Parton Scattering at Small-x and Scaling Violation [1]

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Abstract

Scaling violation of inclusive jet production at small-x in hadron scattering with increasing total collision energy is discussed. Perturbative QCD based on the factorization theorem for hard processes and GLAPD evolution equations predicts a minimum for scaled cross section ratio which depends on jet rapidity. Studes of such a scaling violation can reveal a vivid indication on new dynamical effects in the high-energy limit of QCD. The BFKL effects, which seem to be seen in recent L3 data at CERN LEP2, should give different results from GLAPD preditions.

QCD is an essential ingredient of the Standard Model and is well tested in hard processes when transferred momentum is on the order of the total collision energy (Bjorken limit: $Q^2 \sim s \to \infty$). The cornerstones of perturbative QCD at this kinematic regime (QCD-improved parton model): factorization of inclusive hard processes [1] and the Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD) evolution equation [2] provides a basis for the successful QCD-improved parton model. The factorization theorem [1] for inclusive hard processes ensures that the inclusive cross section factorizes into partonic subprocess(es) and parton distribution function(s). The GLAPD-evolution equation governs the $\log(Q^2)$-dependence (at $Q^2 \to \infty$) of the inclusive hard process cross sections at fixed scaling variable $x = Q^2/s$.

Another kinematic domain which is very important at high-energy is given by the (Balitsky-Fadin-Kuraev-Lipatov) BFKL-limit [3, 4, 5, 6], or QCD Regge-limit, whereby at fixed $Q^2 \gg \Lambda^2_{QCD}$ $s \to \infty$. In the BFKL-limit, the BFKL-evolution in the leading order (LO) governs $\log(1/x)$-evolution (at $x \to 0$) of inclusive processes. Note that the BFKL evolution in the next-to-leading order (NLO) [7, 8, 9, 10], unlike the LO BFKL [3, 4, 5], includes GLAPD-evolution with the running coupling constant of the LO GLAPD, $\alpha_S(Q^2) = 4\pi/\beta_0 \ln(Q^2/\Lambda^2_{QCD})$.

Therefore, the BFKL, and, especially the NLO BFKL [7, 8, 9, 10] including the GLAPD-evolution are anticipated to be important tools for exploring the high-energy limit of QCD. In particular, this importance arises since the highest eigenvalue, ω^{max}, of the BFKL equation [3, 4, 5, 6, 9, 10] is related to the intercept of the Pomeron which in turn governs the high-energy asymptotics of the total cross sections: $\sigma \sim (s/s_0)^{\alpha_{IP}-1} = (s/s_0)^{\omega^{max}}$, where Regge-parameter s_0 defines the approach to the asymptotic regime. The BFKL Pomeron intercept in the LO turns out to be rather large: $\alpha_{IP} - 1 = \omega^{max}_{LO} = 12 \ln 2 \, (\alpha_S/\pi) \simeq 0.54$ for $\alpha_S = 0.2$; hence, it is very important to analyze recently calculated NLO corrections [7, 8] to the BFKL.

One of the striking features of the NLO BFKL analysis [9] is that the NLO value for the intercept of the BFKL Pomeron, improved by the BLM procedure [11], has a very weak dependence on the gluon virtuality Q^2: $\alpha_{IP} - 1 = \omega^{max}_{NLO} \simeq 0.13 - 0.18$ at $Q^2 = 1 - 100$ GeV^2. This agrees with the conventional Regge-theory where one expects an universal intercept of the Pomeron without any Q^2-dependence. The minor Q^2-dependence obtained leads to approximate conformal invariance.

There have been a number of recent papers which analyze the NLO BFKL predictions [12, 13, 14, 15, 16]. Also, a lot of work should be done to clarify the very important issue of the factorization properties of the BFKL regime [17, 18, 19, 20, 21, 22].

