QED Radiative Corrections to $e^+e^- \to \bar{f}f$ with Realistic Cuts at LEP Energies and Beyond

M. Jack
DESY Zeuthen, Platanenallee 6,
D-15738 Zeuthen, Germany
E-mail: jack@ifh.de

Abstract

After 10 years of steadily increasing the experimental precision at LEP/SLC, there is a strong demand on an update of existing programs for fermion pair production. We present a rederivation of the $O(\alpha)$ Bremsstrahlung corrections to $e^+e^- \to \bar{f}f$ for the semi-analytic program `ZFITTER`. We focus on observables like total cross section and forward-backward asymmetry in the leptonic case with combined cuts on acollinearity angle, acceptance angle, and minimal energy of the fermions. The outcome of our analysis is a shift of the predictions by `ZFITTER` at LEP 1 energies off-resonance of a few per mil while at the Z resonance numerical changes can be neglected. Thus we obtain for cross sections and asymmetries at LEP 1 a level of agreement with other programs of better than per mil, like for the kinematically simpler s' cut option. A preliminary analysis of `ZFITTER`, `TOPAZO`, and other codes at LEP 2 energies showing deviations of several per cent with acollinearity cuts enforce a future examination of higher order effects with different cuts. The predictions by LEP/SLC data, however, are not affected within the experimental errors.

1 Introduction

In the light of high precision measurements to the Standard Model (`SM`), fermion pair production in e^+e^- annihilation still plays a very important role, e.g. for the extraction of information from data on the electroweak symmetry breaking sector of the `SM`. This impressing increase of precision by experiment over the last 10 years was nicely illustrated by [1] :

Quantity	LP 89 (233 events)	LP 99 (18×10^6 events)
M_Z (GeV)	91.17 ± 0.18	91.1871 ± 0.0021
Γ_Z (GeV)	$1.95^{+0.40}_{-0.30}$	2.4944 ± 0.0024
N_ν (light)	3.0 ± 0.9	2.9835 ± 0.0083

Table 1: Examples for the development of high precision measurements at the Z resonance.

As one of the most copious processes from LEP to Linear Collider (LC) energies fermion pair production also forms an important incoherent background to W-pair production, or more generally, to 4-fermion final states at higher energies [2, 3]. Another very interesting application of this 'traditional' channel is the possibility to extract limits on extensions of the `SM` in experiments like LEP or SLC when comparing with the theoretical predictions (e.g. Z' and W' boson searches, contact interaction scales, exchange of leptoquarks, preons, or particles in R-parity breaking supersymmetric models [4]).

We want to focus on high precision measurements to the `SM` and `MSSM`. The needed accuracy at energies below the Z resonance region from QED Bremsstrahlung, forming the bulk of the radiative corrections, is relatively small. However, with the high experimental precision now obtained at LEP 1 and LEP 2 energies, it is necessary to estimate theoretically these corrections around and above the Z resonance region. Especially, hard QED corrections which become more and more important with growing c.m. energies and resummed soft and virtual higher order effects have to be calculated precisely taking into account realistic experimental cuts. Having this in mind, the accuracy of the theoretical prediction for single contributions to observables should be typically a factor of 10 or so better than the experimental error. This is moreover true when considering a high luminosity option of a future LC running at the Z resonance ('Giga Z' option) [5]. Looking at the experimental situation (Table 1), we can briefly state that

the theoretical predictions therefore have to be better than 0.015% at the peak, at the order of 0.03% in the resonance region ($\approx M_Z \pm 3\,\mathrm{GeV}$), and around 0.5% starting at LEP 2 and higher energies.

2 Realistic observables and the ZFITTER concept

The semi-analytic approach of the ZFITTER code [6] consists of a fast, one-dimensional numerical integration of analytical formulae for different observables like cross sections, asymmetries, polarizations, and angular distributions with the inclusion of different experimentally relevant cut options and resummed $O(\alpha)$ QED corrections plus dominant higher order effects. The number of analytically calculable cuts in such an aproach is of course limited, in our case to two angular cuts and to cuts on the final state energies and invariant mass squared. Corresponding numerical programs for fermion pair production like ALIBABA [7], BHM [8], KORALZ, KK [9, 10], or TOPAZ0 [11] are in this respect complementary to our approach, as they can in principle treat multi-differential observables with nearly arbitrary cuts to the final state phase space, but this at the expense of a clear increase in computing time.

