BRST-anti-BRST covariant theory for the second class constrained systems.

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Abstract

The BRST-anti-BRST covariant extension is suggested for the split involution quantization scheme for the second class constrained theories. The constraint algebra generating equations involve on equal footing a pair of BRST charges for second class constraints and a pair of the respective anti-BRST charges. Formalism displays explicit $Sp(2) \times Sp(2)$ symmetry property. Surprisingly, the the BRST-anti-BRST algebra must involve a central element, related to the nonvanishing part of the constraint commutator and having no direct analogue in a first class theory. The unitarizing Hamiltonian is fixed by the requirement of the explicit BRST-anti-BRST symmetry with a much more restricted ambiguity if compare to a first class theory or split involution second class case in the nonsymmetric formulation.

1 Introduction

The BFV–BRST theory originally appeared as a tool for quantization of the gauge fields or, from the standpoint of a general Hamiltonian formalism, constrained dynamical systems. Hamiltonian BFV–BRST formalism first developed by Batalin, Fradkin and Vilkovisky [1] (for review see [2, 3]) is well established today to solve, in principle, quantization problem in a general first class constrained system. The anti-BRST symmetry [4] attracted considerable interest due to an elegant structure revealed in quantum theory possessing both BRST and anti–BRST symmetry on equal footing [5, 6, 7, 8]. In the $Sp(2)$ covariant description, BRST and anti-BRST charges form a doublet Ω^a, $a = 1, 2$. In nonsymmetric approach, one of them, say Ω^1, could be identified to a charge, and another one to an anti–charge. An $Sp(2)$ symmetry of the formalism can be retained at all the stages of the quantization procedure although it restricts the choice of gauge fixing term.

In contrast to a first class constraint theory, a second class one does not have today a universal tool for quantization. Three basic trends could be distinguished among the approaches to quantization of second class constrained systems. The first trend is to reduce the phase space eliminating the constraints. This approach is often related to an attempt to find a realization for a Dirac bracket in quantum theory. In the path integral formalism for the theories with nontrivial second class constraints, the idea of reduction results in a singular measure [9, 10]. The second trend, quite opposite to the first one, implies to extend the initial phase space by auxiliary variables to convert the original second class constraints into effective first class ones in the extended manifold. This idea was applied probably first by Stuckelberg [11] to gauge the massive abelian vector field theory introducing an auxiliary scalar field. From the viewpoint of the general Hamiltonian constrained dynamics, the conversion idea was suggested in Ref [12, 13] and the respective general theory was developed in Ref[14, 15]. The weak point in the conversion idea is related to the possible conflict between the space-time covariance and locality and embedding of the original second class surface in the extended manifold. The third trend is an attempt to quantize second class theory as it is, i.e. without recourse to reduction or extension of the original phase space. With this regard we refer to the papers [16, 17, 18, 19]. In particular, in the split involution approach [16, 17], the second class theory is quantized with the original phase space variables (no extra variables introduced) subject to *canonical* commutation relations, whereas the constraints are accounted by weak conditions selecting physical states. These conditions are related to the extended BRST symmetry associated in this approach to the second class constraints. This BRST symmetry is generated for the pure second class constrained theory [16] by a pair of independent BRST charges constructed from the original constraints[1]. The only restriction is that the specific constraint basis should be chosen for this construction to obey the split involution conjecture [16] (which is briefly recalled in section 2).

[1] Generalization of the method to the case of both first and second class is given in [17], the existence theorem for this extended BRST algebra is proven in [20]

This talk is devoted to the BRST-anti-BRST symmetric extension for the split involution method for a pure second class constrained theory first suggested in our paper [21]. This extension shows some essentially new features both from the viewpoint of the algebra and it's dynamical content as compared to the respective construction known in a first class theory. In particular, we observe that the BRST-anti-BRST algebra must have a central extension in the second class theory. The central element originates from the revertible element in the second class constraint commutators. This extended formulation allows to explicitly construct BRST-anti-BRST invariant quantum theory for any second class system if the constraint basis was found subject to the split involution conjecture. Stress once again that, in our approach, *the BRST embedding of the second class system is performed without transformation of the second class constraints into effective first class ones and without any auxiliary variables "gauging" the original theory.*

