An approach to approximate calculations of Green functions[1]

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Abstract

A new approach proposed recently by author for the calculation of Green functions in quantum field theory and quantum mechanics is briefly reviewed. The method is applied to nonperturbative calculations for anharmonic oscillator, ϕ^4-theory, quantum electrodynamics and other models.

1 Toy model

To illustrate the general properties of the calculation scheme, which was proposed recently in works [1]-[3], we consider a toy problem of the calculation of "n-particle Green functions" in zero-dimensional theory with ϕ^4-interaction, i.e., a problem of calculating the following quantities

$$G_n = g_n/g_0, \tag{1}$$

where

$$g_n = \int_{-\infty}^{\infty} d\phi\, \phi^{2n} \exp\{-\frac{m^2}{2}\phi^2 - \lambda\phi^4\}. \tag{2}$$

A generating function for these quantities is

$$g(x) = \int_{-\infty}^{\infty} d\phi \exp\{-\frac{m^2}{2}\phi^2 - \lambda\phi^4 + x\phi^2\}. \tag{3}$$

So that $g_n = \frac{d^n g}{dx^n}\mid_{x=0}$.

The generating function $g(x)$ satisfies the differential equation

$$4\lambda\frac{d^2g}{dx^2} + (m^2 - 2x)\frac{dg}{dx} - g = 0, \tag{4}$$

which is the Schwinger-Dyson equation (SDE) for this toy model.

A solution of this equation can be written as a coupling constant perturbation (CCP) series $g(x) = g^{(0)}_{pert} + \cdots + g^{(i)}_{pert} + \cdots$ where the leading ("free field") approximation is $g^{(0)}_{pert} = \exp\{-\frac{1}{2}\log|m^2 - 2x|\}$ (up to a multiplier which inessential for calculating of $G^{(i)}$) and $g^{(i)}_{pert}$ is a solution of the perturbation iteration scheme equation

$$(m^2 - 2x)\frac{dg^{(i)}_{pert}}{dx} - g^{(i)}_{pert} = -4\lambda\frac{d^2g^{(i-1)}_{pert}}{dx^2}.$$

¿From the point of view of differential equation theory the CCP is attributed to the type of so-called singular perturbations. This fact defines, to a considerable extent, both the poor convergence properties (the CCP series is an asymptotic expansion at best) and the limited nature of the region of its applicability. (In the model under consideration the CCP theory gives good results only in the region $\lambda \leq 0.01m^4$.) A perturbation is named to be singular if it contains a higher derivative term. The perturbation theory in λ is singular in the above sense, since the leading approximation consists in the neglecting of the higher derivative term. (See [4], [2] for more discussion.)

An alternative for the perturbation theory in λ can be another iterative scheme that is based on an approximation of eq. (1) near the point $x = 0$ by an equation with constant coefficients. Take as a leading approximation the equation

$$4\lambda\frac{d^2g^{(0)}}{dx^2} + m^2\frac{dg^{(0)}}{dx} - g^{(0)} = 0. \tag{5}$$

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The term $2x\frac{dg}{dx}$ will be considered as a perturbation. The iteration scheme will consist in step-by step solutions of inhomogeneous equations

$$4\lambda\frac{d^2g^{(i)}}{dx^2}+m^2\frac{dg^{(i)}}{dx}-g^{(i)}=2x\frac{dg^{(i-1)}}{dx}. \tag{6}$$

The leading approximation equation has the solution $g^{(0)}=e^{\alpha x}$, where α's are the roots of the characteristic equation

$$4\lambda\alpha^2+m^2\alpha-1.$$

From a condition of the convergence of integral (3) at $\lambda\to 0$ we choose the root

$$\alpha=\frac{-m^2+\sqrt{m^4+16\lambda}}{8\lambda}=1/m^2+\mathcal{O}(\lambda).$$

A solution of nth-step equation is looked for in the form $g^{(i)}=p^{(i)}g^{(0)}$. Taking into account the leading approximation equation (5), we obtain the equation for $p^{(i)}$

$$4\lambda\Big(\frac{d^2p^{(i)}}{dx^2}+2\alpha\frac{dp^{(i)}}{dx}\Big)+m^2\frac{dp^{(i)}}{dx}=2x\Big(\frac{dp^{(i-1)}}{dx}+\alpha p^{(i-1)}\Big). \tag{7}$$

It is evident from this equation that $p^{(i)}$ is a polynomial of degree $2i$ in x.

