Renormalization Group Flows from AdS Dynamics

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Abstract

5d gauged supergravity is a convenient and general framework to study deformations of N=4 super Yang-Mills theories in the limit of large 't Hooft parameter.

It has been proposed that certain conformal field theories in d dimensions have a dual description in terms of superstring/M theory on $AdS_{d+1} \times X$, where AdS_{d+1} is Anti-de-Sitter space in $d+1$ dimensions and X is an Einstein manifold [1, 2, 3]. Such a duality can also be extended to non-conformal field theories[4, 5] . One can for instance study non-conformal field theories that correspond to deformations of the conformal theory by considering deformations of the AdS geometry.

The example we will be concerned with is N=4 Super Yang-Mills in four dimensions with gauge group $SU(N)$. According to Maldacena conjecture [1], N=4 Yang-Mills theory is dual to Type IIB string on $AdS_5 \times S^5$. This dual description can be used when the superstring is weakly coupled: this corresponds to the large N, large t'Hooft coupling $g = \sqrt{g_{YM}^2 N}$ [6] regime of the N=4 Yang-Mills theory. The aim of this work is to study the dynamics of strongly coupled N=4 Yang-Mills deformed with relevant operators via its supergravity dual.

There are two possible ways of breaking the conformal invariance of N=4 SYM: one can either deform the Lagrangian with a relevant operator $O(x)$ or consider the theory in a vacuum state characterised by the VEV $< O(x) >$ of a certain operator. These two choices both break conformal invariance since they introduce an energy scale. This induces a RG flow for the coupling constant and the parameters of the theory that can then flow to the IR to (a) another conformal fixed point or (b) a non conformal theory.

The supergravity picture dual to field theory RG flow relies on the observation that the fifth coordinate y of AdS can be interpreted as an energy scale: $y \to \infty$ corresponds to the UV regime while $y \to -\infty$ to the IR. We can then look for solutions of the supergravity equations of motion that interpolate between these two asymptotic regions and interpret them as RG flows. Comparison with field theory puts some constraints on the form of such solutions.

In the holographic prescription [2, 3], the set of gauge invariant composite operators of the conformal field theory are associated to the supergravity modes on AdS_5 via boundary values. The conformal dimension Δ of an operator $O(x)$ is related to the mass of the corresponding AdS_5 state [3, 7]. For a scalar we have $m^2 = \Delta(\Delta - 4)$. Deforming N=4 with relevant operator then corresponds in supergravity to exciting scalars with negative mass squared. Such scalars are interpreted as the parameters of the field theory.

The RG flow gives the variation of the parameters of the theory as functions of the energy scale. We then look for supergravity solutions where the fields only depend on the coordinate y.

Conformal invariance of the field theory translates into the AdS geometry of the supergravity background. Since conformal invariance is lost along the flow, we want to break the conformal group $O(4,2)$ to the $4d$ Poincare' group on the boundary. We will then consider Poincare' invariant metric that are only asymptotically AdS

$$ds^2 = dy^2 + e^{2\phi(y)} dx^\mu dx_\mu, \quad \mu = 0,1,2,3. \tag{1}$$

For $\phi(y) = y/R_{AdS}$ one recovers the AdS metric.

In the UV the field theory is N=4 deformed with a relevant operator. Then the supergravity solution, when linearised around $y \to \infty$ should look like the AdS background deformed with the scalar field associated with the operator $O(x)$.

In the IR ($y \to \infty$) the solution can be asymptotic to either to (a) $AdS_5 \times H$ or to (b) a space-time with a singularity or a horizon.

The field theory interpretation of a supergravity solution depends on its asymptotic UV behaviour. Solutions asymptotic to $e^{-(4-\Delta)y}$ describe deformations of a CFT with the operator $O(x)$, while solutions asymptotic to $e^{-\Delta y}$ correspond to the same theory in a different vacuum where the operator $O(x)$ has a non-zero VEV. [8, 9].

10d solutions interpolating between CFT's are difficult to find because of the high non-linearity of the type IIB equations of motion. Perturbation theory around the UV point is not practicable and exact solutions are not easy to construct. However as long as we are interested only in relevant deformations of N=4 SYM, we can consider the 5d gauged supergravity instead of the full 10d theory [10, 11].