2 Split involution construction.

Consider two sets of linearly independent second class constraints $T^a_\alpha(q,p)$, $a=1,2\,;\ \alpha=1,...,m.$[2] The Poisson bracket of the constraints has symmetric and antysymmetric parts in $a\,b$:

$$\{T^a_\alpha, T^b_\beta\} = \frac{1}{2}\{T^{(a}_\alpha, T^{b)}_\beta\} + \frac{1}{2}\{T^{[a}_\alpha, T^{b]}_\beta\}\,, \tag{1}$$

$$A^{\{a\,b\}} = A^{(a\,b)} = A^{ab} + A^{ba}\,, \quad A^{[ab]} = A^{ab} - A^{ba}\,,$$

$$\{T^{[a}_\alpha, T^{b]}_\beta\} \equiv \epsilon^{ab}\Delta_{\alpha\beta}\,. \tag{2}$$

where ε_{ab} is an $Sp(2)$–invariant constant tensor $\epsilon^{12}=1, \quad \epsilon_{12}=-1, \quad \epsilon^{ab}\epsilon_{bc}=\delta^a_c$.

The split involution conjecture [16] implies that it is the antisymmetric part (2) which forms the invertible element in the constraint matrix (1)

$$\exists\Delta^{-1}\,, \qquad \Delta_{\alpha\beta} \equiv \Delta_{\beta\alpha} \equiv \epsilon_{ba}\{T^a_\alpha, T^b_\beta\}\,, \tag{3}$$

The symmetric part of the constraint commutator (1) and the Hamiltonian should obey the split involution relations

$$\{\,T^{(a}_\alpha\,,\,T^{b)}_\beta\,\} = U^{(a\,\gamma}_{\alpha\beta}(q,p)\,T^{b)}_\gamma\,, \quad U^{a\ \gamma}_{\alpha\beta} = -\,U^{a\ \gamma}_{\beta\alpha}\,, \tag{4}$$

$$\{\,H\,,\,T^b_\beta\} = V_\beta{}^\gamma T^b_\gamma\,, \quad V_\beta{}^\gamma = V_\beta{}^\gamma\,(q,p) \tag{5}$$

The split involution relations (4,5) do not actually restrict the second class constraint surface, although they require a special basis for the constraints and a special choice of the Hamiltonian outside the constraint surface. In the simplest case, when $U^{a\ \gamma}_{\alpha\beta}=$ const, the relations (4) define a *pair of Lie algebras* mutually related by a certain compatibility condition involving both sets of the structure constants U^1, U^2 [16]:

$$U^{(a\,\gamma}_{\alpha\beta}U^{b)\ \rho}_{\gamma\nu} + U^{(a\,\gamma}_{\beta\nu}U^{b)\ \rho}_{\gamma\alpha} + U^{(a\,\gamma}_{\nu\alpha}U^{b)\ \rho}_{\gamma\beta} = 0\,, \tag{6}$$

These conditions include, besides the Jacobi identity for U^1 and, independently, U^2, some additional restrictions (appeared when $a\neq b$ in (6)) to the respective Lie algebras. For the case $V_\beta{}^\gamma = const$, it should obey the compatibility condition following immediately from (4), (5):

$$U^{a\ \gamma}_{\alpha\beta}V_\gamma{}^\nu - V_\alpha{}^\gamma U^{a\ \nu}_{\gamma\beta} + V_\beta{}^\gamma U^{a\ \nu}_{\gamma\alpha} = 0.$$

To build the BRST invariant theory, the split involution method implies to introduce Lagrange multipliers $\lambda^a_\alpha\,,\ a=1,2\,,\ \alpha=1,\ldots,m$

$$\{\lambda^a_\alpha,\,\lambda^b_\beta\} = d_{\alpha\beta}\,\epsilon^{ab}\,, \quad d_{\alpha\beta}\equiv d_{\beta\alpha} = \text{const}\,, \quad \exists d^{\alpha\beta}\ :\ d^{\alpha\beta}d_{\beta\gamma} = \delta^\alpha_\gamma\,, \tag{7}$$

and an odd ghost variable set for each the pair T^a_α, $a=1,2$:

$$\{\,C^\alpha\,,\,\bar{\mathcal{P}}_\beta\,\} = \delta^\alpha_\beta\,, \quad \{\,\mathcal{P}^\alpha\,,\,\bar{C}_\beta\,\} = \delta^\alpha_\beta\,, \quad gh(C)=gh(\mathcal{P})=1\,,\ gh(\bar{C})=gh(\bar{\mathcal{P}})=-1\,. \tag{8}$$

[2]In this paper we consider only even Grassmann constraints to avoid sign factors, although no essential problem appears in an odd case as well.

