Light cone formalism in AdS spacetime

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Abstract

Light cone form of field dynamics in anti-de Sitter spacetime is described. We also present light cone reformulation of the boundary conformal field theory representations. AdS/CFT correspondence between the bulk fields and the boundary operators is discussed.

Motivation. A long term motivation to develop light cone formalism in AdS spacetime comes from a number of the following potentially important applications.

One important application is to type IIB superstring in $AdS_5 \times S^5$ background. Inspired by the conjectured duality between the string theory and $\mathcal{N} = 4$, $4d$ SYM theory [1] the Green-Schwarz formulation of strings propagating in $AdS_5 \times S^5$ was suggested in [2] (for further developments see [3]-[5]). Despite considerable efforts these strings have not yet been quantized (some related interesting discussions are in [6],[7]). Alternative approaches can be found in [8]. As is well known, quantization of GS superstrings propagating in flat space is straightforward only in the light cone gauge. The light cone gauge in string theory implies the corresponding light cone formulation for target space fields. In the case of strings in AdS background this suggests that we should first study a light cone form dynamics of *target space fields* propagating in AdS spacetime. Understanding a light cone description of AdS target space fields might help to solve problems of strings in AdS spacetime.

The second application is to a theory of higher massless spin fields propagating in AdS spacetime. Some time ago completely self-consistent interacting equations of motion for higher massless fields of all spins in four-dimensional AdS spacetime have been discovered [9]. For generalization to higher spacetime dimensions see [10]. Despite efforts, the action that leads to these equations of motion has not yet been obtained. In order to quantize these theories and investigate their ultraviolet behavior, it would be important find an appropriate action. Since the higher massless spin theories correspond quantum mechanically to non-local point particles in a space of certain auxiliary variables, it is conjectured that they may be ultraviolet finite (see [11],[12]). We believe that a light cone formulation is what is required to understand these theories better. The situation here may be analogous to that in string theory; a covariant formulation of closed string field theories is non-polynomial and is not useful for practical calculations, while the light cone formulation restricts the string action to cubic order in string fields.

Light cone formulation of field dynamics in AdS spacetime. First let us discuss the forms of AdS algebra, that is $so(d-1,2)$, we are going to use. AdS algebra of d dimensional AdS spacetime consists of translation generators $\hat{P}^A$ and rotation generators $\hat{J}^{AB}$ which span $so(d-1,1)$ Lorentz algebra. The commutation relations of AdS algebra in antihermitean basis are

$$[\hat{P}^A, \hat{P}^B] = \lambda^2 \hat{J}^{AB}\,, \quad [\hat{P}^A, \hat{J}^{BC}] = \eta^{AB}\hat{P}^C - \eta^{AC}\hat{P}^B\,, \quad [\hat{J}^{AB}, \hat{J}^{CE}] = \eta^{BC}\hat{J}^{AE} + 3 \text{ terms}\,,$$

where $\eta^{AB} = (-,+\ldots,+)$, $A,B,C,E = 0,1,\ldots,d-1$. The λ is a cosmological constant of AdS spacetime. As $\lambda \to 0$ the AdS algebra becomes the Poincaré algebra

$$\lim_{\lambda\to 0} \hat{P}^A = P^A_{Poin}\,, \qquad \lim_{\lambda\to 0} \hat{J}^{AB} = J^{AB}_{Poin}\,.$$

This form algebra is not convenient for our purposes. We prefer to use the form provided by nomenclature of conformal algebra. Namely, we introduce new basis which consists of new translations P^a, conformal boosts K^a, dilatation D, and $so(d-2,1)$ algebra generators J^{ab} defined by

$$P^a \equiv \hat{P}^a + \lambda \hat{J}^{d-2a}\,, \qquad K^a \equiv \frac{1}{2}(-\frac{1}{\lambda^2}\hat{P}^a + \frac{1}{\lambda}\hat{J}^{d-2a})\,, \qquad D \equiv -\frac{1}{\lambda}\hat{P}^{d-2} \qquad J^{ab} \equiv \hat{J}^{ab}\,.$$

Flat space limit in this notation is given by

$$\lim_{\lambda\to 0} P^a = P^a_{Poin}\,, \qquad \lim_{\lambda\to 0}(-\lambda D) = P^{d-2}_{Poin}\,, \qquad \lim_{\lambda\to 0}(\frac{1}{2\lambda}P^a + \lambda K^a) = J^{d-2a}_{Poin}\,. \tag{1}$$