N=8 gauged supergravity [12] is the low energy effective action for the "massless" modes of the compactification of Type IIB on $AdS_5 \times S_5$. It is believed to be a consistent truncation of type IIB on S^5 [13] in the sense that every solution of the 5d theory can be lifted to a consistent 10d type IIB solution. Five-dimensional gauged supergravity has 42 scalars, which transform under the N=4 YM R-symmetry $SU(4)$ as $\underline{1}, \underline{20}, \underline{10}$. The singlet is associated with the marginal deformation corresponding to a change in the coupling constant of the N=4 theory. The mode in the $\underline{20}$ has mass square $M^2 = -4$ and is associated with a symmetric traceless mass term for the scalars $\mathrm{Tr}\,\phi_i\phi_j, i,j = 1,...,6$ ($\Delta = 2$). The $\underline{10}$ has mass square $M^2 = -3$ and corresponds to the fermion mass term $\mathrm{Tr}\,\lambda_A\lambda_B, A,B = 1,...,4$, of dimension 3. Thus the scalar sector of N=8 gauged supergravity is enough to discuss at least all mass deformations that have a supergravity description[1].

The effective 5d Lagrangian, restricted to the 42 scalars λ_a, is

$$L = \sqrt{-g}\left[-\frac{R}{4} + \frac{1}{2}g^{IJ}\partial_I\lambda_a\partial_J\lambda_b G^{ab} + V(\lambda)\right]. \tag{2}$$

This is the most general Lagrangian for scalars coupled to gravity[2]. A typical feature of 5d gauged supergravity [12] is the presence of a non trivial potential $V(\lambda)$ which has no flat directions (apart from

[1]The only relevant operators that cannot be described in the contest of 5d supergravity are some potential terms for the scalars and a mass state corresponding to the R-symmetry singlet $\sum_{i=1}^{6} \mathrm{Tr}\,\phi_i\phi_i$, which is associated with a stringy mode.

[2]It is possible choose a convenient parametrisation where the scalar kinetic term is canonical ($G^{ab} = \delta^{ab}$).

the dilaton).

In this picture, RG flows correspond to solutions of the 5d Einstein's equations and the equations of motion of the scalars following from (2):

$$\partial_y^2 \lambda^a + 4\partial_y \phi\, \partial_y \lambda^a = \frac{\partial V}{\partial \lambda^a}, \quad 6(\partial_y \phi)^2 = \sum_a (\partial_y \lambda^a)^2 - 2V. \tag{3}$$

Solutions connecting two minima of the potential represent flows between UV and IR fixed points while solutions going to infinity in the parameter space are associated to flows to IR non conformal field theories.

(a) Flow to conformal field theories

Up to now, we know all critical points of the potential $V(\lambda)$ that have AdS metric and at least $SU(2)$ symmetry[14, 15]. There is a central critical point with $SO(6)$ symmetry and with all the scalars λ_a vanishing: it corresponds to the N=4 YM theory, There are three N=0 theories with residual symmetry $SU(3) \times U(1)$, $SO(5)$ and $SU(2) \times U(1)^2$. They correspond to non-zero VEV for some of the scalars in the $\underline{10}$, $\underline{20}$, and $\underline{10} + \underline{20}$ respectively. Then there is N=2 point with symmetry $SU(2) \times U(1)$, obtained giving VEV to scalars in the $\underline{10} \oplus \underline{20}$ [15].

According to the AdS/CFT correspondence, these other minima should correspond to new IR conformal field theories.

The symmetries of the field theories can be read from those of the supergravity minima according to the correspondence : gauge symmetry in supergravity $\leftrightarrow$ global symmetry in field theory, supersymmetry in supergravity $\leftrightarrow$ superconformal symmetry in field theory.

The scalars with a non-zero VEV, λ_{IR}, at the critical points indicate which operators have been used to deform the UV theory: all those IR theories are obtained as mass deformations of N=4 SYM [10, 11]. All the non supersymmetric critical points might be unstable [11, 16]. On the contrary the N=1 point with symmetry $SU(2) \times U(1)$ [15] is stable due to supersymmetry.

The value of the potential at the critical point (i.e. the cosmological constant of the AdS space) is associated with the central charges $c = a$ of the CFT by [17] [3].

$$c = a \sim (-V(\lambda_{IR}))^{-3/2}. \tag{4}$$

Since the new minima are not continuously connected to the maximally symmetric one, the linearization around the N=4 theory can not be completely trusted. Higher order corrections, giving rise to higher dimensional operators in the deformed N=4 theory, must be included and thus an explicit Lagrangian realization for these theories it not known. However we can use supergravity to show that these theories can be connected to N=4 SYM by an RG, and are therefore local theories.

