Polynomial Gauge Invariants of a Bosonic String

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Abstract

An open bosonic string is considered with the aim to construct a general gauge invariant, as a polynomial in Fubini-Veneziano (FV) fields. The FV fields are transformed as 1-forms on S^1, that allows to formulate the problem in geometric terms. We introduce the most general anzats for these invariants and explicitly resolve the invariance conditions in the framework of the anzats. The invariants are interpreted as integrals of an n-form over a gauge invariant domains in an n-dimensional torus, where these domains are invariant under to the action of the diagonal of the group $\times(Diff\ S^1)^n$.

We also discuss a possibility to get a complete set of gauge invariants which allow an actual dependence on the string zero modes. We find that the complete set can not be restricted to polynomial invariants only.

The classical polynomial invariants, being directly defined in the string Fock space, turn out to break the structure of the respective BRST cohomology even in the critical dimension. We discuss a possibility to restore the BRST invariance of the corresponding operator algebra by a non-trivial quantum deformation of the original invariants.

Introduction

The a im of the work is to find a complete set of gauge invariants of a bosonic string.

A classical gauge invariant is understood as a parametrization independent object that is a physical observable. A quantum gauge invariant is an operator which is well-defined in the respective BRST-cohomology. It is an operator which represents a physical observable.

A complete set of classical gauge invariants is defined as the set in terms of which an arbitrary physical observable can be expressed. A complete set of quantum gauge invariants is the set whose enveloping algebra includes all invariant operators. It will be shown how to find all classical invariants at least in the class of polynomials. The question of quantization of these invariants will be also discussed.

The bosonic string is a well studied model. It allows to apply various methods of quantization, its spectrum can be obtained in different ways. However a structure of the reduced phase space of the model is rather complicated and is not recognized well. It is the set of gauge invariants that can be applied for investigating the structure.

It could be useful for constructing string interaction, for a string field theory. There is also another question less obvious and less well known, namely, to understand how the phase space of the string stratifies into phase spaces of the elementary particles constituting its spectrum. The information about the invariants seems to be rather useful for elaborating the last question.

It is commonly known that the complete set of quantum gauge invariants can be represented by the set of vertex operators [1]. However, the vertex operators have no classical limit. Actually we are looking for another set of invariants which do have a certain classical limit.

For simplicity we shall restrict ourselves to the case of the open bosonic string.

Classical gauge invariants

The complete set of the phase space variables of the open bosonic string consists of the Fubini-Veneziano(FV) fields and the string zero mode

$$V^\mu = V^\mu(\sigma)\ :\ [0\,,2\pi] \to \mathbf{R}^{1\,,D-1} \qquad V^\mu(0) = V^\mu(2\pi) \qquad q^\mu \in \mathbf{R}^{1\,,D-1} \tag{1}$$

They are subject to the first class constraints

$$L(\sigma) = \frac{1}{4} V^\mu(\sigma) V_\mu(\sigma) \tag{2}$$

First of all, let us pose the question whether there are polynomial gauge invariants which depend on the FV fields only. The positive answer to the question can be found in the literature, namely, an infinite set of such invariants was proposed in the works of Pohlmeyer and Rehren [2, 3, 4].

$$I_n^{\mu_1\mu_2\dots\mu_n} = \int_0^{2\pi} V^{\mu_1}(\sigma_1)d\sigma_1 \int_{\sigma_1}^{\sigma_1+2\pi} d\sigma_2 V^{\mu_2}(\sigma_2) \int_{\sigma_1}^{\sigma_2} V^{\mu_3}(\sigma_3)\dots \int_{\sigma_1}^{\sigma_{n-1}} d\sigma_n V^{\mu_n}(\sigma_n) \tag{3}$$

In the paper [4] it was proved that these polynomials (3) exhaust all gauge invariants which depend on the FV fields only. To obtain a complete set of classical gauge invariants, we should include an actual dependence on the string zero mode q^μ .

The most general polynomial expression for a classical gauge invariant is

$$I = \sum C_{\mu_1\mu_2\dots\mu_n} q^{\mu_1} q^{\mu_2} \dots q^{\mu_n} + \sum C^{m_1}_{\nu_1\mu_1\mu_2\dots\mu_{n-1}} \alpha^{\nu_1}_{m_1} q^{\mu_1} q^{\mu_2} \dots q^{\mu_{n-1}} + \dots + \sum C^{m_1 m_2\dots m_n}_{\nu_1\nu_2\dots\nu_n} \alpha^{\nu_1}_{m_1} \alpha^{\nu_2}_{m_2} \dots \alpha^{\nu_n}_{m_n} \tag{4}$$

where

$$\alpha^\mu_n = \frac{1}{2\sqrt{\pi}} \int_0^{2\pi} V^\mu(\sigma) e^{-in\sigma} d\sigma$$