The question about a small parameter for the expansion defined by eqs. (5)-(7) arises. There is no manifest small parameter for this expansion , but it is clear, that the expansion approximates well the exact solution not only for small values of λ. Really, in the strong coupling region $\lambda\to\infty$ the first-step one-particle function G_1 ("propagator") approximates the exact result with accuracy of 26%, and second-step one with accuracy 7%.

To be more exact, the question about the small parameter should be replaced by the question about a convergence of the expansion. But the convergence of this iteration scheme can be easily proved. Notice, the iteration scheme defined by eqs. (5)-(7) is equivalent to iterations of the second kind Volterra equation with the continuous kernel. The convergence of the iteration of this equation is fulfilled by the textbook theorem.

So, this regular expansion possesses the good convergency properties in the framework of this simple zero-dimensional model and, more importantly, is a nonperturbative method of calculations of Green functions.

2 ϕ^4-theory

Let us go to the field theory. Consider the theory of a scalar field $\phi(x)$ in the Euclidean space E_d with the action

$$A(\phi)=\int dx\{\frac{1}{2}(\partial_\mu\phi)^2+\frac{m^2}{2}\phi^2+\lambda\phi^4\} \tag{8}$$

and with the generating functional of Green functions (vacuum expectation values)

$$G(\eta)=\int D\phi\exp\{-A+\phi\eta\phi\}. \tag{9}$$

Here $\eta(x,y)$ is a bilocal source, $\phi\eta\phi\equiv\int dxdy\phi(x)\eta(x,y)\phi(y)$. A normalization constant is omitted. The nth derivative of G over η with the source being switched off is the $2n$-point (n-particle) Green function.

The Schwinger-Dyson equation (SDE) for the generating functional $G(\eta)$ is a corollary of the identity

$$0=\int D\phi\frac{\delta}{\delta\phi(x)}(\phi(y)\exp\{-A+\phi\eta\phi\}). \tag{10}$$

Taking into account the above definitions we get the SDE for ϕ^4-theory

$$4\lambda\frac{\delta^2G}{\delta\eta(y,x)\delta\eta(x,x)}+(m^2-\partial^2)\frac{\delta G}{\delta\eta(y,x)}-2\int\eta(x,u)\frac{\delta G}{\delta\eta(y,u)}du-\delta(x-y)G=0. \tag{11}$$

At $d = 0$ (zero-dimensional theory, or "single-mode approximation") the functional derivatives transform into usual ones, and eq. (11), after obvious redesignations, reduces to the ordinary differential eq. (4). At $d = 1$ the model corresponds to the quantum-mechanical anharmonic oscillator. At $d \geq 2$ (field theory) for the cancellation of ultraviolet divergences the appropriate counterterms should be included in the action. The SDE for the theory with counterterms has the form of eq. (11) with the substitution $\lambda \to \lambda + \delta\lambda, \quad m^2 \to m^2 + \delta m^2, \quad \partial^2 \to (1 + \delta z)\partial^2$, where $\delta\lambda, \delta m^2$ and δz are correspondingly counterterms of coupling, mass and wave function renormalizations.

Let apply to the SDE (11) the same idea about the approximation by an equation with "constant" (i.e., independent from η) coefficients. As the leading approximation equation we will consider the equation

$$4\lambda \frac{\delta^2 G^{(0)}}{\delta\eta\delta\eta} + (m^2 - \partial^2)\frac{\delta G^{(0)}}{\delta\eta} - G^{(0)} = 0, \tag{12}$$

and the term $2\eta\frac{\delta G}{\delta\eta}$ (that contains the source η explicitly) should be treated as a perturbation. Since Green functions are the derivatives of $G(\eta)$ at zero and only the behaviour of G near $\eta = 0$ is essential , such an approximation seems to be acceptable. The iteration procedure for the generating functional $G = G^{(0)} + G^{(1)} + \cdots + G^{(i)} + \cdots$ consists in the step-by-step solution of the equations

$$4\lambda \frac{\delta^2 G^{(i)}}{\delta\eta\delta\eta} + (m^2 - \partial^2)\frac{\delta G^{(i)}}{\delta\eta} - G^{(i)} = 2\eta\frac{\delta G^{(i-1)}}{\delta\eta}. \tag{13}$$

The solution of the leading approximation equation (10) is the functional

$$G^{(0)} = \exp\{\int dxdy\eta(y,x)\,\triangle^{(0)}\,(x-y)\}, \tag{14}$$

where $\triangle^{(0)}$ is a solution of the "characteristic" equation

$$4\lambda\,\triangle^{(0)}\,(0)\,\triangle^{(0)}\,(x-y) + (m^2 - \partial^2)\,\triangle^{(0)}\,(x-y) = \delta(x-y). \tag{15}$$

At $d \geq 2$ the quantity $\triangle^{(0)}(0)$ must be considered as some regularization.