Next is to construct a $Sp(2)$ doublet of the BRST generators Q^a and BRST invariant Hamiltonian $\mathcal{H}$, being defined by the $Sp(2)$ covariant generating equations

$$\{ Q^a , Q^b \} = 0 , \quad \{ Q^a , \mathcal{H} \} = 0, \tag{9}$$

Then the complete Unitarizing Hamiltonian of the theory is built in the $Sp(2)$–symmetric form

$$H_{complete} = \mathcal{H} + \frac{1}{2}\varepsilon_{ab} \{ Q^b , \{ Q^a , B \} \} , \tag{10}$$

where B is known as a "gauge–fixing" Boson, $gh(B) = -2$. The simplest possible choice for B is as follows

$$B = \bar{\mathcal{P}}_\alpha \bar{C}^\alpha \tag{11}$$

Being the physical quantities defined in an invariant way, they do not depend on a particular choice of the B. This independence is quite a nontrivial feature of the split involution scheme [16, 17], because we have pure second–class constraints which do not generate an actual gauge symmetry.

The solution for the Fermions Q^a, $a = 1, 2$ and Boson $\mathcal{H}$ is sought in the form of a series expansion in ghost powers

$$Q^a = C^\mu T^a_\mu + \mathcal{P}^\alpha \lambda^a_\alpha + \dots , \quad \mathcal{H} = H + \dots . \tag{12}$$

These relations can be thought about as a boundary condition to the generating equations (9) Mention that the Poisson bracket between Q^a does not include an antisymmetric combination a and b, so the commutativity of the charges (9) has no contradiction to the noncommutativity of the corresponding constraints (3).

We would like to note again that the extended BRST symmetry in the split involution theory (9, 10, 12) has no relation to any gauge transformations as all the constraints T^a_α, being involved into Q^a (12), are of the second class indeed (3). The commutativity of the charges and Hamiltonian (9) is possible in spite of noncommutativity of the constraints themselves (3). The BRST symmetry emerges here not from the gauge identities of the original theory, it follows from the remarkable polarization of the second class constraint surface revealed in the split involution relations (4, 5).

3 BRST–anti–BRST symmetric description for the second class theory

In this section, we introduce an anti–BRST charge for each of the charges Q^a and formulate the BRST–anti–BRST symmetric extension for the generating equations (9, 12) of the split involution method.

At first we need the covariant notation for the ghost anti-ghost variables themselves. Introduce a doublet of the odd Grassman ghost canonical pairs $C^{\dot{a}\,\alpha}, \mathcal{P}_{\dot{b}\,\alpha}$, $\dot{a} = 1, 2$ for each the constraint pair T^a_α. The indices of the $Sp(2)$ group, which specify ghost or anti–ghost canonical pair in this doublet, are denoted by dots in distinction from the indices related to the second class constraints T^a_α and the corresponding Lagrange multipliers (7)

$$\{ C^{\dot{a}\,\alpha} , \mathcal{P}_{\dot{b}\,\beta} \} = \delta^{\dot{a}}{}_{\dot{b}} \ \ \delta^\alpha{}_\beta , \quad gh(C^{\dot{a}\,\alpha}) = -gh(\mathcal{P}_{\dot{a}\,\alpha}) = (-1)^{\dot{a}+1}. \tag{13}$$

We find sometimes convenient to use both lower and upper indices $\dot{a} = 1, 2$ and $\alpha = 1, \dots , m$:

$$\mathcal{P}^{\dot{b}\,\beta} = \mathcal{P}_{\dot{c}\,\gamma}\epsilon^{\dot{c}\dot{b}} d^{\gamma\beta} , \quad \{ C^{\dot{a}\,\alpha} , \mathcal{P}^{\dot{b}\,\beta} \} = \epsilon^{\dot{a}\dot{b}} d^{\alpha\beta} .$$

Finally, we have the following complete set of canonical variables:

$$\Gamma^A = (q^i , p_j ; \ C^{\dot{a}\,\alpha} , \mathcal{P}_{\dot{b}\,\beta} ; \ \lambda^{a\,\alpha}) , \qquad a = 1, 2 , \quad \dot{a} = 1, 2 , \quad \alpha = 1, \dots , m .$$

which includes, besides the original canonical coordinates and momenta q^i, p_j, a doublet of canonical ghost–anti–ghost pairs (13), and one canonical pair of the Lagrange multipliers (7), for each the constraint pair.