In the ZFITTER approach [12, 13, 14, 15, 6], we calculate:

$$\sigma(s) \sim \int \frac{ds'}{s}\ \sigma^0(s')\ \rho(s'/s), \tag{1}$$

with $s' = m^2_{f\bar{f}}$ as the invariant mass squared of the final state fermion pair. Cross sections and asymmetries like $\sigma_T(s)$ and $A_{FB}(s)$ are treated in an *improved Born approximation*, convoluting *effective Born observables*, $\sigma^0(s)$ and $A^0(s)$, over s' with a flux function ρ (*radiator*) containing the photonic corrections. The electroweak and QCD corrections are described with effective couplings in the effective Born terms; this is valid due to their smallness at LEP 1 energies. [1] With ZFITTER, three different cut options are available: (i) no cut [13], (ii) cuts on s' and on the scattering angle ϑ of one fermion [14, 15], or (iii) cuts on the fermions' acollinearity angle, θ_{acol}, on their energies, $E^f = E^{\bar{f}}$, and on $\cos\vartheta$ [16, 17]. For a detailed description of the phase space with cuts on final state acollinearity, minimal energies, and/or s' please refer to [18, 4]. The effective Born cross sections, $\sigma^0(s')$, may also be chosen according to following approaches: (A) Standard Model, (B) Model Independent, (C) Others [12, 19, 6]. For the latest updates and afs-accounts for the ZFITTER code and other programs please consult [6, 10, 11, 20].

3 Different codes – comparison and problems

Focussing first on LEP 1 energies and the s' cut branch (see Section 2), the situation of the ZFITTER code up to versions v.5.x (1998) in comparison with the code TOPAZ0 can be stated as quite satisfactory. At $M_Z \pm 3\,\mathrm{GeV}$ the agreement is better than 10^{-4} [21, 22]. And also at LEP 2 energies and higher we can meet the demands by experiment with a deviation of the codes of not more than 1 or 2 per mil for different cut values, and with a substantial decrease of this difference below 1 per mil in case of a sufficiently large s' cut. This situation does not change when an extra cut on the maximal scattering angle $\cos\vartheta$ is applied [17, 20].

If we introduced an acollinearity cut instead of the s' cut, we obtained a comparable agreement around the Z resonance as long as we only considered initial state Bremsstrahlung. But as soon as we included the initial-final state corrections this agreement deteriorated to $O(3\times10^{-3})$ which grows to an unbearably large discrepancy between the two codes of several per cent at larger energies ($\sqrt{s} \approx 100 \ldots 200\,\mathrm{GeV}$) [4]. An earlier comparison of the ZFITTER code with the ALIBABA code for the s-channel part of the Bhabha scattering branch had already shown similar deviations [23, 17].

4 QED Corrections with Acollinearity Cut

The formulae for photonic $O(\alpha)$ corrections in ZFITTER with s' cut have been analytically and numerically multiply checked at LEP energies [13, 14, 15, 2, 22]. For the acollinearity cut branch, however, there

[1] At higher energies, ZZ and WW box corrections contribute at the 1 to 2 per cent level at LEP 2 energies so the validity of our non-gauge invariant, effective Born approximation will have to be carefully reexamined there.

has been no independent check until recently and only little literature available on the exact $O(\alpha)$ final state corrections to the total cross section and forward-backward asymmetry [24] and on some formulae related to the initial state corrections (and its combined exponentiation with final state radiation) for the angular distribution [25, 26]. So, a recalculation and documentation of the acollinearity cut situation was absolutely mandatory with the main focus first at energies around the Z resonance. The slightly more involved Bhabha scattering case with extra t channel contributions is kept for a later analysis.

