Massive Rarita-Schwinger field in the AdS/CFT correspondence [1]

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Abstract

In the frame of the so called AdS/CFT correspondence, we consider a massive Rarita-Schwinger field on the Anti-de Sitter space and solve its equations of motion. We show that appropriate boundary terms calculated on-shell give correct two-point correlation function for operators with spin 3/2 in the conformal field theory on the AdS boundary. The relation between Rarita-Schwinger field masses and conformal dimensions of corresponding operators is established.

Recently much attention was attracted by the new relation between string theories and conformal field theories (so called AdS/CFT correspondence), proposed in [1] and elaborated in [2, 3] (for review see [4]). In its general form it states that string theory on Anti-de Sitter (AdS) space is equivalent (dual) to appropriate conformal field theory (CFT) on its boundary. Large number of examples was investigated, however, the most important of them is provided by IIB superstring theory on $AdS_5 \times S^5$ background which is dual to $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory (SYM), appearing on the AdS_5 boundary. More precisely, this equivalence means that all physical exitations of the AdS bulk theory can be put in one-to-one correspondence with the composite operators of the conformal theory on the boundary. For example, the dilaton field in the bulk theory corresponds to $\mathrm{Tr}F^2$. Also there are relations between string theory and conformal theory parameters, namely

$$g_s = \frac{g_{YM}^2}{4\pi}, \qquad \frac{R^2}{\alpha'} = \sqrt{g_{YM}^2 N}. \tag{1}$$

Here α' and g_s are string tension and string coupling constant respectively, g_{YM} is SYM coupling constant. R denotes the radius of AdS space. Unfortunately, all real calculations could be performed only in appropriate limits, namely $g_s \to 0$, $\alpha' \to 0$. The first requirement means that there are no string loop corrections, the second requirement is the low energy limit in which string theory could be described by effective supergravity theory. On the SYM side we have the so called 't Hooft limit $N \to \infty$, in which 't Hooft coupling constant is fixed and large $g_{YM}^2 N \gg 1$. Thus, in the case of IIB superstrings we find the duality between IIB supergravity on $AdS_5 \times S^5$ background and $\mathcal{N} = 4$, $D = 4$ SYM with the gauge group $SU(N)$ in the 't Hooft limit.

Except one-to-one correspondence between bulk fields and boundary operators there is a prescription how to find CFT correlators performing calculations in supergravity [2, 3]. Namely, according to AdS/CFT correspondence the generating functional for CFT operators is equal to the exponentiated classical supergravity action I calculated on-shell with the given boundary conditions specified by the function ϕ_0

$$\left\langle \exp\left(\int_{\partial AdS_{d+1}} d^d x \mathcal{O} \phi_0 \right) \right\rangle_{CFT} = \exp(-I(\phi)). \tag{2}$$

Thus, in order to find the correlation functions of operators in conformal field theory, one has to solve the classical Dirichlet problem for the corresponding supergravity field on the AdS space. It gives the direct way to check the correspondence: one should compare the correlation functions expected in conformal field theories and those obtained via supergravity. In this way two-point functions of local operators corresponding to the scalar fields [3, 5, 6, 7, 8], vector fields [3, 7, 9, 10], spinor fields [11, 9], metric tensor fields [12, 13, 14], massive symmetric rank 2 tensor fields [15], antisymmetric form fields [16, 17, 18] and massless Rarita-Schwinger fields [19, 20] were investigated and the agreement with the conformal field theory results was established. Also various three-point correlation functions for these fields were examined [5, 6, 7, 9, 21, 22]. In all cases the results are in agreement with those expected in the conformal field theory.

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Our work continues this activity and is devoted to consideration of the massive Rarita-Schwinger field and finding two-point correlation function of corresponding CFT operators.

The general action for the massive Rarita-Schwinger field Ψ_μ in a $d+1$-dimensional curved space is [23]

$$S = \int d^{d+1}x\sqrt{g}(\bar{\Psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \Psi_\rho - m_1 \bar{\Psi}_\mu \Psi^\mu - m_2 \bar{\Psi}_\mu \Gamma^{\mu\nu} \Psi_\nu). \tag{3}$$