As a phenomenological application of the NLO BFKL improved by BLM procedure, with its effective resummation of the conformal-violating β_0-terms into the running coupling in all orders of the perturbation theory, one can consider the gamma-gamma scattering [23, 10]. This process is attractive because

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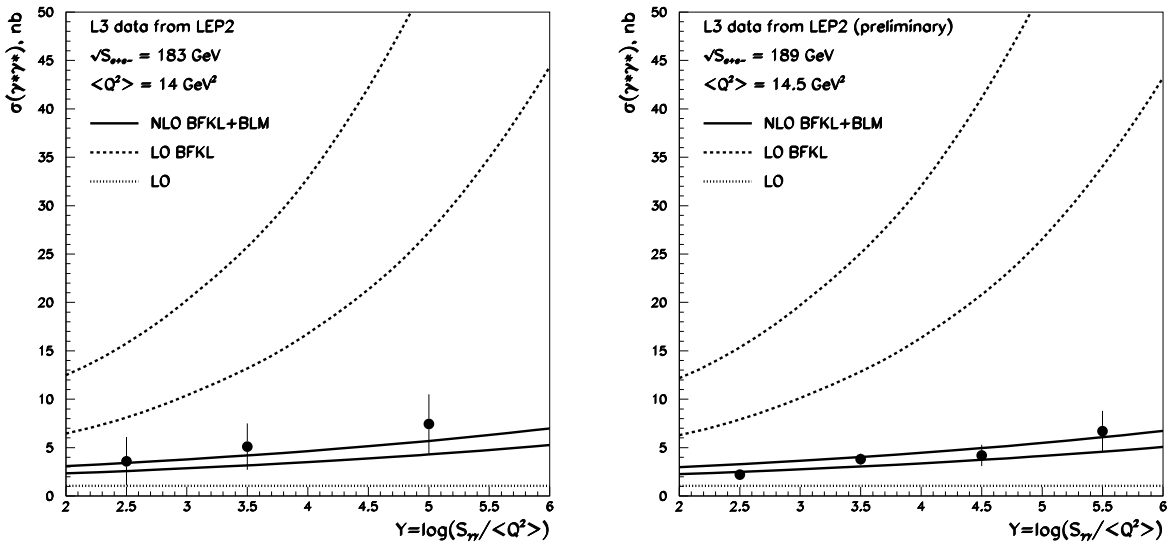

Figure 1: Virtual gamma-gamma total cross section by the BFKL Pomeron versus L3 Collaboration data at energies: (a) 183 GeV and (b) 189 GeV of e^+e^- collisions. Solid curves correspond to NLO BFKL in BLM; dashed: LO BFKL and dotted: LO Born contribution. Two different curves are for two different choices of the Regge scale: $s_0 = Q^2/2$, $s_0 = 2Q^2$

it is theoretically more under control than the hadron-hadron and lepton-hadron collisions, where non-perturbative hadronic structure functions are involved. In addition, for the gamma-gamma scattering the unitarization (screening) corrections due to multiple Pomeron exchange would be less important than in hadron collisions.

The gamma-gamma cross sections with the BFKL resummation in the LO were considered in [4, 24, 25]. In the NLO BFKL case one should obtain a formula analogous to LO BFKL [23].

In Fig. 1 we present the comparison of BFKL predictions for LO and NLO BFKL improved by the BLM procedure with L3 data [26] from CERN LEP. The different curves reflect uncertainty of the theoretical predictions with the choice of the Regge scale parameter, s_0, which defines the transition to the asymptotic regime. For the present calculations two variants have been choosen $s_0 = Q^2/2$ and $s_0 = 2Q^2$, where Q^2 is the virtuality of the photons. One can see from Fig. 1 that the agreement of the NLO BFKL improved by the BLM procedure is reasonably good at energies of LEP2 $\sqrt{s_{e^+e^-}} = 183$ - 189 GeV. One can notice also that sensitivity of the NLO BFKL results to the Regge parameter, s_0, is much smaller than in the case of the LO BFKL.

It was shown in Refs. [27, 28] that the unitarization corrections in hadron collisions can lead to a higher value of the (bare) Pomeron intercept than the effective intercept value. Since the hadronic data fit yields about 1.1 for the effective intercept value [29], the bare Pomeron intercept value should be above this value. Therefore, assuming small unitarization corrections in the gamma-gamma scattering at large Q^2 one can accomodate the NLO BFKL Pomeron intercept value 1.13 - 1.18 [9] in the BLM optimal scale setting along with larger unitarization corrections in hadronic scattering [28, 29], where they can lead to a smaller effective Pomeron intercept value of about 1.1 for hadronic collisions. The above intercept value of the NLO BFKL Pomeron is in a good agreement with analysis [30] of the diffractive dijet production at the Tevatron.