In our semi-analytic approach it is technically feasible to calculate observables with cuts on one of the final state fermions' scattering angles, ϑ, their energies, $E_{\bar{f},f}$, their acollinearity angle, θ_{acol}, and/or their invariant mass squared, s'. The perturbative calculation of the $O(\alpha)$ hard corrections to total (differential) cross sections then consists of an analytical integration over 3, or respectively 2, angles of phase space. The corresponding formulae are finally numerically integrated over s'.

We observe that neglecting the initial and final state masses at the mentioned high energies necessitates a separation of the phase space formed by the remaining two angles of integration, $\cos\vartheta$ and $\cos\theta_{acol}$, into several different regions. Only where necessary, the masses are kept in order to regularize the mass singularities from collinear radiation of Bremsstrahlung photons. This 'slicing' of phase space delivers for each region different analytical expressions for the calculated observables [17]. For the special cases of either full angular acceptance, i.e. no cut on $\cos\theta$, or no cut on the acollinearity angle θ_{acol}, the number of different expressions can be substantially reduced and very compact formulae can be obtained [16]. This allows one to install a numerically fast second branch in the Fortran code when demanding fewer cuts, important for quick data-fitting routines needed by experiment.

The Fortran package `acol.f` contains a complete collection of the analytical formulae for the $O(\alpha)$ corrections and is called from `ZFITTER` v.6.04/06 [27] onwards. The angular distribution will be available in versions v.6.2x. A complete collection of all analytical expressions is in preparation [28].

5 Results

The main modifications in the new coding are corrected terms in the QED initial state and interference radiator parts. For different regions of phase space, these are different, non-logarithmic contributions proportional to $\cos\vartheta$ in the symmetric part of the angular distribution $d\sigma/d\cos\vartheta$ and terms of the type $a + b\cos^2\vartheta$ in the antisymmetric part with corresponding changes in the integrated results [17, 28]. Omitting these terms in earlier versions is justified with the then anticipated experimental precision of only 5×10^{-3} at LEP 1, but not with the higher accuracy now at the Z resonance peak.

The net effect of the corrected initial state, final state, and interference terms is depicted for σ_T and A_{FB} in Fig. 1 for the energy range 30 to 300 GeV (default flag setting): We compared the cross section ratios and the absolute differences of the asymmetries of `ZFITTER` v.6.11 [6], containing the new results, with v.5.20 [29], still with the old coding, for different acollinearity cut values. The net corrections are largest where the radiative return to the Z starts to be prevented by the acollinearity cut. For $\theta_{acol} < 10^\circ$ or 25° this sets in at roughly $\sqrt{s} > 100\,\mathrm{GeV}$, or 115 GeV respectively. These corrections to the code are at most roughly 0.5% for σ_T and 1% for A_{FB} and shrink below 1% at higher energies.

The corrections are mainly due to the new initial-final state interference contributions. This is illustrated for a maximal acollinearity angle of $\theta_{acol} < 10^\circ$ in Table 2 where the shifts of the initial state corrected total cross sections and asymmetries are shown when switching on the interference contributions. One can show that the numerical effects stay below 1% at LEP 2 energies with such a cut value [17, 20]. At LEP 1 they are much smaller, of the order of few per mil.

The corrected initial state and final state terms only have minor effects on σ_T and A_{FB} and amount at most to corrections at the order of $0.1\% - 0.2\%$ for A_{FB} at LEP 2 energies for different cuts. A detailed analysis of all new modifications to the code can be found in [17].

The numerical comparison of the newly updated `ZFITTER` version v.6.11 [6] with `TOPAZ0`'s latest release version v.4.4 [11] now delivers for LEP 1 energies the same high level of agreement as for the s' cut (Fig. 2): At the peak itself we have a deviation of the codes of $O(10^{-4})$ or less for σ_T and A_{FB}, with an acceptable increase to $O(3 \times 10^{-4})$ for σ_T and a slightly worse value for A_{FB} of $O(7 \times 10^{-4})$ with a maximal acollinearity angle of 10°.