Introduce the generating operators $\Omega^{\dot a\, a}(\Gamma)$ and $\mathcal{H}(\Gamma)$ with the following distribution of the ghost numbers

$$\varepsilon(\Omega^{\dot a\, a}) = 1\,, \quad gh(\Omega^{\dot a\, a}) = (-1)^{\dot a+1}\,, \qquad \varepsilon(\mathcal{H}) = 0\,, \quad gh(\mathcal{H}) = 0\,.$$

to be a power series in ghosts, anti–ghosts and Lagrange multipliers

$$\Omega^{\dot a\, a} = C^{\dot a\,\mu}\, T^a_\mu \,-\, \mathcal{P}^{\dot a\,\alpha}\lambda^a_\alpha \,+\, \dots\,, \quad \mathcal{H} = H + \dots\,. \tag{14}$$

Thus, the pair $\Omega^{\dot 1\, a}\,, a=1,2$ can be considered as BRST generators (12), while the other pair $\Omega^{\dot 2\, a}\,, a=1,2$ should serve as an $Sp(2)$ doublet of anti-BRST generators.

The Poisson bracket between $\Omega^{\dot a\, a}$ is decomposed into totally symmetric and antisymmetric part with respect to permutations of the indices $a=1,2$ and, independently, $\dot a = 1,2$:

$$\{\,\Omega^{\dot a\, a}\,,\,\Omega^{\dot b\, b}\,\} \equiv \{\,\Omega^{\dot a\, a}\,,\,\Omega^{\dot b\, b}\,\}_{sym} + \epsilon^{\dot a\dot b}\epsilon^{ab}\,\Delta\,,$$

where $\{\,,\,\}_{sym}$ is a symmetric part:

$$\{\,\Omega^{\dot a\, a}\,,\,\Omega^{\dot b\, b}\,\}_{sym} \equiv \frac{1}{4}\{\,\Omega^{(\dot a\,\{a}\,,\,\Omega^{\dot b)\,b\}}\,\}\,,$$

$$\Delta \equiv \frac{1}{4}\,\epsilon_{\dot a\dot b}\epsilon_{ab}\{\,\Omega^{\dot a\, a}\,,\,\Omega^{\dot b\, b}\,\}\,, \quad \varepsilon(\Delta)=0\,, \quad gh(\Delta)=0\,. \tag{15}$$

We observe, in contrast to the nonextended case, that the Poisson bracket between the extended BRST generators may involve antisymmetric part in a and b which emerges when anyone of the charges is taken together with the anti–charge to another "half" of the constraints. As the different constraint "halves" T^1_α, T^2_β do not commute to each other (3), the totally antisymmetric part (15) can not vanish. That is why, the commutation relations between BRST and anti-BRST charges have to involve the new generator Δ (15) which has direct analogue neither in a first class theory nor in the nonextended BRST formulation for a split involution second class system. As to the symmetric part of the commutator between the generators $\Omega^{\dot a\, a}$, the formalism contains no Sp(2) covariant tensors to be involved in the r.h.s. Moreover, comparison to the nonextended theory reveals obstructions to noncommutativity in the symmetric sector.