In the conformal algebra basis one has the following well known commutation relations

$$[D,P^a]=-P^a\,,\qquad [D,K^a]=K^a\,,\qquad [P^a,P^b]=0\,,\qquad [K^a,K^b]=0\,, \tag{2}$$

$$[P^a,J^{bc}]=\eta^{ab}P^c-\eta^{ac}P^b\,,\qquad [K^a,J^{bc}]=\eta^{ab}K^c-\eta^{ac}K^b\,, \tag{3}$$

$$[P^a,K^b]=\eta^{ab}D-J^{ab}\,,\qquad [J^{ab},J^{ce}]=\eta^{bc}J^{ae}+3\text{ terms}\,,\qquad \eta^{ab}=(-,+,\dots+)\,, \tag{4}$$

where $a,b,c,e=0,1,\dots,d-3,d-1$. In this form the AdS algebra is known as the algebra of conformal transformations in $(d-1)$-dimensional Minkowski spacetime. Below we shall be interested in realization of this algebra as the one of transformations of massless bulk fields propagating in d-dimensional AdS spacetime as well as in realization of this algebra as the algebra of conformal transformations on appropriate CFT operators in $(d-1)$-dimensional Minkowski spacetime.

To develop light cone formulation we shall use Poincaré parametrization of AdS spacetime in which

$$ds^2=\frac{1}{z^2}(-dt^2+dx_i^2+dz^2+dx_{d-1}^2)\,,\qquad z>0\,.$$

Here and below we set cosmological constant λ equal to unity. The boundary at spatial infinity corresponds to $z=0$.[1] The Killing vectors in these coordinates are given by[2]

$$\xi^{P^a,\mu}=\eta^{a\mu}\,,\quad \xi^{K^a,\mu}=-\frac{1}{2}x_\nu^2\eta^{a\mu}+x^a x^\mu\,,\quad \xi^{D,\mu}=x^\mu\,,\quad \xi^{J^{ab},\mu}=x^a\eta^{b\mu}-x^b\eta^{a\mu}\,, \tag{5}$$

while the corresponding generators are defined as $G=\xi^{G,\mu}\partial_\mu$. Then we introduce light cone variables $x^\pm$, $x^I=(x^i,x^z)$

$$x^\pm\equiv\frac{1}{\sqrt{2}}(x^{d-1}\pm x^0)\,,\qquad x^0\equiv t,\ \ x^{d-2}\equiv z\,,\qquad I,J,K,L=1,\dots,d-2\,,\qquad i,j,k,l=1,\dots,d-3\,.$$

In this notation scalar product of tangent space vectors is decomposed as

$$X^AY^A=X^+Y^-+X^-Y^++X^IY^I\,,\qquad X^IY^I=X^iY^i+X^zY^z\,,$$

i.e. we use the convention $X^{d-2}=X^z$. The coordinate x^+ is considered as an evolution parameter. Here and below to simplify our expressions we will drop the metric tensors η_{AB}, η_{ab} in scalar products.

In light cone basis the AdS algebra splits into generators

$$P^+,\,P^i,\,J^{+i},\,K^+,\,K^i,\,D,\,J^{+-},\,J^{ij}\,, \tag{6}$$

which we refer to as kinematical generators and

$$P^-,\,J^{-i}\,,K^-\,, \tag{7}$$

which we refer to as dynamical generators. For $x^+=0$ the kinematical generators are realised quadratically in physical fields while the dynamical generators receive corrections in interaction theory. In this paper we deal with free fields. The light cone form of AdS algebra can be obtained from (2)-(4) with the light cone metric having the following non vanishing elements $\eta^{+-}=\eta^{-+}=1$, $\eta^{ij}=\delta^{ij}$.

Now our primary goal is to find realization of this algebra on the physical fields. For definiteness, we will be interested in spin s totally symmetric fields. To keep formulas as simple as possible, let us start with spin one Maxwell field. Instead of target space field A^μ with the equations of motion $D_\mu F^{\mu\nu}=0$, we introduce tangent space field Φ^A defined by

$$\Phi^A\equiv e_\mu^A A^\mu,\qquad e_\mu^A=\frac{1}{z}\delta_\mu^A \tag{8}$$

[1] Poincaré coordinates cover half of AdS spacetime. Because we are interested in infinitesimal transformation laws of physical fields the global description of AdS spacetime is not important for our study.