For a wider energy range, we compared `ZFITTER` v.6.11 (1999) with `TOPAZ0` v.4.3 (1998) and v.4.4 (1999) [30, 11, 22] and `ALIBABA` v.2 (1990) [7] in Fig. 3. An older comparison can be found in [4]. All

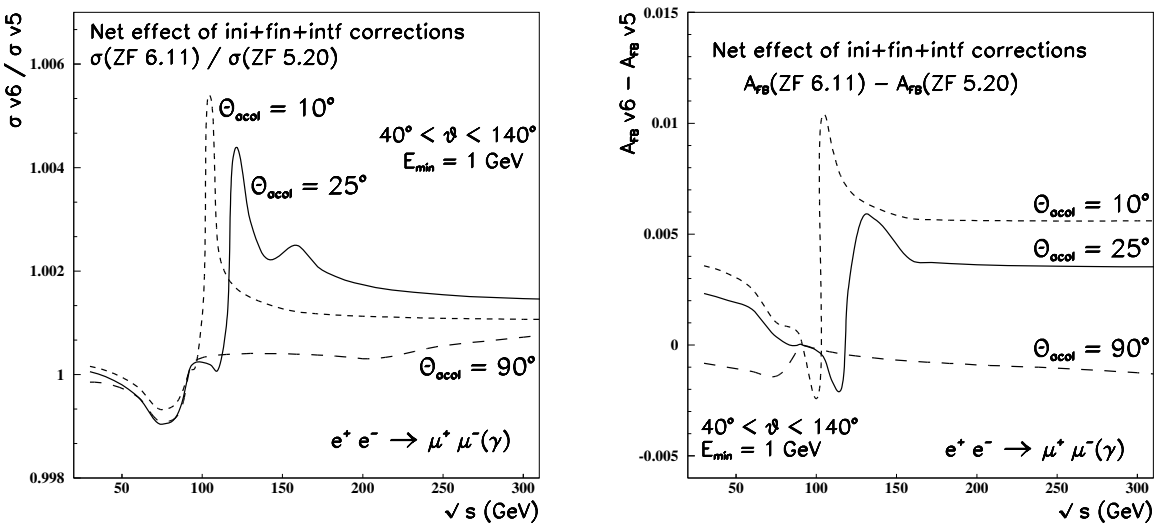

Figure 1: Net ratios of muon pair production cross sections and differences of forward-backward asymmetries predicted from ZFITTER v.6.11 and v.5.20 with three different acollinearity cuts: $\theta_{\rm acol} < 10°, 25°, 90°$; $E_{min} =$ 1 GeV; $40° < \vartheta < 140°$; all corrections included.

numbers have been produced with the default settings of the programs.

At LEP 2 energies the deviation of ZFITTER v.6.11 and TOPAZ0 v.4.4 is at the order of 1% or less for different acollinearity cuts and an acceptance cut of $40° < \vartheta < 140°$. A cross check with comparable s' cuts delivers a much better agreement of about 1 per mil or better [17].

In both cases, however, there is a clear peak of the cross section ratios at energies where the Z radiative return is not prevented by the cuts. While for the s' cut this discrepancy stays moderate at the per cent level, it grows up to several per cent for the acollinearity cut. [2] This effect is now similar to the observed peak in the ALIBABA - ZFITTER comparison at energies above roughly 100 GeV. Interestingly enough, when switching off two-loop contributions present in ALIBABA, but not in ZFITTER, the agreement improved considerably. Corrections by initial state pair production or different exponentiation of initial and final state higher orders, however, do not have a large effect here [20].

Preliminary studies show that a correct description of hard two-loop QED corrections, especially for the acollinearity cut option in the ZFITTER code, together with a correct resummation of the soft and virtual initial-final state interference contribution, not contained in the ZFITTER code so far, seem to play a key role here. Since the acollinearity cut is not as effective in preventing the radiative return to the Z as the s' cut, these deviations also survive more profoundly for the acollinearity cut at higher energies than for the s' cut. Further comparisons in this respect for both cut options, especially including programs KORALZ [9] and KK [31], are intended (see also [32]).