The RG flow connecting the N=4 theory to one of these IR CFT's corresponds in supergravity to a kink solution of eqs.(3) [10], which interpolates between the two critical points. We consider the metric

[3] In the case of the supersymmetric point $SU(2) \times U(1)$ the central charge predicted by supergravity agrees with quantum field theory expectation [18]. Supersymmetry allows to compute c and a using the results in [19].

(1) and we look for solutions that are asymptotic to AdS_5 spaces (but with different radii R_{UV} and R_{IR}) both for $y \to \infty$ and $y \to -\infty$: $\phi(y) \to y/R_{UV}$ + const for $y \to \infty$, and $\phi(y) \to y/R_{IR}$ + const for $y \to -\infty$. Analogously, $\lambda(y) \to 0$ for $y \to \infty$ (N=4 SYM in the UV), while $\lambda(y) \to \lambda_{IR}$ for $y \to -\infty$ (IR CFT).

One can show that for the the $SU(3) \times U(1)$, and $SO(5)$ critical points such a kink solution always exists [10]. Indeed eliminating ϕ from one of the equations, the problem reduces to the analysis of the motion of a classical particle in a potential with never-vanishing, positive damping: the particle is rolling down from the top of a hill (the IR CFT) and because of friction it will stop at the bottom of the hill (the N=4 point)[4]. An analogous result was obtained for the supersymmetric point [20].

The interpretation of the kink as a RG flow is strengthened by exhibiting a candidate c-function [10],

$$c(y) = \text{const}\,(T_{yy})^{-3/2}, \tag{5}$$

where $T_{yy} = 6(\dot{\phi})^2 = \sum_a (\dot{\lambda}_a)^2 - 2V$ is the y component of the stress-energy tensor. Indeed $c(y)$ reduces to the central charge eq. (4) at the critical points, where $\dot{\lambda}_a = 0$ and is monotonic along the flow. One way to see this is to observe that $c(y)$ is inversely proportional to the energy of a classical particle in a potential subject to a positive damping: under this conditions energy always decreases.[5]. This proof of the c-theorem is valid for all the supergravity flows involving the metric and an arbitrary number of scalars; an extension to generic Poincaré invariant supergravity solutions is given in [20]. All these kink solutions generically correspond to deformations of the UV fixed point; this can be explicitly checked for the $SU(2) \times U(1)$ point.

(b) Flow to non conformal theories.

Even though we have been able to prove that a kink solution always exist, this requires a certain amount of fine tuning. Most of the solutions starting at the N=4 point run to infinity in the space of parameters described by supergravity. It is natural to interpret them as RG flows to non-conformal field theories [21]. This is the case of QCD like theories obtained deforming N=4 SYM with fermion mass terms [21, 22], or of solutions corresponding to the Coulomb branch of N=4 SYM [23]. Similar behaviour is also exhibited by certain 10d solutions [24, 25, 27, 28].

Let us consider N=4 SYM deformed with a generic mass term for the fermions. The scalar fields may also have a mass term or they can acquire it via loop corrections. In the IR we expect these theories to flow to QCD like theories. Even though standard QCD is outside the regime of validity of supergravity,[6] we expect to find from supergravity the qualitative properties of the theory: in particular, confinement and mass gap [5].

[4] In this interpretation we go from IR to UV.

[5] From the equations of motion $6\ddot{\phi} = -4\sum_a (\dot{\lambda}_a)^2$ and it is easy to check that, due to the boundary conditions, $\dot{\phi}$ is always positive. Note that y increases when the energy increases, but $c(y)$ decreases going from UV to IR.

[6] To get standard QCD would indeed require a fine tuning of the UV parameters, in which the mass m diverges while the 't Hooft coupling constant, x, goes to zero as an (inverse) logarithm of m.