[2] The target space indices μ,ν take the values $0,1,\dots,d-1$.

and use equations of motion in tangent space $D_B F^{BA} = 0$, $F_{AB} = D_A\Phi_B - D_B\Phi_A$, where F^{AB} is the field strength in the tangent space while D_A is covariant derivative which in Poincaré coordinates take the form

$$D_A\Phi_B = \hat{\partial}_A\Phi_B + \delta^z_B\Phi_A - \eta_{AB}\Phi_z\,, \qquad F_{AB} = \hat{\partial}_A\Phi_B - \hat{\partial}_B\Phi_A + \delta^z_B\Phi_A - \delta^z_A\Phi_B\,, \quad \hat{\partial}_A \equiv e^\mu_A\partial_\mu$$

From these relations one gets the following second order equations of motion for the gauge field Φ^A

$$(\hat{\partial}^2 + (1-d)\hat{\partial}_z + d - 2)\Phi^A - \hat{\partial}^A(\hat{\partial}\Phi) + (d-3)\hat{\partial}^A\Phi^z + (2-d)\delta^A_z\Phi^z + 2\delta^A_z(\hat{\partial}\Phi) = 0\,, \tag{9}$$

where $\hat{\partial}^2 \equiv \hat{\partial}^2_A$ and $\hat{\partial}\Phi \equiv \hat{\partial}^A\Phi^A$. Since these equations are invariant with respect to the gauge transformation $\delta\Phi^A = \hat{\partial}^A\Lambda$, we can impose the light cone gauge

$$\Phi^+ = 0\,. \tag{10}$$

Inserting this into equations (9) we get the following constraint[3]

$$\hat{\partial}^A\Phi^A = (d-3)\Phi^z\,. \tag{11}$$

From (11) we express the Φ^- in terms of the physical field[4]

$$\Phi^- = -\frac{\partial^I}{\partial^+}\Phi^I + \frac{d-3}{\hat{\partial}^+}\Phi^z\,. \tag{12}$$

Note that the second term in r.h.s. of equation (12) is absent in flat space. It is this term that breaks $so(d-2)$ manifest invariance and reduce it to $so(d-3)$ one. By virtue of the constraint (11), the equations of motion (9) take the form

$$(\hat{\partial}^2 + (1-d)\hat{\partial}_z + d - 2)\Phi^A + (d-4)\delta^A_z\Phi^z = 0\,.$$

From this we get the following equations for the physical fields Φ^i, Φ^z:

$$(\hat{\partial}^2 + (1-d)\hat{\partial}_z + d - 2)\Phi^i = 0\,, \qquad (\hat{\partial}^2 + (1-d)\hat{\partial}_z + 2d - 6)\Phi^z = 0\,.$$

Since this form of equations of motion is not convenient we introduce new physical field ϕ^I defined by[5]

$$\Phi^I = z^{(d-2)/2}\phi^I\,. \tag{13}$$

In terms of ϕ^I the equations of motion take the form

$$(\partial^2 - \frac{1}{4z^2}(d-2)(d-4))\phi^i = 0\,, \qquad (\partial^2 - \frac{1}{4z^2}(d-4)(d-6))\phi^z = 0\,. \tag{14}$$

Dividing by ∂^+ these equations can be rewritten in the Schrodinger form

$$\partial^-\phi^I = P^-\phi^I\,, \tag{15}$$

where the action of P^- on physical fields is defined by

$$P^-\phi^i = \Big(-\frac{\partial^2_I}{2\partial^+} + \frac{(d-2)(d-4)}{8z^2\partial^+}\Big)\phi^i\,, \qquad P^-\phi^z = \Big(-\frac{\partial^2_I}{2\partial^+} + \frac{(d-4)(d-6)}{8z^2\partial^+}\Big)\phi^z\,. \tag{16}$$

Few comments are in order. i) From equations of motion (14) we see that in $d = 4$ the mass-like terms cancel. This fact reflects the conformal invariance of spin one field in four dimensional AdS spacetime; ii) In the equations above the fields ϕ^i and ϕ^z have different mass terms. This is reflection of breaking the

[3]Recall that in the Minkowski spacetime the Maxwell equations in gauge $\Phi^+ = 0$ lead to the Lorentz constraint $\partial^A\Phi^A = 0$. This is not the case in AdS spacetime. Here, by virtue of the relation $D_A\Phi^A = \hat{\partial}\Phi + (1-d)\Phi^z$, the constraint (11) does not coincide with the Lorentz constraint $D^A\Phi^A = 0$.