As we see all the terms in the expression (4) are of the same order in the phase space variables. One can take the anzats in such a form simply because the gauge transformations are homogeneous in the phase space variables:

$$\delta_\varepsilon V^\mu = (\varepsilon(\sigma)V^\mu(\sigma))' \qquad \delta_\varepsilon q^\mu = \int_0^{2\pi} d\sigma \varepsilon(\sigma) V^\mu(\sigma) \qquad \delta I = \{ L[\varepsilon], I \} \tag{5}$$

If one requires the polynomial (4) to be gauge invariant, the respective structure coefficients are subject to following conditions:

$$C_{\mu_1\mu_2\dots\mu_n} = 0\,,$$
$$C^{n_1 n_2\dots n_l}_{\nu_1\nu_2\dots\nu_l\mu_1\dots\mu_{n-l}} = 0$$

if

$$n_1 \neq 0\,, n_2 \neq 0\,, \dots\,, n_l \neq 0\,,$$
$$C^{n_1 n_2\dots n_l\, 0\ \dots\ 0}_{\nu_1\nu_2\dots\nu_l(\nu_{l+1}\dots\nu_s\mu_1)\dots\mu_{n-s}} = 0\,. \tag{6}$$

One of the examples of such invariants which depends on the whole set of phase space variables is the momentum tensor of the string:

$$\mathcal{M}^{\mu\nu} = q^\mu \alpha^\nu_0 - q^\nu \alpha^\mu_0 + \sum_{n\neq 0} \frac{i}{n} \alpha^\mu_n \alpha^\nu_{-n} \tag{7}$$

Using the relations (6) one proves that an abitrary polynomial gauge invariant can be expressed, modulo constraints, in terms of the momentum tensor (7) and the polynomials (3). It turns out that the proposed polynomial invariants form only a subalgebra of the algebra of physical observables. Actually they do not exhaust the complete set of string gauge invariants because there are physicaly different points on the constraint surface of the string that cannot be distinguished with the help of these polynomials. This means that the complete set of string gauge invariants must include observables which are not polynomial in the phase space variables. Unfortunately, no such invariants are known yet.

Quantization problem

Let us discuss quantization of the polynomial invariants. As we know, the momentum tensor (7) of the bosonic string can be quantized without any problems. The same situation takes place with the polynomials (3) whise $n < 4$. The invariants (3) with $n \geq 4$ the situation drastically changes. Namely, the invariants, being directly defined in the Fock space of the string, do not commute with the Virasoro generators, because of the quantum corrections. This means that the respective operators are not defined

in the space of the physical states. This situation is related to the usual quantization problem of the systems with constraints. It would be rather strange if quantum corrections did not destroy some key relations of a classical theory. In some cases this leads us to the true values of the critical parameters, in other this means that it is not possible to construct a consistent quantum theory. There is however the third case, when we can say that some relations do not have a consistent quantum interpretation, but the quantum theory does exist.

We think that the problem we face with can be solved. Firstly let us note that the classical invariants are defined ambiguously off the constraint surface. Namely, one can add to the above polynomial an expression, which vanishes on constraints. The terms, which vanish classically, may contribute to the quantum commutator between the invariant and the BRST charge.

$$\Omega = \sum_n L_n C_{-n} + \sum_{n\,m} m P_n C_m C_{-n-m}\,, \tag{8}$$

where C_{-n} and P_n are canonical ghosts and

$$L_n = \frac{1}{2}\sum_k \alpha_k^\mu \alpha_{n-k}^\nu \eta_{\mu\nu} \tag{9}$$

are the Virasoro generators.

The arbitrariness can be used for constructing genuine quantum BRST invariants polynomial in string operators and ghosts.

Let us summarize the things to be done.

i)It is necessary to add to the naive invariant the most general expression which vanishes on constraints

$$I = I[V] \qquad [\underbrace{I, L_n}] = \textstyle\sum_m W_{nm} L_m \tag{10}$$

ii) to construct a quantum operator with ghosts using the BFV method

$$\tilde{I} = I + C_{-m} P_n W_{nm} + \dots\,,$$

iii) to evaluate the commutator between the constructed operator and the BRST charge

$$[\tilde{I}, \Omega] = \textstyle\sum_n [\underbrace{I, L_n}] C_{-n} + \sum_{n\,k\,l} m [W_{kl} C_{-l} \underbrace{P_k\,, L_n C_{-n}}] + \sum_{n\,m\,k\,l} m W_{kl} [C_{-l} \underbrace{P_k\,, P_n C_m C_{-n-m}}] + \dots \tag{11}$$

where dots stand for the terms with higher structure functions.

Doing the last step it is necessary to account for one-loop contributions only, because higher corrections vanish. Thus, we obtain the equation for the additional terms. This equation could be solvable, because the ghost terms give quantum corrections of the same order as those which appear from the anomaly.

Conclusion

We have considered the set of physical observables which exhaust all polynomial invariants. We have proved that the set of the polynomial invariants is not complete. Also we pose the question whether it is possible to realize the BRST cohomology of the string operator algebra in terms of the operators polynomial in string modes and ghosts. These invariants, unlike the vertex operators can clarify the connection between the reduced phase space of the bosonic string and the BRST cohomologies of the corresponding quantum theory [5] .

References

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