Equation (15) is similar to the equation for the propagator in the leading approximation of the $1/N$-expansion. Certainly, the similarity is completely superficial, since the principle of the construction of the approximation scheme is different.

The solution of equation (15) is the free propagator $\triangle^{(0)} = (\mu^2 - \partial^2)^{-1}$ with the renormalized mass $\mu^2 = m^2 + 4\lambda\triangle^{(0)}(0)$. The quantity $\triangle^{(0)}(0)$ is defined from the self-consistency condition. The propagator is the first derivative of $G(\eta)$ over the source η: $G_1 \equiv \triangle = \frac{\delta G}{\delta\eta}\,|_{\eta=0}$. As can be easily seen, it is simply $\triangle^{(0)}$ at the leading approximation.

Notice, that the generating functional (14) of the leading approximation does not possess the complete Bose-symmetry. Really, as follows from the definition of generating functional, the Bose-symmetry implies for *full* generating functional the condition

$$\frac{\delta^2 G}{\delta\eta(y,x)\delta\eta(y',x')} = \frac{\delta^2 G}{\delta\eta(y',x)\delta\eta(y,x')}. \tag{16}$$

Evidently condition (16) is not fulfilled for $G^{(0)}$ defined by eq.(14). The violation of this condition in particular leads to the violation of connected structure of the leading approximation two-particle (four-point) Green function.

Such a situation is rather typical for nonperturbative calculational schemes with bilocal source (for example, for $1/N$-expansion in the bilocal source formalism), but the discrepancy of such type is not an obstacle for using these iteration schemes. Really, condition (16) should be satisfied by the *full* generating functional G which is an *exact* solution of SDE. It is clear that an approximate solution may not possess all the properties of an exact one. In given case we have just this situation. Properties of connectivity and Bose-symmetry of higher Green functions, which are not fulfilled at first steps of the iteration scheme, are improved at subsequent steps. For example, the structure of disconnected part of the two-particle function is reconstructed as already at the first step of the iteration scheme. At subsequent

steps, the correct connected structure of many-electron functions and other corollaries of Bose-symmetry are reconstructed. Such stepwise reconstruction of the properties exact solution is very natural for the given iteration scheme as it is based on an approximation of the generating functional $G(\eta)$ in vicinity of zero. The Green functions are coefficients of the generating functional expansion in the vicinity of zero, therefore only the lowest functions are well-described at first steps of the approximation – at the leading approximation the propagator only. Higher-order many-particle functions come into play later, at following steps, and relation (16) is fulfilled more and more exactly when we go toward the exact solution.

In the general case, the solution of equation for the i-th step of the iteration scheme is the functional $G^{(i)} = P^{(i)}G^{(0)}$, where $P^{(i)}$ is a polynomial in η of a degree $2i$. Therefore, at the i-th step the computation of Green functions reduces to solving a system of $2i$ linear integral equations.

A solution of the first step equation is $G^{(1)} = P^{(1)}G^{(0)}$ where $P^{(1)} = \frac{1}{2}F\eta^2 + \triangle^{(1)}\eta$. The function F is two-particle (four-point) function of the first step, and $\triangle^{(1)}$ is the first-step correction to the propagator. Eq.(13) at $i = 1$ gives us a system of equations for F and $\triangle^{(1)}$. Equations for F and $\triangle^{(1)}$ are simple linear integral equations. The exact form of solutions of these equations see in [1]. At $\lambda \to 0$ the first step propagator reproduces correctly the first term of the usual CCP theory.

At $d = 1$ the model with action (6) describes the quantum-mechanical anharmonic oscillator. Ultraviolet divergences are absent, the quantities of $\triangle^{(0)}(0)$ type are finite and the above formulae are applied directly for the computation of Green functions.