[4]We assume, as usual in light cone formalism, that the operator ∂^+ has trivial kernel.

[5]Note that it is the field ϕ^I that has conventional canonical dimension $\Delta_0 = (d-2)/2$.

usual flat space transverse $so(d-2)$ algebra. On the other hand we see that AdS light cone equations of motion keep manifest $so(d-3)$ symmetry. Thus light cone formalism in AdS spacetime breaks manifest $so(d-2)$ symmetry and keeps manifest $so(d-3)$ one.

Before continuing with our main theme, let us note that gauge invariant action for spin one Maxwell field $S = -\frac{1}{4}\int d^dx\sqrt{g}F^2_{AB}$ takes the following simple form in terms of physical field ϕ^I

$$S_{l.c.} = \int d^dx\partial^+\phi^I(-\partial^- + P^-)\phi^I\,. \tag{17}$$

Now let us turn to our primary aim, that is transformation of physical degrees of freedom ϕ^I. Toward this aim we start, as usual, with original global AdS symmetries, supplemented with compensating gauge transformations required to maintain the gauge

$$\delta_{tot}A^\mu = \xi^\nu\partial_\nu A^\mu - A^\nu\partial_\nu\xi^\mu + \partial_\mu\Lambda\,.$$

As usual, the gauge transformation parameter Λ can be found from the requirement $\delta_{tot}\Phi^+ = 0$. From these relations transformation of physical field ϕ^I is fixed uniquely. The expressions can be simplified if we use creation and annihilation operators α^I, $\bar\alpha^I$, $[\bar\alpha^I, \alpha^J] = \delta^{IJ}$, and introduce Fock vector for the physical field $|\phi\rangle \equiv \phi^I\alpha^I|0\rangle$. The transformation laws of the physical field can be then cast into the following form

$$\delta_{tot}|\phi\rangle = \Big(\xi\partial + \frac{d-2}{2z}\xi^z + \frac{1}{2}\partial^I\xi^J M^{IJ} + M^{IJ}\partial^+\xi^I\frac{\partial^J}{\partial^+} - \frac{\partial^+\xi^I}{2z\partial^+}\{M^{zj}, M^{jI}\}\Big)|\phi\rangle \tag{18}$$

where the spin operator M^{IJ} is given by

$$M^{IJ} \equiv \alpha^I\bar\alpha^J - \alpha^J\bar\alpha^I\,. \tag{19}$$

Generalization to the case of arbitrary spin s totally symmetric field is relatively straightforward (see [15]). In this case the physical degrees of freedom are described by traceless totally symmetric tensor field $\phi^{I_1\dots I_s}$. To simplify expressions it is useful as above to collect physical degrees of freedom in one Fock vector

$$|\phi\rangle \equiv \phi^{I_1\dots I_s}\alpha^{I_1}\dots\alpha^{I_s}|0\rangle\,, \qquad \bar\alpha^2_I|\phi\rangle = 0\,.$$

With this notation the equations of motion and corresponding hamiltonian P^- take the form

$$\Big(z^2\partial^2 + \frac{1}{2}M^2_{ij} - \frac{(d-4)(d-6)}{4}\Big)|\phi\rangle = 0, \qquad P^- = -\frac{\partial^2_I}{2\partial^+} + \frac{1}{2z^2\partial^+}(-\frac{1}{2}M^2_{ij} + \frac{(d-4)(d-6)}{4})\,. \tag{20}$$