Another possible application of the BFKL approach can be the collision energy dependence of the *inclusive* jet production [31, 22]. Unlike the case with the selection of most forward/backward (Mueller-Navelet) jets [19, 32, 33, 34, 35, 36, 37], the usual *inclusive* jets [20, 21, 22, 31, 38, 39] can be more reliable at the limited acceptance in rapidity detectors.

The advent of the Fermilab Tevatron and the CERN LHC provides a new testing ground for the parton model—the kinematic conditions when the energies of the produced hadrons are large enough to be described by perturbation theory and, at the same time, are much smaller than the total energy of the collision (BFKL semihard kinematics). Because the parton model was originally invented and

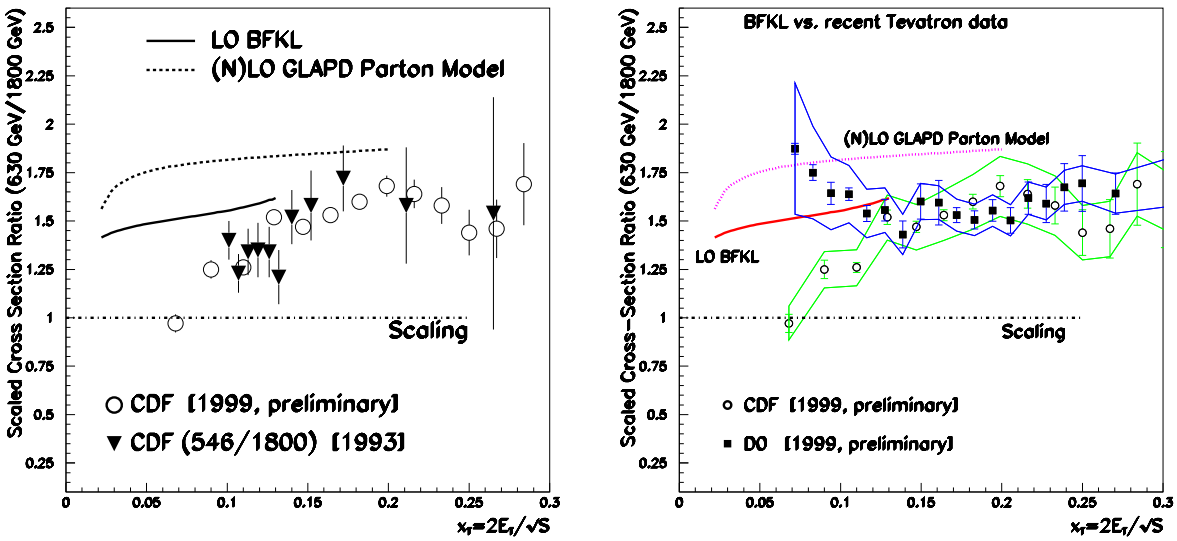

Figure 2: Inclusive single jet production: scaled cross section ratio at the Tevatron. BFKL [31, 22] and GLAPD [48] predictions versus Tevatron data: (a) CDF data [45]. (b) preliminary data from CDF and DØ Collaborations [47]

subsequently tested for the hard kinematics, the second condition makes it plausible that a substantial modification of the parton model will be needed to describe this BFKL semihard kinematic region.

The range of applicability of the QCD-improved parton model is a subject of controversy at the moment. There are statements (see, *e.g.*, [40, 41]) that the fitting capacity of the conventional QCD-improved parton model is sufficient to accomodate all the data on deep inelastic scattering (DIS) parton structure functions available at the small-x kinematics. On the other hand, the same data from HERA on DIS structure functions can be described by NLO BFKL [15, 16]. In addition, the data from HERA [42] and Tevatron [43] on most forward(backward) jet production may be interpreted as a manifestation of the BFKL Pomeron [33, 19], which is beyond the conventional QCD-improved parton model. The situation is further complicated by the observation that the range of applicability of the QCD-improved parton model may be different for different observables. In particular, the cross sections of processes with specific kinematics exhibit breakdown of the applicability of finite order perturbative QCD via the development of sensitivity to the choice of the normalization scale. On the other hand, some dedicated combinations (ratios) of cross sections may be less sensitive to the inclusion of the higher order corrections. An example is the scaled cross section ratio [44, 45, 46, 47] since, as it follows from Ref. [48], it is relatively insensitive to the inclusion of the NLO correction.