Here D_μ is a covariant derivative, $\Gamma^{\mu\nu\rho} = \Gamma^{[\mu}\Gamma^\nu\Gamma^{\rho]}$, $\Gamma^{\mu\nu} = \Gamma^{[\mu}\Gamma^{\nu]}$, where Γ_μ are curved space gamma matrices connected with the flat space gamma matrices γ_a via vielbein e^a_μ: $\Gamma_\mu = e^a_\mu \gamma_a$, μ, $a = 0, 1, .., d$. The flat space gamma matrices are subjects of the anticommutation relations $\{\gamma_a, \gamma_b\} = 2\delta_{ab}$. We suppose that the background is $d+1$-dimensional Euclidean Anti-de Sitter space with the coordinates $x = (x_0, \vec{x}) = (x_0, x_i)$, where $x_0 > 0$, $i = 1, .., d$. The hypersurface $x_0 = 0$ will be considered as the AdS boundary. The corresponding AdS metric has the form

$$ds^2 = \frac{1}{x_0^2}(dx_0^2 + dx_i dx^i) \tag{4}$$

and we can specify the vielbein as

$$e^a_\mu = \frac{1}{x_0}\delta^a_\mu. \tag{5}$$

Below we will investigate the action (3) with rather general mass parameters m_1 and m_2. However the most interesting results are related with the Rarita-Schwinger field on AdS_5 appearing in the compactification of IIB supergravity on $AdS_5 \times S^5$. The corresponding mass spectrum was found in [24] and consists of three sets of states:

$$\text{I.} \quad m_1 = -(3/2 + k), \quad m_2 = 0, \quad k = 0, 1, 2, .. \tag{6}$$

$$\text{II.} \quad m_1 = 7/2 + k, \quad m_2 = 0, \quad k = 0, 1, 2, .. \tag{7}$$

$$\text{III.} \quad m_1 = 0, \quad m_2 = -3/2. \tag{8}$$

$$\tag{9}$$

The action (3) leads to the equation of motion which can be written in the following form

$$\Gamma^\nu(D_\nu \Psi_\rho - D_\rho \Psi_\nu) - m_- \Psi_\rho + \frac{m_+}{d-1}\Gamma_\rho \Gamma^\nu \Psi_\nu = 0, \tag{10}$$

where $m_\pm = m_1 \pm m_2$. For the future convenience we project the Rarita-Schwinger field on the vielbein introducing

$$\psi_a = e^\mu_a \Psi_\mu. \tag{11}$$

Following [3], in order to solve this equation we will find a solution which depends only on x_0 coordinate and has a singularity on the infinity and then apply the inversion (accompanied by the rotation of the vielbein), which moves the singularity to the boundary $x_0 = 0$. The fields depending only on x_0 coordinate are the subjects of the following equations

$$x_0 \gamma^0 \partial_0 \psi_0 - (\frac{d}{2} + 1)\gamma_0 \psi_0 - m_- \psi_0 = x_0 \partial_0 \chi - \frac{m_+}{d-1}\gamma_0 \chi - \chi, \tag{12}$$

$$x_0 \gamma^0 \partial_0 \psi_i - \frac{d}{2}\gamma_0 \psi_i - m_- \psi_i = \frac{1}{2}\gamma_0 \gamma_i \chi - \frac{m_+}{d-1}\gamma_i \chi + \gamma_i \psi_0, \tag{13}$$

where we denote $\chi = \gamma^a \psi_a$. It is easy to see that these equations put the constraints on the fields

$$(d - 1 - 2m_- \gamma_0)\psi_0 = (d - 1 + 2m_2 \gamma_0)\chi. \tag{14}$$

Taking (14) into account we get the following solution for the equations (12), (13)

$$\psi_0 = x_0^{d/2+C} b_0^+ + x_0^{d/2-C} b_0^-, \tag{15}$$

$$\psi_i = x_0^{d/2+m_-} b_i^+ + x_0^{d/2-m_-} b_i^- -$$
$$-\frac{2m_1}{d(d-1-2m_2)} x_0^{d/2-C} \gamma_i b_0^- - \frac{2m_1}{d(d-1+2m_2)} x_0^{d/2+C} \gamma_i b_0^+, \tag{16}$$

where

$$C = \frac{d(d-1)}{4m_1} + \frac{(m_1-m_2)(m_1+dm_2)}{m_1(d-1)} \tag{17}$$

and $b_0^\pm$, $b_i^\pm$ are constant spinors with a definite chirality ($\gamma_0 b_a^+ = b_a^+$, $\gamma_0 b_a^- = -b_a^-$). Also $b_0^\pm$, $b_i^\pm$ are the subject of the additional relations

$$\gamma^i b_i^+ = 0, \quad \gamma^i b_i^- = 0. \tag{18}$$

Applying O(d+1,1) transformation of the AdS_{d+1} isometry group we can map infinity to an arbitrary point on the boundary $x_0 = 0$. It introduces the dependence on x_i coordinates. The corresponding transformation is

$$x_\mu \longrightarrow \frac{x_\mu}{x_0^2 + \vec{x}^2}. \tag{19}$$

Note that an application of this transformation changes the vielbein (5), so in order to get a solution in an old local Lorentz frame we have to transform a vielbein as follows

$$e_\mu^a \longrightarrow L_b^a e_\mu^b, \quad \text{where} \quad L_b^a = \delta_b^a - \frac{2x^a x_b}{x_0^2 + \vec{x}^2}. \tag{20}$$