The transformations of physical fields under the action of global AdS symmetries take the same form as in (18) where we have to use Schrodinger equation of motion (15) and P^- given in (20).[6] Making use of the AdS transformations (18) we can represent them as differential operators acting on the physical massless field $|\phi\rangle$. Plugging the Killing vectors (5) in transformation laws (18) we get corresponding differential form of generators. Light cone form of AdS algebra kinematical generators is given by

$$P^i = \partial^i\,, \qquad P^+ = \partial^+\,, \qquad D = x^+P^- + x^-\partial^+ + x^I\partial^I + \frac{d-2}{2}\,, \tag{21}$$

$$J^{+-} = x^+P^- - x^-\partial^+\,, \qquad J^{+i} = x^+\partial^i - x^i\partial^+\,, \qquad J^{ij} = x^i\partial^j - x^j\partial^i + M^{ij}\,, \tag{22}$$

$$K^+ = -\frac{1}{2}(2x^+x^- + x^2_I)\partial^+ + x^+D\,, \qquad K^i = -\frac{1}{2}(2x^+x^- + x^2_J)\partial^i + x^iD + M^{iI}x^I + M^{i-}x^+\,, \tag{23}$$

where

$$M^{-i} = M^{ij}\frac{\partial^j}{\partial^+} - \frac{1}{\partial^+}M^i\,, \quad M^i \equiv M^{zi}\partial_z + \frac{1}{2z}\{M^{zj}, M^{ji}\}. \tag{24}$$

[6] Note that light cone action for arbitrary spin s field takes the following elegant and extremely simple form $S_{l.c.} = \int d^dx\langle\partial^+\phi|(-\partial^- + P^-)|\phi\rangle$ where P^- is given by (20). In $d = 4$ because of $M_{11} = 0$ the mass like term in (20) cancels. This fact reflects conformal invariance of arbitrary spin s totally symmetric massless field in AdS_4 spacetime.

P^- is given in (20) and remaining dynamical generators are given by

$$J^{-i} = x^- \partial^i - x^i P^- + M^{-i}\,, \tag{25}$$

$$K^- = -\frac{1}{2}(2x^+x^- + x_I^2)P^- + x^- D + \frac{1}{\partial^+}x^I \partial^J M^{IJ} - \frac{x^I}{2z\partial^+}\{M^{zJ}, M^{JI}\}\,. \tag{26}$$

Light cone form of conformal field theory. Now the $so(d-1,2)$ algebra is considered as algebra of conformal transformations of $(d-1)$-dimensional Minkowski spacetime and we are interested in light cone reformulation of (free) conformal field theory. The reason for doing this is that we are going to establish AdS/CFT correspondence between bulk massless fields and conformal field theory operators. As the bulk massless fields have been studied within the framework of the light cone formalism the most adequate form for comparison is the light cone form of conformal field theory. To keep our presentation as simple as possible we restrict our attention to the case of arbitrary spin totally symmetric operators $\mathcal{O}^{a_1\ldots a_s}(x)$ that have canonical conformal dimension $\Delta = s + d - 3$.[7] These operators, by definition, are traceless $\mathcal{O}^{aaa_3\ldots a_s} = 0$ and divergence free $\partial_{x^a}\mathcal{O}^{aa_2\ldots a_s} = 0$. As above to simplify our presentation we consider Fock space vector (generating function) $|\mathcal{O}_{cov}\rangle \equiv \mathcal{O}^{a_1\ldots a_s}\alpha^{a_1}\ldots\alpha^{a_s}|0\rangle$. In terms of generating function the traceless and divergence free conditions take the following form

$$\bar\alpha^a\bar\alpha^a|\mathcal{O}_{cov}\rangle = 0\,, \qquad \bar\alpha^a\partial_{x^a}|\mathcal{O}_{cov}\rangle = 0\,. \tag{27}$$

Realization of conformal algebra generators on the space of operators $|\mathcal{O}_{cov}\rangle$ is given by [14]

$$P^a = \partial^a\,, \qquad K^a = -\frac{1}{2}x_b^2\partial^a + x^a(x^b\partial_{x^b} + \Delta) + M^{ab}x^b\,, \tag{28}$$

$$J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}\,, \qquad D = x^a\partial_{x^a} + \Delta\,, \qquad M^{ab} = \alpha^a\bar\alpha^b - \alpha^b\bar\alpha^a\,, \tag{29}$$

where $\partial^a \equiv \eta^{ab}\partial_{x^b}$ and M^{ab} is the $so(d-2,1)$ algebra spin operator. Because in the bulk the $so(d-1,2)$ algebra was realized on the space of unconstrained physical fields it is reasonable to solve the second constraint in (27) and formulate boundary conformal theory also in terms of unconstrained operators which we shall denote by $|\mathcal{O}\rangle$. One can choose a basis which makes J^{+i}, J^{+-} and K^+independent of M^{ab}. The final light cone form of the generators realized on conformal theory operators is (see [15])