Under these circumstances, it is crucial to have *qualitative* predictions from the conventional QCD-improved parton model (without BFKL resummation of the energy logarithms) for the new kinematic domain. If the predictions would turn out qualitatively incorrect, a generalization, and a substitute in this kinematical domain of the QCD-improved parton model would become indispensable.

Here we discuss such a prediction of Ref. [49]. It is a prediction for the ratio of inclusive single jet production at a smaller energy $\sqrt{s_N}$ of the hadron collision to the one at a higher energy $\sqrt{s_D}$:

$$R(x,y,s_N,s_D)=\frac{s_N d\sigma}{dxdy}(x,y,s_N)\Big/\frac{s_D d\sigma}{dxdy}(x,y,s_D). \tag{1}$$

Here the cross section is made dimensionless by the rescaling with the corresponding total invariant energy of the collision squared $s_N(s_D)$. The ratio depends on the (pseudo)rapidity $y=1/2\ln(k_+/k_-)$, where $k_\pm=E\pm k_3$ are the light-cone components of the momentum of the produced jet, and on the fraction of the energy $x=(k_++k_-)/\sqrt{s_i}$, $i=N,D$ deposited in the jet produced (s_N is used for the definition of x in the numerator, s_D in the denominator, so x varies from zero to unity for both energies). Note that this scaling variable coincides in the center of mass system with $x_R=E/E_{max}=2E/\sqrt{s}$, the radial Feynman variable, or for $y=0$, *i.e.*, for ($\theta_{CMS}=\pi/2$), the scaling variable becomes the transverse Feynman variable: $x=x_\perp=2E_\perp/\sqrt{s}$.

The ratio R (taken at $y \simeq 0$, *i.e.*, for jets perpendicular to the collision axes) was used in Refs. [44, 45, 46, 47] as a means to test QCD predictions for scaling violations.

$$R(x, y=0, s_N, s_D) = E_\perp^4 \frac{Ed\sigma}{d^3p}(x_\perp, y=0, s_N) \Big/ E_\perp^4 \frac{Ed\sigma}{d^3p}(x_\perp, y=0, s_D) \quad (2)$$

$$= E_\perp^4 \frac{d\sigma}{dyd E_\perp^2}(x_\perp, y=0, s_N) \Big/ E_\perp^4 \frac{d\sigma}{dyd E_\perp^2}(x_\perp, y=0, s_D). \quad (3)$$

Note that without scaling violations the ratio R is *exactly* unity. At fixed y and $x_\perp$ the dependence of R from s_N, s_D comes from the presence of fundamental QCD scale, Λ_{QCD}, in the running coupling and in the parton distribution functions.

To be more explicit, we remark that the QCD-improved parton model, based on the factorization theorem for hard processes and the logarithm-Q^2 GLAPD-evolution, presents the inclusive jet scaled cross section in hadron collsions as

$$E_\perp^4 \frac{Ed\sigma}{d^3p}(x_\perp, y=0, s) = \int_{x_{A,min}}^1 \int_{x_{B,min}}^1 dx_A dx_B F_A(x_A, Q^2) F_B(x_B, Q^2) E_\perp^4 \frac{\hat{s}}{\pi} \frac{d\hat{\sigma}}{d\hat{t}} \delta(\hat{s} + \hat{t} + \hat{u}), \quad (4)$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are Mandelstam variables for the partonic subprocess, the scale of the hard partonic subprocess $-\hat{t} = Q^2 \sim E_\perp^2 \sim x_\perp^2 s$, F_A and F_B are parton distribution functions with the GLAPD-evolution following from perturbative $\alpha_S(Q^2)$-expansion, and the scaled partonic subprocess is being as

$$E_\perp^4 \frac{d\hat{\sigma}}{d\hat{t}} \sim \alpha_S^2(Q^2)[1 + C_{NLO}\alpha_S + ...] = \alpha_S^2(x_\perp s^2)[1 + C_{NLO}\alpha_S + ...].$$

Hence, within the QCD-improved parton model the scaled cross section ratio for inclusive jet production at fixed x and y is the dimensionless function of α_S. The GLAPD scaling violation due to the interacting QCD-partons appears as the *logarithmic*[2] dependence on the total energy of collision though the coupling constant α_S. We note here that, the BFKL leads to a *power-like* scaling violation, the strength of which depends on the Regge-scale s_0.