To calculate a ground state energy E one can use the well-known formula

$$\frac{dE}{d\lambda} = G_2(0,0,0,0),$$

where G_2 is the four-point (or two-particle) function. Integrating the formula with a boundary condition $E(\lambda = 0) = m/2$ taken into account, one can calculate the ground state energy for all values of the coupling (see [1]).

At $\lambda \to 0$ the first step calculation reproduces the perturbation theory up to the second order. At $\lambda \to \infty$: $E = \epsilon_0\lambda^{1/3} + O(\lambda^{-1/3})$, and $\epsilon_0 = 0.756$. The coefficient ϵ_0 differs by 13% from the exact numerical one $\epsilon_0^{exact} = 0.668$. At $\lambda/m^3 = 0.1$ the result of the calculation differs from the exact numerical one by 0.8% and at $\lambda/m^3 = 1$ differs by 6.3%. Therefore, the first step calculations approximate the ground state energy for all values of λ with the accuracy that varies smoothly from 0 (at $\lambda \to 0$) to 13% (at $\lambda \to \infty$).

At $d \geq 2$ action (6) should be supplemented by counterterms for the elimination of ultraviolet divergences. There is no need to add a counterterm of wave function renormalization at the leading approximation, and the equation of the leading approximation will be

$$4(\lambda + \delta\lambda_0)\frac{\delta^2 G^{(0)}}{\delta\eta\delta\eta} + (\delta m_0^2 + m^2 - \partial^2)\frac{\delta G^{(0)}}{\delta\eta} - G^{(0)} = 0. \tag{17}$$

At $i \geq 1$ the counterterms $\delta\lambda_i$, δm_i^2 and δz_i should be considered as perturbations. Therefore, the corresponding terms should be added to the r.h.s. of equation (11). So, the first step equation will be

$$\begin{aligned} &4(\lambda + \delta\lambda_0)\frac{\delta^2 G^{(1)}}{\delta\eta\delta\eta} + (\delta m_0^2 + m^2 - \partial^2)\frac{\delta G^{(1)}}{\delta\eta} - G^{(1)} = \\ &= 2\eta\frac{\delta G^{(0)}}{\delta\eta} - \delta m_1^2\frac{\delta G^{(0)}}{\delta\eta} + \delta z_1\partial^2\frac{\delta G^{(0)}}{\delta\eta} - 4\delta\lambda_1\frac{\delta^2 G^{(0)}}{\delta\eta\delta\eta}. \end{aligned} \tag{18}$$

For the super-renormalizable theory ($d = 2$ and $d = 3$) it is sufficient to add counterterms of mass renormalization and wave function renormalization, i.e. $\delta\lambda_i = 0$ for all n. The normalization condition on the physical renormalized mass μ^2 gives us a counterterm of the mass renormalization in the leading approximation. This counterterm diverges logarithmically at $d = 2$ and linearly at $d = 3$. The counterterm δz_1 is finite at $d = 2, 3$. The counterterm δm_1^2 diverges as that of the leading approximation does, namely, logarithmically at $d = 2$ and linearly at $d = 3$.

At $d = 4$, besides the renormalizations of the mass and the wave function, a coupling renormalization is necessary. Due to the presence of the counterterm $\delta\lambda$ the normalization condition on the renormalized

mass μ^2 for the leading approximation becomes the connection between counterterms δm_0^2 and $\delta\lambda_0$. Counterterm $\delta\lambda_0$ (and, consequently, δm_0^2) will be fixed at the *following* step of the iteration scheme.

A solution of the equation for the four-point function F at $d = 4$ diverges logarithmically, and a renormalization of the coupling is necessary. The equation for F contains the counterterm $\delta\lambda_0$ only. Therefore, by defining a renormalized coupling λ_r as a value of the amplitude at a normalization point, we obtain the counterterm of the coupling renormalization $\delta\lambda_0$ and the renormalized amplitude. Taking the renormalization of the two-particle amplitude in such a manner, one can solve the equation for $\triangle^{(1)}$ and renormalize the mass operator in correspondence with the general principle of normalization on the physical mass. But in four-dimensional case one gets an essential obstacle. As the regularization is removed, $\delta\lambda_0 \to -\lambda$, the coefficient $\lambda + \delta\lambda_0$ in the leading approximation equation (17) vanishes. The same is true for all subsequent iterations. The theory is trivialized. One can object that an expression

$$(\lambda + \delta\lambda_0) \cdot \frac{\delta^2 G}{\delta\eta(y,x)\delta\eta(x,x)} \tag{19}$$

is really an indefinite quantity of $0 \cdot \infty$ type, and the renormalization is, in the essence, a definition of the quantity. But it does not save a situation *in this case* since the renormalized amplitude possesses a nonphysical singularity in a deep-euclidean region (it is a well-known Landau pole). The unique noncontradictory possibility is a choice $\lambda_r \to 0$ at the regularization removed. This is the triviality of the theory again. The triviality appears almost inevitably in an investigation of ϕ_4^4-theory beyond the CCP theory and is a practically rigorous result. Notice, that contrary to the CCP theory which is absolutely nonsensitive to the triviality of the theory, the proposed method leads to the triviality already at the first step.