$$P^a = \partial^a\,, \qquad D = x^+\partial^- + x^-\partial^+ + x^i\partial^i + \hat\Delta\,, \tag{30}$$

$$J^{+i} = x^+\partial^i - x^i\partial^+\,, \qquad J^{+-} = x^+\partial^- - x^-\partial^+\,, \qquad J^{ij} = x^i\partial^j - x^j\partial^i + M^{ij}\,, \tag{31}$$

$$K^+ = -\frac{1}{2}(2x^+x^- + x_i^2)\partial^+ + x^+D\,, \qquad J^{-i} = x^-\partial^i - x^i\partial^- + M^{ij}\frac{\partial^j}{\partial^+} - \frac{1}{\partial^+}M^i\,, \tag{32}$$

where the spin operator M^{ij} is the same as in (29). The operator M^i transforms in vector representation of the spin operator M^{ij} and satisfies the commutation relations[8]

$$[M^i, M^{jk}] = \delta^{ij}M^k - \delta^{ik}M^j\,, \qquad [M^i, M^i] = \Box M^{ij}\,,$$

where $\Box$ is the Dalamber operator in $(d-1)$ dimensional Minkowski spacetime $\Box \equiv \partial^2_{x^a}$. A representation of spin part $\hat\Delta$ of the dilatation operator D on conformal operator $|\mathcal{O}\rangle$ is determined to be

$$\hat\Delta \equiv \alpha^i\bar\alpha^i + d - 3\,. \tag{33}$$

AdS/CFT correspondence. After we have derived the light cone formulation for both the bulk field $|\phi\rangle$ and the boundary conformal theory operator $|\mathcal{O}\rangle$ we are ready to demonstrate explicitly the AdS/CFT correspondence. We demonstrate that boundary values of normalizable solutions of bulk equations of motion are related to conformal operators $|\mathcal{O}\rangle$ (see [16],[15]).

[7] The fact that this expression is nothing but the lowest energy value of spin s massless fields propagating in d dimensional AdS spacetime has been demonstrated in [13].

[8] Detailed description of operator M^i which should not be confused with M^i given in (24) may be found in [15].

To this end we decompose the field $|\phi\rangle$ into irreducible representations of $so(d-3)$ subalgebra $|\phi\rangle = \sum_{s'=0}^{s} \oplus |\phi_{s'}\rangle$, $(\alpha^i\bar{\alpha}^i - s')|\phi_{s'}\rangle = \bar{\alpha}_i^2|\phi_{s'}\rangle = 0$, and rewrite equations of motion (20) in the form

$$(-\partial_z^2 + \frac{1}{z^2}(\kappa^2 - \frac{1}{4}))|\phi_{s'}\rangle = q^2|\phi_{s'}\rangle\,, \qquad \kappa \equiv s' + \frac{d-5}{2}, \qquad q \equiv \sqrt{\Box}\,, \tag{34}$$

where we have used the relation

$$\frac{1}{2}M_{ij}^2|\phi_{s'}\rangle = -s'(s'+d-5)|\phi_{s'}\rangle$$

Normalizable solution to the equation (34) is

$$|\phi_{s'}(x,z)\rangle = \sqrt{qz}J_\kappa(qz)q^{-(\kappa+\frac{1}{2})}|\mathcal{O}_{s'}(x)\rangle \tag{35}$$

where an operator $|\mathcal{O}\rangle$[9] does not depend on z and the J_κ is Bessel function. In (35) we use the notation $|\mathcal{O}\rangle$ since we are going to demonstrate that this is indeed the CFT operator discussed above. Namely, we are going to prove that AdS transformations for $|\phi\rangle$ lead to conformal theory transformations for $|\mathcal{O}\rangle$

$$G_{ads}|\phi_{s'}\rangle = Z_\kappa(qz)q^{-(\kappa+\frac{1}{2})}G_{cft}|\mathcal{O}_{s'}\rangle\,, \qquad Z_\kappa(z) \equiv \sqrt{z}J_\kappa(z)\,. \tag{36}$$

Here and below we use the notation G_{ads} and G_{cft} to indicate the realization of $so(d-1,2)$ algebra generators on the bulk field (21)-(26) and conformal operator (30)-(33) respectively.