This inversion of a vielbein is supplemented by an appropriate transformation of the spinor field [25]

$$\psi_a \longrightarrow L_a^b \frac{x_0\gamma^0 + x_i\gamma^i}{\sqrt{x_0^2 + \vec{x}^2}} \psi_b. \tag{21}$$

Note that this transformation changes the sign of gamma-matrices, so in order to recover the invariance of equations of motion, we have to change all mass signs. The above procedure, applied to the solution (15), (16) gives us bulk-to-boundary propagator for the gravitino field. The contraction of this propagator with arbitrary functions yields

$$\begin{aligned}
\psi_0 = \int d^d y \Bigg(& \frac{(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2 - C\gamma_0}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2 - C\gamma_0}} (1 - \frac{2x_0^2}{x_0^2 + (\vec{x}-\vec{y})^2}) b_0(\vec{y}) \\
& - \frac{2(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2+1-m_-\gamma_0}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2+1-m_-\gamma_0}} (x-y)_i b_i(\vec{y}) \\
& - \frac{4m_1}{d(d-1+2m_2)} \frac{(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2+1+C}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2+1+C}} (x-y)_i \gamma_i b_0^-(\vec{y}) \\
& - \frac{4m_1}{d(d-1-2m_2)} \frac{(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2+1-C}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2+1-C}} (x-y)_i \gamma_i b_0^+(\vec{y}) \Bigg),
\end{aligned} \tag{22}$$

$$\begin{aligned}
\psi_i = \int d^d y \Bigg(& \frac{(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2 - m_-\gamma_0}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2 - m_-\gamma_0}} (\delta_{ij} - \frac{2(x-y)_i(x-y)_j}{x_0^2 + (\vec{x}-\vec{y})^2}) b_j(\vec{y}) \\
& - \frac{2(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2+1-C\gamma_0}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2+1-C\gamma_0}} (x-y)_i b_0(\vec{y}) \\
& + \frac{2m_1}{d(d-1+2m_2)} \frac{(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2+C}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2+C}} (\delta_{ij} - \frac{2(x-y)_i(x-y)_j}{x_0^2 + (\vec{x}-\vec{y})^2}) \gamma_j b_0^-(\vec{y}) \\
& + \frac{2m_1}{d(d-1-2m_2)} \frac{(x_0\gamma^0 + (x-y)_i\gamma^i) x_0^{d/2-C}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2-C}} (\delta_{ij} - \frac{2(x-y)_i(x-y)_j}{x_0^2 + (\vec{x}-\vec{y})^2}) \gamma_j b_0^+(\vec{y}) \Bigg).
\end{aligned} \tag{23}$$

Eq. (22) and (23) give the general solution to the equations of motion (10). In the same way we construct the solution for the conjugated spinors $\bar{\psi}_a$, that has the same form except for the change of all mass signs.

Now we have to impose the appropriate boundary conditions at the infinity and at the hypersurface $x_0 = 0$. Namely, in the solution (22), (23) we keep only the terms, which vanish as x_0 goes to infinity. Note that the asymptotic behaviour of the fields essentially depends on the values of the constants d, m_1 and m_2. Moreover, different terms in (22), (23) have different limits as x_0 tends to zero. In order to be concrete, in all formulae below we suppose m_1 and $m_1 - m_2$ to be positive (all other cases could be studied in the same way). We denote $\psi_a = \psi_a^{'}[b_i^{\pm}] + \psi_a^{''}[b_0^{\pm}]$, where we collect in $\psi_a^{'}$ all terms depending on b_i and in $\psi_a^{''}$ all terms depending on b_0. The fields $b_i^{\pm}(\vec{y})$ define the asymptotics of $x_0^{-d/2+m_-}\psi_a^{'}[b_i^{\pm}]$. As in the Dirac case [11] in order to obtain a square integrable asymptotics on the boundary we have to put $b_i^{+}(\vec{y}) = 0$. The asymptotic behaviour of the terms $x_0^{-d/2+m_-}\psi_a^{''}[b_0^{\pm}]$ is determined by the relation between two constants m_- and C. In general, due to these relations the functions b_0 could not be fixed by the boundary data, since all the terms $x_0^{-d/2+m_-}\psi_a^{''}[b_0^{\pm}]$ vanish or go to infinity as x_0 approaches zero. Thus, only b_i^- are determined by the boundary conditions. Note that in the particular case $b_0 = 0$ we have the following solution to the Dirichlet problem for the Rarita-Schwinger field on the AdS_{d+1} space