On the supergravity side we then look for solutions with only one asymptotically AdS_5 region, corresponding to the UV N=4 super Yang-Mills $SU(N)$ theory: $\phi(y) \to y/R_{UV}$ + const, $\lambda_a(y) \to 0$ and $\lambda_0(y) \to cost$ for $y \to \infty$. Here λ_a are scalars in the $\underline{10}$ of $SU(4)$ and λ^0 is the five-dimensional dilaton. One can show [21] that eqs.(3) have a universal IR solution as long as the potential $V(\lambda_a)$ can be neglected:

$$\begin{aligned}\lambda_a &= -K_a \log|y-a| + \text{const.}, \ \sum K_a a^2 = \frac{3}{8} \\ ds^2 &= dy^2 + |y-a|^{1/2} dx^\mu dx_\mu. \end{aligned} \tag{6}$$

The assumption that the potential is irrelevant is justified whenever V is polynomial in λ^a. This in general not the case in gauged supergravity, where the potential is exponential at least in some of the scalars. This means that the constants K^a in Eq. (3) must obey some model-dependent inequalities.

Depending on the value of the parameters K_a, the solutions may exhibit conformal behaviour, confinement, or screening in the IR. Confinement corresponds to an area law for the Wilson loop of a quark-antiquark pair. In supergravity, the Wilson loop associated to a given boundary contour C is obtained by minimising the classical action for a string free to move in the five-dimensional space but with endpoints on C [29]. In an appropriate coordinate system, we have [21]:

$$E = S/T = \int dx \sqrt{(\partial_x u)^2 + f(u)}, \tag{7}$$

with $\frac{\partial u}{\partial y} = T(y)e^{\phi(y)}$, $f(u) = T^2(u)e^{4\phi(u)}$. $T(y)$ is the tension of the fundamental (the case of a quark) or of the D1 string (monopole) in five dimensions. On the basis of naive dimensional reduction from 10d, they are given by the coefficients of the kinetic terms for the anti-symmetric tensors B. Because of the several Weyl rescaling we need to reduce the 10d equations of motion in the string frame to the five-dimensional Einstein frame, the five-dimensional tensions are in general non-trivial functions of the supergravity scalars and model-dependent. In general we expect a power behaviour for the tensions: $T_{F1} = y^{-\sum d_a K_a - K_0}$ and $T_{D1} = y^{-\sum d_a K_a + K_0}$; the coefficients d_a depend on the specific model.

The knowledge of the asymptotic behaviour for $u \sim 0$ of the function $f(u)$ is sufficient for extracting qualitative information about the Wilson loop [21]. Roughly speaking we expect confinement when the string feels a barrier that prevents it from penetrating the deep IR region. This happens when $f(u)$ goes as negative power of u or when it is bounded from below.

One can show that for the class of solutions (6) confinement is generic, in the sense that there is a large range of values of K^a for which quarks confine and monopoles are screened. This is expected from the physical interpretation as a mass deformation of N=4 super Yang-Mills.[7]

[7] It is indeed quite plausible that solutions where the scalars in the $\underline{10}$ of $SU(4)$ are turned on correspond to mass deformation of N=4 SYM rather than to different vacua. In general we can not explicitly check this claim, since we do not know the full solution connecting the UV with the singular IR region. A scalar fluctuation on the AdS_5 background has the UV asymptotic behaviour (see above for the physical interpretation of this expression.): $\lambda(y) = Ae^{-(4-\Delta)y} + Be^{-\Delta y}$. Then in order to describe a different vacuum of N=4 SYM, we should set $A = 0$. For an operator of dimension $\Delta = 3$ this would require a fine tuning, or in other words a non generic solution.

The singular IR behaviour of the solutions (6) is common to many ten dimensional solutions from type IIB [24, 25] to type OB [26] and even to non-critical strings [27]. Agreement with ref. [24] is encouraging, since that paper studies a deformation of type IIB theory on $AdS_5 \times S_5$; i.e. a model that falls into the class we are studying. More surprising is agreement with refs. [26, 27]. It has however at least a technical reason: all these models can be reduced to some effective theory for five-dimensional scalars. The common behaviour of the IR metric in all these different cases may mean that the result we obtained is truly universal, and largely insensitive to the UV theory used to derive the IR confining behaviour.

Finally some comments about the validity of the solution are in order. Since the metric and the kinetic terms for scalars blow up in the IR, we expect that the supergravity solution is significantly corrected by the higher derivatives terms in the Lagrangian. In the case of the Coulomb branch discrepancies between supergravity and field theory results have been found [23]. It does not mean however that all supergravity solutions could not be trusted. Screening is more sensitive to infrared physics than confinement: confinement is obtained when the fundamental string does not probe the far IR region, whose details become therefore irrelevant. It is possible then that the particles wave-functions and the fundamental string world-sheet, needed for computing the mass spectrum and the Wilson loops, always live in a region where curvature and higher derivatives terms are still under control.

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