Perturbative QCD calculations of Ref. [48] with hard kinematics ($Q^2 \sim s$) predict for R at $y = 0$ a steep increase around the value of 1.8-1.9 for x growing in the range above 0.1 (for the case $\sqrt{s_N}/\sqrt{s_D} =$ 0.63 TeV/1.8 TeV). For moderate x, the prediction is in reasonable agreement with CDF data [45, 46]. For $x < 0.1$ calculations are above the preliminary data of CDF [46]. This was among the reasons for the conclusion of Ref. [48] that NLO GLAPD [52] with hard kinematics is insufficient for the description of the absolute cross section of jets with transverse energy less than 50 GeV within an accuracy of 10%. It was shown in Ref. [31] that resummation of the energy logarithms, *i.e.*, BFKL, restores the agreement between theory and experiment.

In Ref. [49] we have found the following result: the QCD-improved parton model predicts that R is not a monotonic function of its arguments, *i.e.*, the single jet production cross section, if measured in the natural units of the same cross section taken at another (higher) energy of the collision, has extrema. Namely, it has minima ("dips"): there is a value of x for each y with the smallest ratio of jets produced. The reason this fact was overlooked is that for $y = 0$ (the only value for which the calculations were reported earlier) the minimum is at a value of x too small to be inside the acceptance of the existing detectors ($x_{dip}(y = 0) < 0.01$ at the Tevatron).

Fig. 3(a) presents the ratio for energies 0.63 TeV/1.8 TeV at the Fermilab Tevatron, Fig. 3(b) for energies 6 TeV/14 TeV at the CERN LHC. Each curve on the plots presents the dependence of the ratio on x at different values of rapidity y. Each curve ends at a lowest value of x where $\alpha_S(Q^2)$ has the value of about 0.5 (it corresponds to $Q = 0.7$ GeV, and Q was taken to be half of the transverse energy of the jet produced). For lower values of x, perturbative theory becomes unreliable because the coupling approaches unity.

[2]Indeed, the small-x asymptotics of the GLAPD-evolution gives a growth of parton structure functions which is faster than any power of a logarithm, but slower than any power - so-called double logarithmic asymptotics [50, 6, 51, 40].