6 Summary

We presented a rederivation of analytical formulae for the $O(\alpha)$ hard QED Bremsstrahlung corrections to $e^+e^- \to \bar{f}f$ in the case of leptonic final states with cuts to the fermions' acollinearity angle and energies ($f \neq e$). This was done in the context of the semi-analytic program ZFITTER calculating radiatively corrected observables with realistic experimental cuts, e.g. for LEP/SLC applications. These corrections to all contributions – initial state, final state, and initial-final state interference – are certain

[2] The flip of sign of these effects compared to the older versions, TOPAZ0 v.4.3 and ZFITTER v.6.05, is mainly due to a corrected interference contribution in the TOPAZ0 code. Changes to code ZFITTER v.6.05 were negligible here.

σ_μ [nb] with $\theta_{\rm acol} < 10^\circ$					
$\theta_{\rm acc} = 0^\circ$	$M_Z - 3$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3$
TOPAZ0	0.21932	0.46287	1.44795	0.67725	0.39366
	0.21776	0.46083	1.44785	0.67894	0.39491
	−7.16	**−4.43**	**−0.07**	**+2.49**	**+3.17**
ZFITTER	0.21928	0.46284	1.44780	0.67721	0.39360
	0.21772	0.46082	1.44776	0.67898	0.39489
	−7.16	**−4.40**	**−0.03**	**+2.60**	**+3.27**
$A^\mu_{\rm FB}$ with $\theta_{\rm acol} < 10^\circ$					
$\theta_{\rm acc} = 0^\circ$	$M_Z - 3$	$M_Z - 1.8$	M_Z	$M_Z + 1.8$	$M_Z + 3$
TOPAZ0	−0.28450	−0.16914	0.00033	0.11512	0.16107
	−0.28158	−0.16665	0.00088	0.11385	0.15936
	+2.92	**+2.49**	**+0.55**	**−1.27**	**−1.71**
ZFITTER	−0.28497	−0.16936	0.00024	0.11496	0.16083
	−0.28222	−0.16710	0.00083	0.11392	0.15926
	+2.75	**+2.27**	**+0.60**	**−1.03**	**−1.56**

Table 2: A comparison of predictions from ZFITTER v.6.11 and TOPAZ0 v.4.4 for muonic cross sections and forward-backward asymmetries around the Z peak. First row is without initial-final state interference, second row with, third row the relative effect of that interference in per mil.

non-logarithmic terms which could be neglected earlier but have to be considered now with the new high level of experimental precision (e.g. $\delta M_Z/M_Z = 2.2 \times 10^{-5}$ [1]).

The older versions of ZFITTER, i.e. versions v.5.20 [29] and earlier, derive the $O(\alpha)$ QED corrections to σ_T with acollinearity cut with a numerical accuracy of about 0.4 % in the Z resonance region ($M_Z \pm 3\,\mathrm{GeV}$), and similarly for A_{FB} with about 0.13 %. The limiting factor here are mainly the initial-final state interference corrections. The new and improved coding in ZFITTER v.6.11 [6] now reproduces the very nice agreement with program TOPAZ0 v.4.4 [11] already obtained for the s' cut [22] of better than 0.03% (0.1%) for σ_T (A_{FB}) at LEP 1 and better than 0.01% at the Z resonance peak. [3] At higher energies, $\sqrt{s} > 200\,\mathrm{GeV}$, the agreement with s' cut is better than per mil, but still only 1 to 2% for the acollinearity cut. Especially in the intermediate energy range, where the Z radiative return events are not prevented, this discrepancy peaks and amounts up to several per cent. A similar effect is also visible for the s' cut, although to a much lesser extent (below 1%).

Preliminary studies indicate that higher order corrections, in particular, hard two-loop corrections together with an acollinearity cut and a correct resumming of soft and virtual interference effects not included in the ZFITTER code seem to be the major underlying cause for this last point [31, 32]. Further studies hereof, also for the angular distribution, and a similar update of the code for the Bhabha scattering case are in preparation.

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[3] At the Z peak itself, the accuracy had already been quite satisfactory before, i.e. better than 10^{-4}, due to suppressed hard-photon radiation.