$$\psi_0(x_0, \vec{x}) = -\int d^d y \frac{2(-x_0 + (x-y)_i\gamma^i)x_0^{d/2+1+m_-}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2+1+m_-}}(x-y)_i b_i^-(\vec{y}), \tag{24}$$

$$\psi_i(x_0, \vec{x}) = \int d^d y \frac{(-x_0 + (x-y)_i\gamma^i)x_0^{d/2+m_-}}{(x_0^2 + (\vec{x}-\vec{y})^2)^{(d+1)/2+m_-}} \left(\delta_{ij} - \frac{2(x-y)_i(x-y)_j}{x_0^2 + (\vec{x}-\vec{y})^2}\right) b_j^-(\vec{y}). \tag{25}$$

The general scheme of AdS/CFT correspondence [3] prescribes us to calculate the classical action on-shell with given boundary conditions. The action for the Rarita-Schwinger field (3) vanishes on-shell, so, as in the Dirac case, [11, 13, 26] it should be supplemented with a special boundary term which should be covariant and quadratic in the Rarita-Schwinger fields [19, 20]. We take such term in the form $I = I_1 + I_2$, where

$$I_1 = \text{const} \cdot \lim_{\varepsilon \to 0} \int_{M_d^\varepsilon} d^d x \sqrt{g_\varepsilon} \bar{\Psi}_i(\vec{x}) g_\varepsilon^{ij} \Psi_j(\vec{x}), \tag{26}$$

$$I_2 = \text{const} \cdot \lim_{\varepsilon \to 0} \int_{M_d^\varepsilon} d^d x \sqrt{g_\varepsilon} \bar{\Psi}_i(\vec{x}) \Gamma^i \Gamma^j \Psi_j(\vec{x}). \tag{27}$$

M_d^ε is the hypersurface $x_0 = \varepsilon$, $\sqrt{g_\varepsilon}$ is a volume element in the induced metric g_{ij} on this hypersurface. The second structure (27) does not contribute to the correlation function of operators coupled to b_i^-, since on-shell we have $\gamma^a\psi_a \sim b_0$. The constants in (26) and (27) were determined in [27], were the approach of [28] was used.

In order to find I_1 we will substitute (23) and the corresponding solution for the conjugated spinor in (26) and rewrite it as a sum of the integrals with the similar structure. So the integral to be calculated is

$$\begin{aligned} G(y_1, y_2, \epsilon) = \int d^d x \varepsilon^{-d} \left(\delta_{im} - 2\frac{(x-y_1)_i(x-y_1)_m}{\varepsilon^2 + (\vec{x}-\vec{y}_1)^2}\right) \times \\ \times \left(\frac{\varepsilon\gamma^0 + (x-y_1)_k\gamma^k}{(\varepsilon^2 + (\vec{x}-\vec{y}_1)^2)^{d/2+1/2+\sigma}}\varepsilon^{d/2+\sigma}\right) \left(\frac{\varepsilon\gamma^0 + (x-y_2)_j\gamma^j}{(\varepsilon^2 + (\vec{x}-\vec{y}_2)^2)^{d/2+1/2+\rho}}\varepsilon^{d/2+\rho}\right) \times \\ \times \left(\delta_{in} - 2\frac{(x-y_2)_i(x-y_2)_n}{\varepsilon^2 + (\vec{x}-\vec{y}_2)^2}\right). \end{aligned} \tag{28}$$

This integral is singular in the limit $\epsilon \to 0$, if the difference between ρ and σ is not integer. Except very special cases of masses, the difference between C and m_- is not integer, so the terms mixing b_0 and b_i give a singular contribution to I_1. Therefore, if we perform the "minimal" renormalization [5], i.e. simply drop the singular terms, we see that the generating functional does not depend on the terms containing simultaneously b_0 and b_i. So the correlation functions of operators coupled to b_i are determined by the term involving only b_i. The result for the integral turns out to be

$$I_1 = \text{const} \cdot \int d^d y_1 d^d y_2 \bar{b}_i^+(\vec{y}_1) \frac{(y_1-y_2)_k\gamma^k}{|\vec{y}_1-\vec{y}_2|^{d+1+2m_-}}(\delta_{ij} - \frac{2(y_1-y_2)_i(y_1-y_2)_j}{(\vec{y}_1-\vec{y}_2)^2}) b_j^-(\vec{y}_2). \tag{29}$$

