \to \gamma + \gamma$ taking into account the interaction between quarks in the intermediate state

Other amplitudes of the "box" type diagrams on fig.2 can be presented in the similar form. After account divergences in $\bar{\rm MS}$ scheme framework the total amplitude is gauge invariant. It's structure is analogous to (7) in the frame of system, in which $p_1\epsilon(k_1) = p_1\epsilon(k_2) = 0$:

$$M_1^{tot.} = e^2[\left(g^{\mu\nu}\frac{(k_1k_2)}{m_\pi^2} - \frac{k_1^\nu k_2^\mu}{m_\pi^2}\right)\sum_{i=1}^{4} A_i + g^{\mu\nu}\sum_{i=1}^{4} B_i]\epsilon^\mu(k_1)\epsilon^\nu(k_2). \tag{10}$$

The coefficient functions take the kind of integrals over Feynman parameters x, y, z (8)-(9). Omitting exact expressions of these quantities, we give their numerical values $\sum_{i=1}^4 A_i = 0.006$, $\sum_{i=1}^4 B_i = 0.243$, which were used for the calculation of decay rate $A_{2\pi} \to \gamma + \gamma$.

There is intensive interaction of u-, d-quarks in the intermediate state (fig.2 (c)). Here we present some estimation of such effects in the decay amplitude. The interaction potential of quarks is determined by the sum of perturbative part, which can be found on the basis of QCD, and nonperturbative term [21]. The amplitude of such process contains product of free two particle propagator $G_{0\ 12}$ and quark scattering amplitude T_{12} in the form: $G_{0\ 12}T_{12}G_{0\ 12}$. In the momentum region $|k^2 - w^2| \ll |2m_\pi k^0|$ we can simplify $G_{0\ 12}$:

$$\frac{1}{(k+p_1)^2 - m^2} \times \frac{1}{(k-p_2)^2 - m^2} \approx \frac{\pi i \delta(k^0)}{2m_\pi(\vec{k}^2 + w^2)}, \quad w^2 = m^2 - m_\pi^2. \tag{11}$$

Projecting T_{12} on positive frequency states by means of Dirac bispinors, we can transform the corresponding amplitude of decay $A_{2\pi} \to \gamma + \gamma$ as follows:

$$M_2 = \frac{3g_\pi^2(Q_u^2 + Q_d^2)m^2(m_\pi + m)^2}{2m_\pi^2(m_\pi^2 + m^2)}\int\frac{d\vec{k}}{(2\pi)^3}\left[\frac{1}{\vec{k}^2 + w^2} - \frac{1}{\vec{k}^2 + m^2}\right] \times \tag{12}$$

$$\times \int \frac{d\vec{k}\,'}{(2\pi)^3(\vec{k}\,'^2 + w^2)} V(\vec{k}, \vec{k}\,') g^{\mu\nu}\epsilon^\mu(k_1)\epsilon^\nu(k_2),$$

where calculating bispinor averages, we have used nonrelativistic approximation for Dirac bispinors: $\bar{u}(k+p_1)\gamma_5(\hat{k}+m)\gamma_5 u(k-p_2) \approx m(m+m_\pi)$. Second bispinor average may be simplified in a similar way. The quark interaction operator $V(\vec{k}, \vec{k}\,')$ was obtained in the framework of quasipotential quark model in [19, 21]. To calculate integrals of (12) it is suitable to make transformation to coordinate space:

$$M_2 = \frac{3g_\pi^2(Q_u^2 + Q_d^2)m^2(m + m_\pi)^2}{8\pi m_\pi^2(m_\pi^2 + m^2)} g^{\mu\nu}\epsilon^\mu(k_1)\epsilon^\nu(k_2)\int e^{-wr}\left[e^{-wr} - e^{-mr}\right]\left[V^c(r) + V^{conf.}(r)\right] dr = \tag{13}$$

$$= \frac{5g_\pi^2 \alpha m^2 (m+m_\pi)^2}{6m_\pi^2(m_\pi^2+m^2)}\left[\alpha_s\left(\ln 2+\ln\frac{w}{w+m}\right)+\frac{(m-w)[2Bw(m+w)+A(m+3w)]}{4w^2(m+w)^2}\right] g^{\mu\nu}\epsilon^\mu(k_1)\epsilon^\nu(k_2),$$

where the confining part of quasipotential $V^{conf.}(r) = Ar+B$, and the parameters A, B were taken from [19].

Using expressions (2)-(4), (10) (13) and the relation for differential decay rate $A_{2\pi} \to \gamma+\gamma$ [23]

$$d\Gamma = \frac{|\vec{p}_\gamma|}{E^2_{A_{2\pi}}}\frac{d\Omega_\gamma}{64\pi^2}|F|^2, \tag{14}$$

we obtain the expression for total width of pionium decay on two photons:

$$\Gamma = \frac{m_\pi \alpha^5}{4}\left[1+0.302\pm 0.028\right] \tag{15}$$

where in square brackets there are contributions of amplitudes (3), (10) and (13), accounting interference terms. As it follows from (15), the sum of Born diagrams of fig.1 give the main contribution to the decay width. The box graphs give basic part of correction to this result, which reaches 30%. It is important to note that the contributions of perturbative and nonperturbative parts of quasipotential to the amplitude M_2 have similar values but opposite signs. The uncertainty of theoretical result (15) is determined by the approximation of small quark momenta ($|\vec{k}| \sim w$) in the intermediate state, which we used calculating diagram (c) of fig.2. Correction, obtained by us, is twice that was obtained in the paper [13], where the experimental values of pion polarizabilities were used for calculation of pionium decay rate. As noted earlier, the polarizabilities $\alpha_{\pi^\pm}$ and $\beta_{\pi^\pm}$ are known at present time with large error: $\alpha_{\pi^\pm} = -\beta_{\pi^\pm} =$ 6.8±1.4 ± 1.2 [13, 15]. So, experimental investigation of pionium decays, including rare decay on two photons, open the possibility to define more precisely the values of $\alpha_{\pi^\pm}$, $\beta_{\pi^\pm}$.

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