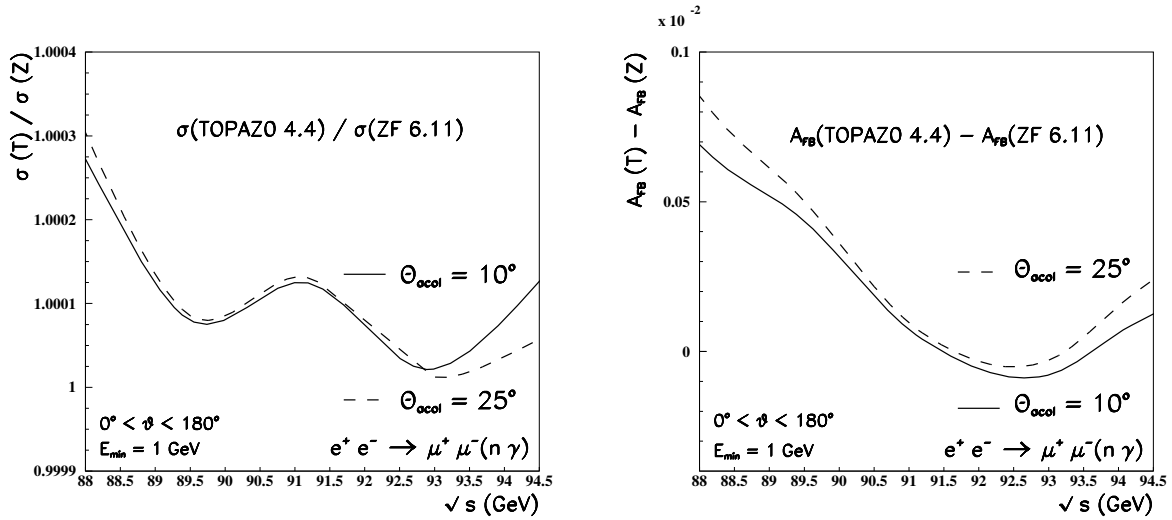

Figure 2: Ratios of muon pair production cross sections and differences of forward-backward asymmetries at the Z resonance: ZFITTER v.6.11 vs. TOPAZ0 v.4.4 for different acollinearity cuts (full acceptance).

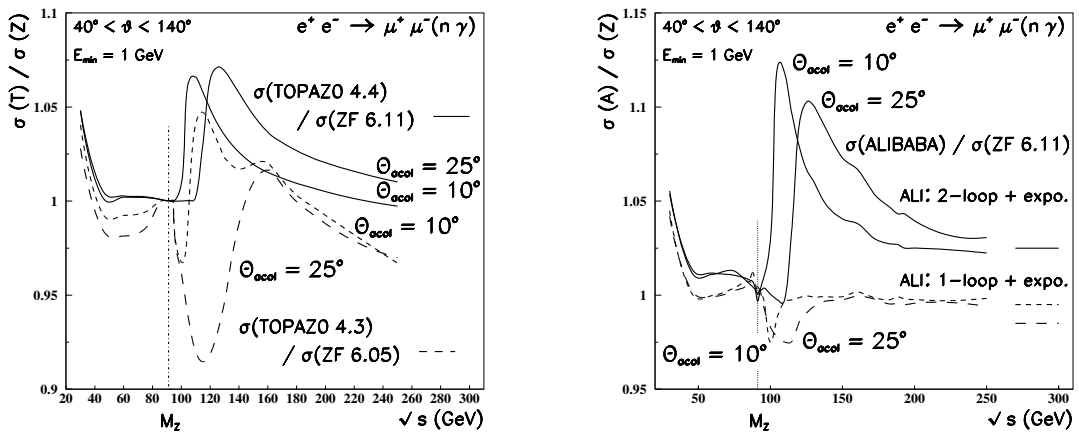

Figure 3: Comparison of predictions from ZFITTER v.6.04/06 and v.6.11, TOPAZ0 v.4.3 and v.4.4, and ALIBABA v.2 for muon pair production cross sections.

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