With the account of all said above, one may impose the following extended system of generating equations

$$\{\,\Omega^{\dot a\, a}\,,\,\Omega^{\dot b\, b}\,\}_{sym} = 0\,, \tag{16}$$

$$\{\,\Omega^{\dot a\, a}\,,\,\mathcal{H}\,\} = 0\,, \tag{17}$$

which reveals an explicit $Sp(2)$ symmetry under ghost–antighost $Sp(2)$–rotations, and independently, it has an invariant form with respect to the $Sp(2)$ transformations of the split involution constraint basis T^a_α. The equations (16), (17) should be supplemented by the boundary conditions following the definitions (14)

$$\left.\frac{\delta\Omega^{\dot a\, a}}{\delta C^{\dot b\,\alpha}}\right|_{C=\mathcal{P}=\lambda=0} = T_\alpha\delta^{\dot a}{}_{\dot b}\,, \quad \left.\frac{\delta\Omega^{\dot a\, a}}{\delta\lambda^{b\,\alpha}}\right|_{C=\lambda=0} = -\mathcal{P}^{\dot a}_\alpha\delta^a{}_b = -\mathcal{P}_{\dot b\,\alpha}\epsilon^{\dot b\dot a}\,\delta^a{}_b\,, \tag{18}$$

$$\mathcal{H}|_{C=\mathcal{P}=\lambda=0} = H. \tag{19}$$

In principle, one may omit the symmetrization in the left hand side of the equation (16), including simultaneously Δ (15) into the r.h.s. Then the generating equations (16) read

$$\{\,\Omega^{\dot a\, a}\,,\,\Omega^{\dot b\, b}\,\} = \epsilon^{\dot a\dot b}\epsilon^{ab}\,\Delta\,, \tag{20}$$

and involve, in a sense, the revertible element of the constraint Poisson bracket matrix $\Delta_{\alpha\beta}$ (3) entering Δ (15). This unusual mixture of the revertible element and involution relations, being encoded in the equations (20), contains no contradiction as the compatibility condition

$$\{\,\Omega^{\dot a\, a}\,,\,\Delta\,\} = 0 \tag{21}$$

is automatically fulfilled by virtue of the definition Δ (15) and Jacobi identity. In fact, the totally antisymmetric part of the first of the relations (20) is not a new equation but just the definition for the quantity Δ. Mention that the equations (17) and (20) imply the compatibility condition

$$\{\,\mathcal{H}\,,\,\Delta\,\} = 0\,, \tag{22}$$

which is not a new restriction as well. Relations (20,21,22) show that the split involution BRST - anti - BRST algebra is extended by the central element Δ (15), which emerges from the noncommutativity of the second class constraints.

The absence of the algebraic contradictions among the generating equations (16) (or (20)) and (17) does not automatically provide the existence for the solution with regard of the given boundary conditions (18), (19). The question is: whether the split involution relations, being imposed onto the original constraints and Hamiltonian (4), (5), are sufficient to provide the solution to (16), (17), or one should strengthen the restrictions to the constraint algebra. In general, the problem remains to be investigated, but for the case of an algebra (more precisely, when the structure functions $U^a_{\alpha\beta}{}^\gamma$ (4), $V_\beta{}^\gamma$ (5) are constants) we have obtained an explicit solution to the generating equations. As is shown below in this Section, the solution exists for a general split involution algebra (4) and no restrictions appear to the structure constants $U^a_{\alpha\beta}{}^\gamma$ besides the respective Jacobi identity (6). However, the group of the BRST-anti-BRST symmetric generating equations for the Hamiltonian $\mathcal{H}$ (17) restricts the structure constants $V_\beta{}^\gamma$ (5) to obey an extra symmetry requirement. This new condition (which is necessary to provide an explicit BRST-anti-BRST symmetry) does not restrict the Hamiltonian on the constraint surface, and could be always met by adding to H certain contributions squared in constraints. In a sense, this symmetry requirement to structure constants $V_\beta{}^\gamma$ (5) is an analog to the restriction imposed to the gauge fixing condition to provide the BRST-anti-BRST symmetric description in a first class theory [5, 6, 7, 8].