As it was noted in [5, 8], the integrals obtained after calculating the actions on-shell and taking the limit $\epsilon \to 0$ are divergent and need the regularization. It was suggested to make renormalization or to consider the integral (29) as a value of the distribution

$$\frac{(y_1-y_2)_k\gamma^k}{|\vec{y}_1-\vec{y}_2|^{d+1+2m_-}}(\delta_{ij}-\frac{2(y_1-y_2)_i(y_1-y_2)_j}{(\vec{y}_1-\vec{y}_2)^2})$$

on a test function. The Fourier transform of this distribution is

$$F\left[\frac{y_k\gamma^k}{|\vec{y}|^{d+2m_-+1}}(\delta_{ij}-\frac{2y_iy_j}{\vec{y}^2})\right]=\text{const}\cdot p^{2m-1}p_k\gamma^k\left(\delta_{ij}+\frac{2}{(d+2m_--3)(d+2m_--1)}\frac{p_ip_j}{p^2}\right) \qquad (30)$$

for $2m_-+1\neq 2k$, where $k=0,1,..$ and $p=|\vec{p}|$. The overall factor in (30) is not important for us. Also we take into account that the test functions in (29) are subjected to the restriction $\gamma^i b_i=0$. When $2m_-+1$ is even, the Fourier transform includes a logarithmic term

$$F\left[\frac{y_k\gamma^k}{|\vec{y}|^{d+2m_-+1}}(\delta_{ij}-\frac{2y_iy_j}{\vec{y}^2})\right]=\text{const}\cdot p^{2m-1}\log\frac{p}{c}p_k\gamma^k\left(\delta_{ij}+\frac{2}{(d+2m_--3)(d+2m_--1)}\frac{p_ip_j}{p^2}\right). \qquad (31)$$

As it was noted in [8], the distributions of this type are not conformal invariant.

The general scheme prescribes to vary the unconstrained fields on the boundary, however, we have the restriction $\gamma_i b_i(\vec{y})=0$. It means that we should introduce a projection operator on this constraint, namely

$$b_i(\vec{y})=(\delta_{ij}-\frac{1}{d}\gamma_i\gamma_j)\tilde{b}_j(\vec{y})=\Pi_{ij}\tilde{b}_j(\vec{y}), \qquad (32)$$

where $\tilde{b}_j(\vec{y})$ is an unconstrained field. Taking it into account and using (29) we obtain

$$\left\langle\Sigma^+_{i\alpha}(\vec{y}_1)\Sigma^-_{j\beta}(\vec{y}_2)\right\rangle=\text{const}\cdot\Pi_{ik}\frac{(y_1-y_2)_i(\gamma^i)_{\alpha\beta}}{|\vec{y}_1-\vec{y}_2|^{d+1+2m_-}}(\delta_{kl}-\frac{2(y_1-y_2)_k(y_1-y_2)_l}{(\vec{y}_1-\vec{y}_2)^2})\Pi_{lj}, \qquad (33)$$

$\Sigma_{i\alpha}(\vec{y})$ is a conformal field operator which corresponds to the Rarita-Schwinger field in the supergravity. Note that the second projection operator in (33) could be eliminated by the trivial algebraic transformations, so only the first one is really necessary. The obtained correlation function coincides with the projected two-point correlation function expected in the conformal field theory for operators with spin-3/2 [29] and with the conformal dimension

$$\Delta=\frac{d}{2}+m_-. \qquad (34)$$

For the massless case $m_1=0$ these results coincide with those obtained in [19, 20].

We have considered the action for the massive Rarita-Schwinger field in the Anti-de Sitter space and solved the corresponding equations of motion. According to the prescriptions of AdS/CFT correspondence we have required the appropriate boundary behaviour of the field, and using the special boundary terms we have obtained the two-point correlation function for the conformal field theory operators with spin-3/2. In our approach we did not specify the AdS dimension d and the parameters m_1 and m_2 of the Rarita-Schwinger field. However, we implied that these parameters are subjected to the special relations: we supposed that the difference between C and m_- is not integer. It is easy to see that for the Rarita-Schwinger field masses in IIB supergravity on $AdS_5\times S^5$ [24] the above condition is satisfied.

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