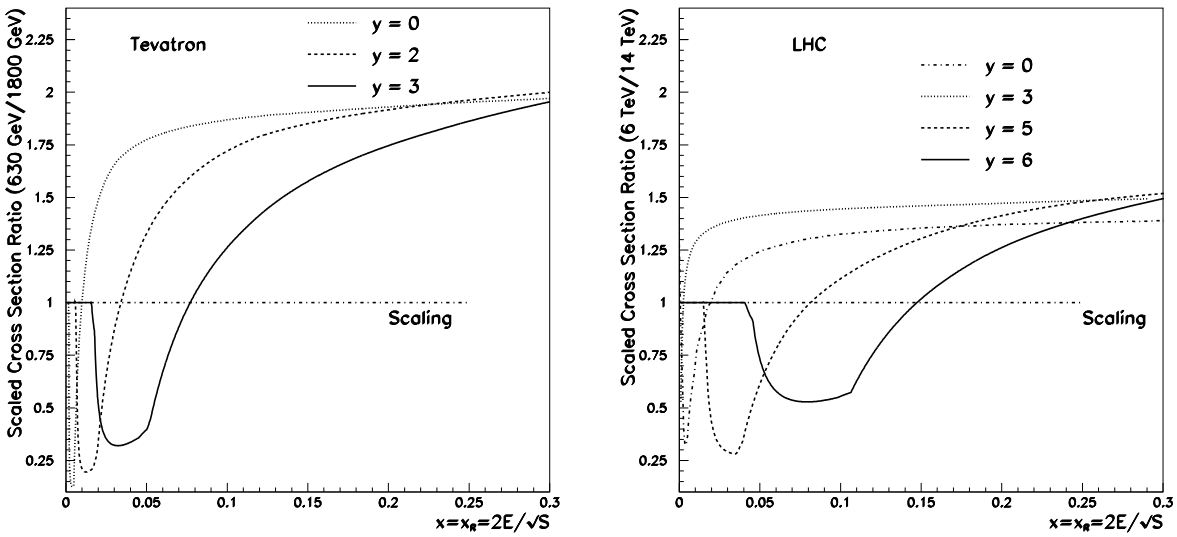

Figure 3: Ratio of scaled cross sections of inclusive single jet production (a) for Fermilab energies; (b) for the LHC energies

There is another lower bound on the values of x at which our plots make sense, because there is a lowest energy for which the jet may be resolved. This energy is accepted now to be around 5 GeV [3] which corresponds to $x > 0.016$ for the Tevatron (Fig.3a), and to $x > 0.0017$ for the LHC (Fig.3b).

There is an important issue concerning the accuracy of the present LO calculation. The most important advantage of the scaled cross section ratio is that this is the ratio of two perturbative series with the same coefficients and with different scales in the running coupling. These scales are defined by the two initial collision energies at fixed scaling variable. One can show that the theoretical accuracy of the ratio in LO of perturbative QCD is not less than the accuracy of NLO calculations for absolute cross sections.

The minima in Fig. 3 originate from a competition between the running of the parton distribution functions and the running of the coupling constant. Namely, the ratio with frozen parton distribution functions is decreasing monotonously (this tendency is realized at small x), while the one with frozen coupling constant is growing monotonously (this latter tendency is realized for x larger than the position of the minimum).

We suggest the following potential implications of the minima we have predicted with the parton model: (i) If one observes the minima experimentally, one employs the orthodox QCD-improved parton model and tries to account for observed positions and depths of the minima by taking into account higher order corrections, in particular, resummation of the energy logarithms. (ii) If one does not observe the minima experimentally, more radical changes are motivated such as an alternative model of the effective constituents inside the hadrons for the BFKL semihard asymptotics. One example might be the color dipole model [54].

Finally, we comment on the possibility of searching for the minima at the Fermilab Tevatron and at the CERN LHC: Positions of the minima for the Tevatron energies (Fig.3a) seem to be reached by both D$\emptyset$ and CDF detectors. The minima of the LHC plot (Fig.3b) seem to be well inside the acceptance of, *e.g.*, the FELIX [55], the ALICE [56] and the CMS [57] detectors.

We take the ratio of 6 TeV/14 TeV for the LHC, because, in addition to 14 TeV pp collisions, lead-lead collisions at the LHC are planned with the collision energy of 6 TeV per nucleon-nucleon collision. Since nuclear collisions bring in nuclear effects which can distort our predicted curves, we also considered the ratio 6 TeV/100 TeV (see Ref. [49]).

Further consideration should be given for which pair of energies and value of rapidity are most convenient for an experimental search of the minima. Also, more work is needed to make quantitative predictions for the locations and the shapes of the dips with the NLO corrections taken into account. It

[3] At the HERA, lepton-hadron collider, jets are resolved from $E_\perp = 3$ GeV, and at the Tevatron, hadron-hadron collider, the CDF Collaboration tags jets from $E_\perp = 8$ GeV [53].

is interesting to observe a resemblance of the dips presented here] with the non-monotonic behavior of parton distribution functions in DIS [58].

It is also worth noting that, in the case of nuclear collisions the effects of initial nuclear parton distributions (small-x EMC-effect [59]) and dynamical effects like quark-gluon plasma, jet quenching, *etc.* [60], phase structure of the QCD vacuum [61, 62] will demand special consideration.

Before making conclusions, we would like to note that many of the above ideas can be studied also for the case of heavy quarkonium production, where similar phenomena should be present [63].

To sum up, we find a new qualitative prediction of the QCD-improved parton model for hadron collisions and suggest its use to test the applicability of the parton model for certain regions of high energy hadron collisions. Study of the scaling violation of the scaled cross section ratio can reveal such new dynamic effects as the BFKL asymptotics.

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