Let us expose the solution to generating equations with the split involution structure coefficients $U^a_{\alpha\beta}{}^\gamma$, $V_\beta{}^\gamma$(4), (5) being constants:

$$\begin{aligned}
\Omega^{\dot a\,a} \;&=\; C^{\dot a\,\alpha}T^a_\alpha \;-\; \mathcal{P}^{\dot a}_\alpha\lambda^{a\,\alpha} \;+\; \frac{1}{2}C^{\dot a\alpha}C^{\dot b\beta}\mathcal{P}^{\dot c}_\nu\epsilon_{\dot b\dot c}\,U^a_{\alpha\beta}{}^\nu \;+\\
&+\; \frac{1}{2}C^{\dot a\,\nu}\lambda^{a\,\alpha}\lambda^c_\beta\epsilon_{cd}U^d_{\nu\alpha}{}^\beta \;+\; \frac{1}{24}C^{\dot b\,\alpha}C^{\dot c\,\beta}C^{\dot a\,\nu}\lambda^c_\rho\epsilon_{\dot b\dot c}\epsilon_{cd}\,U^{(a\,\gamma}_{\alpha\nu}U^{d)\,\rho}_{\gamma\beta} \;+\\
&+ C^{\dot b\,\alpha}C^{\dot c\,\beta}C^{\dot a\,\nu}\lambda^{a\,\rho}\epsilon_{\dot c\dot b}\,Z_{\alpha\beta\nu\rho} \;+\; C^{\dot b\,\alpha}C^{\dot c\,\beta}C^{\dot k\,\mu}C^{\dot l\,\nu}C^{\dot a\,\rho}\epsilon_{\dot b\dot c}\epsilon_{\dot k\dot l}\,R^a_{\alpha\beta\mu\nu\rho}\,,
\end{aligned} \tag{23}$$

$$\mathcal{H} \;=\; H \,+\, C^{\dot a\,\alpha}\mathcal{P}^{\dot b}_\beta V_\alpha{}^\beta\epsilon_{\dot a\dot b} \,+\, \frac{1}{2}\lambda^{a\,\alpha}\lambda^b_\beta\,V_\alpha{}^\beta\epsilon_{ba}\,. \tag{24}$$

The coefficients $U^a_{\alpha\beta}{}^\gamma$ (23) are identified to the respective constants of the split involution (4) and required to obey only the identity (6). The constants $V_\alpha{}^\beta$ (24) should coincide to the corresponding structure coefficients of the split involution relations for the original Hamiltonian (5) and, besides that, they should be antisymmetric

$$V_{\alpha\beta} \;=\; -\,V_{\beta\alpha}\,, \quad V_{\alpha\beta} = V_\alpha{}^\gamma\,d_{\gamma\beta}\,. \tag{25}$$

The relation (25) does not automatically follow from the respective split involution relations (5), so it is a restriction to the definition of the Hamiltonian H off the constraints.

The higher order structure coefficients $Z_{\alpha\beta\nu\rho}$ and $R^a_{\alpha\beta\mu\nu\rho}$ (23) are expressed via the original structure constants $U^a_{\alpha\beta}{}^\gamma$ as follows

$$\begin{aligned}
Z_{\alpha\beta\nu\rho} = \epsilon_{bc}\Bigg\{&\frac{1}{24}U^b_{\nu\alpha}{}^\gamma U^c_{\gamma\beta}{}^\sigma d_{\sigma\rho} + \frac{1}{12}U^b_{\nu\alpha}{}^\gamma U^c_{\gamma\rho}{}^\sigma d_{\sigma\beta} +\\
&+ \frac{1}{12}U^b_{\beta\rho}{}^\gamma U^c_{\gamma\alpha}{}^\sigma d_{\sigma\nu} + \frac{1}{6}U^b_{\alpha\rho}{}^\gamma U^c_{\gamma\nu}{}^\sigma d_{\sigma\beta} + \frac{1}{8}\,U^b_{\rho\beta}{}^\gamma U^c_{\nu\alpha}{}^\sigma d_{\sigma\gamma}\Bigg\};
\end{aligned} \tag{26}$$

$$R^a_{\alpha\beta\mu\nu\rho} = \epsilon_{bc}\Big[\, \frac{1}{48} U^b_{\alpha\mu}{}^{\gamma} U^c_{\gamma\nu}{}^{\sigma} U^a_{\rho\beta}{}^{\theta} d_{\theta\sigma} + \frac{1}{16} U^b_{\rho\nu}{}^{\gamma} U^a_{\gamma\beta}{}^{\sigma} U^c_{\mu\alpha}{}^{\theta} d_{\theta\sigma} + $$
$$ + \frac{1}{24} U^b_{\mu\alpha}{}^{\gamma} U^c_{\gamma\rho}{}^{\sigma} U^a_{\sigma\beta}{}^{\theta} d_{\theta\nu} + \frac{1}{24} U^b_{\rho\mu}{}^{\gamma} U^c_{\gamma\nu}{}^{\sigma} U^a_{\sigma\alpha}{}^{\theta} d_{\theta\beta}\Big]. \tag{27}$$