3 Quantum Electrodynamics

The Lagrangian of Quantum Electrodynamics (QED) in Minkowski space-time with a gauge fixing term has the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2d_l}(\partial_\mu A_\mu)^2 + \bar\psi(i\hat\partial - m + e\hat A)\psi. \tag{20}$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\hat A \equiv A_\mu\gamma_\mu$, $\bar\psi = \psi^*\gamma_0$, m is an electron mass, e is a charge (coupling constant), d_l is a gauge parameter, γ_μ are Dirac matrices. For notational simplicity we write all vector indices as low ones.

The generating functional of Green functions is

$$G(J,\eta) = \int D(\psi,\bar\psi,A)\exp i\Big\{\int dx\Big(\mathcal{L} + J_\mu(x)A_\mu(x)\Big) - \int dxdy\bar\psi^\beta(y)\eta^{\beta\alpha}(y,x)\psi^\alpha(x)\Big\}. \tag{21}$$

Here $J_\mu(x)$ is a source of the gauge field, and $\eta^{\beta\alpha}(y,x)$ is a bilocal source of the spinor field (α and β are spinor indices). Normalization constant omitted.

Functional derivatives of G with respect to sources are vacuum expectation values

$$\frac{\delta G}{\delta J_\mu(x)} = i < 0 \mid A_\mu(x) \mid 0 >, \quad \frac{\delta G}{\delta\eta^{\beta\alpha}(y,x)} = i < 0 \mid T\Big\{\psi^\alpha(x)\bar\psi^\beta(y)\Big\} \mid 0 > . \tag{22}$$

SDEs for the generating functional of Green functions of QED are

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G}{\delta J_\nu(x)} + ie\,\mathrm{tr}\Big\{\gamma_\mu\frac{\delta G}{\delta\eta(x,x)}\Big\} + J_\mu(x)G = 0, \tag{23}$$

$$\delta(x-y)G + (i\hat\partial - m)\frac{\delta G}{\delta\eta(y,x)} + \frac{e}{i}\gamma_\mu\frac{\delta^2 G}{\delta J_\mu(x)\delta\eta(y,x)} - \int dx'\eta(x,x')\frac{\delta G}{\delta\eta(y,x')} = 0. \tag{24}$$

(Here and everywhere below ∂_μ denote a differentiation with respect to variable x.) Let us resolve SDE (23) with regard to the first derivative of the generating functional with respect J_μ:

$$\frac{1}{i}\frac{\delta G}{\delta J_\mu(x)} = -\int dx_1 D^c_{\mu\nu}(x-x_1)\Big\{J_\nu(x_1)G + ie\,\mathrm{tr}\,\gamma_\nu\frac{\delta G}{\delta\eta(x_1,x_1)}\Big\}, \tag{25}$$

and substitute it into the second SDE (24). As a result, we obtain the "integrated over A_μ" (in the functional-integral terminology) equation

$$\delta(x-y)G + (i\hat{\partial} - m)\frac{\delta G}{\delta\eta(y,x)} + \frac{e^2}{i}\int dx_1 D^c_{\mu\nu}(x-x_1)\gamma_\mu \frac{\delta}{\delta\eta(y,x)}\,\mathrm{tr}\,\gamma_\nu \frac{\delta G}{\delta\eta(x_1,x_1)} = \\ = \int dx_1 \Big\{\eta(x,x_1)\frac{\delta G}{\delta\eta(y,x_1)} + eD^c_{\mu\nu}(x-x_1)J_\nu(x_1)\gamma_\mu\frac{\delta G}{\delta\eta(y,x)}\Big\}. \qquad (26)$$