Toward this purpose let us make a comparison of generators for bulk field $|\phi\rangle$ and boundary operator $|\mathcal{O}\rangle$. Important technical simplification is that it is sufficient to make comparison only for the part of the algebra spanned by generators P^a, J^{ab}, D, K^+. It is straightforward to see that if these generators match then the remaining generators K^- and K^i shall match due to commutation relations of the $so(d-1,2)$ algebra. We start with a comparison of the kinematical generators (6). As for the generators P^+, P^i, J^{+i}, J^{ij}, they already coincide on both sides (see (21),(22) and (30),(31)). In fact, this implies that the coordinates p^+, p^i, we use on AdS and CFT sides match.

Now let us consider P^-_{ads} (20) and P^-_{cft} (30). Taking into account that the solutions to equations of motion (20), by definition, satisfy the relation $P^-_{ads}|\phi\rangle = \partial^-|\phi\rangle$ we get that P^-_{ads} and P^-_{cft} satisfy the desired relation (36). So P^-_{ads} and P^-_{cft} also match. Taking this into account it is straightforward to see that the generators J^{+-}_{ads}, D_{ads} (22),(21) and J^{+-}_{cft}, D_{cft} (31), (30) also satisfy the relation (36). Next we consider the kinematical generators K^+_{ads} (23) and K^+_{cft} (32). Making use of the following relation

$$K^+_{ads}q^{-(\kappa+\frac{1}{2})}Z_\kappa(qz) = q^{-(\kappa+\frac{1}{2})}Z_\kappa(qz)(K^+_{cft} + x^+z\partial_z) - q^{-(\kappa+\frac{1}{2})}\frac{\partial^+}{q}(\partial_q Z_\kappa(qz))z\partial_z\,,$$

the relation (35) and the fact that $|\mathcal{O}\rangle$ in (35) does not depend on z, we get immediately that K^+_{ads} and K^+_{cft} satisfy the relation (36). The last step is to match the generators J^{-i}_{ads} and J^{-i}_{cft}. Using the relation $P^-_{ads} = P^-_{cft}$ and comparing the above expression for J^{-i}_{ads} with J^{-i}_{cft} given in (25) and (32), we conclude that all that remains to do is to match M^i_{ads} (24) and M^i_{cft}. Technically, this is the most difficult point of matching which can be proved by direct calculation (see [15]).

Finally let us write AdS/CFT correspondence for bulk symmetric spin s fields and corresponding boundary conformal theory operators. From (35) we can read, up to a factor, the following relation

$$\lim_{z\to 0} z^{-\hat{\Delta}+\Delta_0}|\phi_{s'}(x,z)\rangle = |\mathcal{O}_{s'}(x)\rangle$$

where in this expression $\hat{\Delta}$ is the spin part of dilatation generator given in (33) while Δ_0 is a canonical dimension of bulk massless field in AdS_d spacetime $\Delta_0 = (d-2)/2$. Above analysis is straightforwardly generalized to the case of non-normalizable solutions and shadow operators (see [15]).

Conclusions We have developed the light cone formalism in AdS spacetime. Here we discussed application of this formalism to the study of AdS/CFT correspondence but it is applicable to discussion

[9] Note that we decompose the operator $|\mathcal{O}\rangle$ into irreducible representations of $so(d-3)$ algebra $|\mathcal{O}\rangle = \sum_{s'=0}^{s}|\mathcal{O}_{s'}\rangle$, where $|\mathcal{O}_{s'}\rangle$ satisfy the relations $(\alpha^i\bar{\alpha}^i - s')|\mathcal{O}_{s'}\rangle = \bar{\alpha}_i^2|\mathcal{O}_{s'}\rangle = 0$.

of various interesting problems, some of which are: (i) generalization to supersymmetry and applications to supergravity in AdS background and then to strings in this background; (ii) extension of light cone formulation of conformal field theory to the level of OPE's and study of light-cone form of AdS/CFT correspondence at the level of correlation functions; (iii) application of light cone formalism to the study of the S-matrix along the lines of [17]–[21]; (iv) applications to interaction vertices for higher massless spin fields in AdS spacetime. Because the formalism we presented is algebraic in nature, it allows us to treat fields with arbitrary spin on equal footing. Comparison of this formalism with other approaches available in the literature leads us to the conclusion that this is a very efficient formalism. Application of the formalism above to study of IIB supergravity in $AdS_5 \times S^5$ background may be found in [22].

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