In comparison to the respective solution of the split involution generating equations without an explicit anti-BRST symmetry (9) [16], we observe that the terms are engaged of a new type. In particular, the solution for $\Omega^{\dot{a}a}$ (23) includes contributions of higher orders in ghosts $C^{\dot{a}\alpha}$ and Lagrange multipliers λ^a_α. Of course, the solution (23) obeys the nonextended equations (9) with the identification $\Omega^{\dot{1}a} \equiv Q^a$, but for Q^a one may find another solution without higher orders in ghosts and Lagrange multipliers. Usually only these simplest solutions are considered in the nonextended case, although the general solution to (9) contains an arbitrary part [20] which may include more complicated terms like those constitute the higher order contributions in (23).

Now we are in the position to discuss the invariant total Hamiltonian which has to involve the terms with Lagrange multipliers and constraints, terms like Faddeev-Popov matrix, etc. Consider the value Δ (15) which corresponds the particular solution (23) related to the BRST-anti-BRST generators with the abelian constraints $U^1 = U^2 = 0$:

$$\Omega^{\dot{a}\,a} = C^{\dot{a}\,\alpha} T^a_\alpha - \mathcal{P}^{\dot{a}}_\alpha \lambda^{a\,\alpha} \,,$$
$$\Delta \equiv \frac{1}{4}\,\epsilon_{\dot{a}\dot{b}}\epsilon_{ab}\{\Omega^{\dot{a}\,a},\ \Omega^{\dot{b}\,b}\} = -\frac{1}{4} C^{\dot{a}\,\alpha} C^{\dot{b}\,\beta} \epsilon_{\dot{a}\dot{b}} \Delta_{\alpha\beta} + \epsilon_{ab} T^a_\alpha \lambda^{b\beta} - \frac{1}{2}\mathcal{P}^{\dot{a}}_\beta \mathcal{P}^{\dot{b}}_\alpha \epsilon_{\dot{a}\dot{b}} d^{\alpha\beta} \,.$$

One may see that the later expression contains all the necessary "gauge–fixing" terms. As Δ commutes to $\Omega^{\dot{a}\,a}$ in general case (21) (with different nonvanishing structure functions U^1, U^2), it could be used to construct the sought–for unitarizing Hamiltonian in the following way:

$$H_{complete} = \mathcal{H} + \Delta \,, \tag{28}$$

the Hamiltonian $\mathcal{H}$ is defined by equations (17), Δ conserves with respect to this total Hamiltonian,

$$\{\, \Delta \,,\, H_{complete} \,\} = 0 \,.$$

In the rank-1 theory (more precisely when all the structure functions U^1, U^2, V are constants), the explicit expression for $\mathcal{H}$ has been obtained in (24), the explicit expression for Δ has been obtained in [21].

4 Concluding remarks

The suggested extension for the BRST-anti-BRST algebra, being underlaid by the split involution relations, involves on equal footing 2 BRST generators and 2 anti-BRST ones, and it should contain a new central element Δ which appears in the commutator of a charge and another anti-charge. This central element involves the nonvanishing part of the second class constraint commutator and, thereby it could be never eliminated from the algebra. What is more, it is the central element Δ which determines the unitarizing Hamiltonian $H_{complete}$ along with $\mathcal{H}$, being a direct BRST-anti-BRST invariant extension of the original Hamiltonian H.

We observe that both BRST and anti–BRST symmetry may appear in a pure second class constrained theory, having no genuine gauge invariance. It shows that the common recognition of the BRST invariance as a residual symmetry of the gauge theory is not quite exact in the sense that the original second class constraint surface, being conserved in time, may give rise to the corresponding conservation of the BRST charges, when the ghosts and Lagrange multipliers are introduced in the formalism. Thus, the suggested formulation enables to employ all the tools of the BRST theory to analyze the second class constrained systems. The explicit derivation of the extended BRST symmetry generators for the second class theories, including self–dual nonabelian model and massive Yang Mills theory, is presented in [21].

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