Exploiting Fermi-symmetry condition

$$\frac{\delta^2 G}{\delta\eta^{\beta\alpha}(y,x)\delta\eta^{\beta'\alpha'}(y',x')} = -\frac{\delta^2 G}{\delta\eta^{\beta'\alpha}(y',x)\delta\eta^{\beta\alpha'}(y,x')}. \qquad (27)$$

let us transform eq.(26) in following manner:

$$\delta(x-y)G + (i\hat{\partial} - m)\frac{\delta G}{\delta\eta(y,x)} + ie^2\int dx_1 D^c_{\mu\nu}(x-x_1)\gamma_\mu \frac{\delta}{\delta\eta(x_1,x)}\gamma_\nu \frac{\delta G}{\delta\eta(y,x_1)} = \\ = \int dx_1 \Big\{\eta(x,x_1)\frac{\delta G}{\delta\eta(y,x_1)} + eD^c_{\mu\nu}(x-x_1)J_\nu(x_1)\gamma_\mu\frac{\delta G}{\delta\eta(y,x)}\Big\}. \qquad (28)$$

From the point of view of *exact* solutions equations (26) and (28) are fully equivalent since the transition from eq.(26) to eq.(28) is, in essence, an identical transformation. However, it is not the case for the used iteration scheme since, as for Bose-symmetry condition (16), Fermi-symmetry condition (27) is fulfilled only approximately at any finite step of the iteration scheme. Therefore, eqs. (26) and (28) lead to different expansions. Eq.(26) gives the calculational scheme which in the language of Feynman diagrams of perturbation theory is analog of summation of chain diagrams with fermion loop. This version is named "calculations over perturbative vacuum" since a unique connected Green function of the leading (vacuum) approximation is the free electron propagator. This scheme leads, as ϕ^4-theory (see above, section 2), to Landau pole and triviality at the first step of the iteration scheme (see [5] for more details).

The second version of the iteration scheme, which is based on eq.(28), gives us a fruitful and "insensitive to triviality" scheme of calculation of physical quantities. This version is named "calculations over nonperturbative vacuum" since the electron propagator of the leading vacuum approximation is a solution of a non-trivial nonlinear equation. For this scheme a calculation of two first terms of expansion of the vertex function in photon momentum for chiral-symmetric vacuum have been performed in work [5]. This calculation has allowed to obtain a simple formula for anomalous magnetic moment: $f_2 = \alpha/(2\pi - \alpha)$, where α is the fine structure constant. Also, for a linearized version of the theory (see, for instance, [6]) the problem of dynamical chiral symmetry breaking have been investigated in work [5]. The calculations are performed for renormalized theory in Minkowski space. In the strong coupling region $\alpha \geq \pi/3$ the results correspond to earlier investigations performed in Euclidean theory with cutoff (see [6]): there arise solutions with breakdown of chiral symmetry. But for the renormalized theory a solution with breakdown of chiral symmetry is also possible in the weak coupling region $\alpha < \pi/3$ with a subsidiary condition on the value of α which follows from the gauge invariance (see [5] for more details).

4 Other models

Some other models have been investigated by proposed method in works [2]-[3] and [7]-[9]. Famous Gross-Neveu model with the Lagrangian

$$\mathcal{L} = \bar{\psi}_j i\hat{\partial}\psi_j + \frac{\lambda}{2N}(\bar{\psi}_j\psi_j)^2 \qquad (29)$$

have been investigated by this method in work [3] at $D = 2, 3, 4$ and finite N, where D is space dimension and N is a number of flavors. The results were following: a spontaneous symmetry breaking is shown to exist in $D = 2, 3$ and the running coupling constant is calculated. The four-dimensional theory turns to be trivial. These results exhibit the efficiency of the method and are the finite N generalization of the known results obtained in the framework of $1/N$ expansion.

The greatest interest from the physical point of view presents applications of the method to study gauge theories in nonperturbative region. First steps in this direction were made in works [7]-[9]. In work [7] a generalization of the Higgs mechanism which takes into account the contributions of gauge field vacuum configuration into the formation of the physical vacuum was considered. For the Abelian Higgs model the triviality bound $m_H \leq 1.15 m_A$ was found. In works [8]-[9] a non-Abelian $SU(2)$ gauge theory was considered, and a mechanism for the dynamical mass generation of a non-Abelian gauge field which was based on taking into account the contributions of the gauge field vacuum configurations for formation of the physical vacuum was proposed. A most winning field of application of proposed method seems to be a problem of dynamical chiral symmetry breaking in gauge theories.

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