# Explicit $N = 2$ supersymmetry for higher-spin fields.

A.G. Sibiryakov[1]
Quantum Field Theory Department, Tomsk State University,
pr. Lenina 36, 634050 Tomsk, Russia

**Abstract**

We develop $N = 2$ superfield formulations of free equations of motion for the joint (generating) model of all $D = 4$ massless higher-superspin fields in AdS space. The explicit $Osp(2|4)$ supersymmetry is achieved with adding no auxiliary component fields to those of $N = 1$ superfields. The formulation is developed in the coset space $Osp(2|4)/U(2)$ with dimension $(7|4)$. The $N = 2$ covariant derivatives in this space are expressed in terms of $N = 1$ ones which gives simple rules for the component analysis.

The report is mainly based on the work [1] devoted to the development of the generating formulation for higher-spin superfields in $D = 4$ AdS superspace proposed in the previous paper [2] and to the construction of an $N = 2$ superfield description of free equations of motion. The equations are described in terms of superfields in either of two different coset spaces for $N = 2$ AdS supergroup $Osp(2|4)/U(2)$ or $Osp(2|4)/\mathcal{H}$, where $\mathcal{H}$ is a *supersubgroup* with even part $U(2) \in \mathcal{H}$. Some years ago it was shown by Fradkin and Vasiliev [3] that AdS space is the natural background for massless higher-spin fields. To our mind the recent achievements in the AdS/CFT correspondence show that the connection between string theories and massless higher-spin theories can be closer then it was believed before. Due to technical difficulties arising in the study of higher-spin problem, many authors seek for some generating approach to investigation of underlying algebraic structures. Our approach differs in that the higher-spin fields appear in expansion of usual scalar superfields, defined in a homogeneous superspace of larger dimension (9).

A motivation to develop superfield approach to higher-spin fields follows from a consideration of the question of global symmetries for the Fradkin-Vasiliev theory. This question has never been studied in detail and even the usual supersymmetry for this theory is not explicit and has never been found in the generating form. Here we start with the explicitly $N = 1$ supersymmetric model and construct $N = 2$ supersymmetric one. $N = 2$ AdS superalgebra has the special significance for higher spins because it is the maximal finite-dimensional subalgebra in the superalgebra of higher spins and auxiliary fields [4]. Another motivation to develop our formulation arises due to possible comparisons to the the results of the known $N = 2$ superfield models (see, e.g. [5]).

**1. Superfield generating formulation.** Almost all the information about the geometry of the homogeneous superspaces considered in this work follows directly from the algebra of the covariant derivatives in these manifolds. Let us describe the general structure of covariant derivatives in a coset space $M = G/H$ for any group $G$ and its subgroup $H$. We denote $S_a$ $(a = 1, \ldots, \dim H)$ generators of the subgroup $H$ and $X_\alpha$ $(\alpha = 1, \ldots, \dim G - \dim H)$ complementary generators of $G$. We suppose that the algebra of the generators can be brought to the following form:

$$[S_a, S_b] = C^c_{ab} S_c, \qquad [S_a, X_\alpha] = C^\beta_{a\alpha} X_\beta, \tag{1}$$
$$[X_\alpha, X_\beta] = C^\gamma_{\alpha\beta} X_\gamma + C^a_{\alpha\beta} S_a. \tag{2}$$

This means not only $C^\alpha_{ab} = 0$ ($H$ is subgroup) but also $C^a_{\alpha b} = 0$ that is $X_\alpha$ are in a sense orthogonal to $S_a$. For the coset space $M = G/H$ generators $X_\alpha$ turn into covariant derivatives $\mathcal{D}_\alpha$, which are differential operators of the form

$$\mathcal{D}_\alpha = E_\alpha + \Omega_\alpha{}^a S_a, \tag{3}$$

where $E_\alpha$ are vector fields in $M$ and the connection $\Omega$ is given by certain functions in $M$. The generators $S_a$ in the connection terms (3) act on the fields in $M$ taking their values in a representation $T$ of the subgroup $H$, $S_a \psi_i = T^j_{ai} \psi_j$. In particular $\mathcal{D}_\alpha$ carry the index of special, maybe reducible representation $[S_a, \mathcal{D}_\alpha] = C^\beta_{a\alpha} \mathcal{D}_\beta$. Covariant derivatives satisfy the commutation relations

$$[\mathcal{D}_\alpha, \mathcal{D}_\beta] = C^\gamma_{\alpha\beta} \mathcal{D}_\gamma + C^a_{\alpha\beta} S_a. \tag{4}$$

[1]e-mail: **sib@phys.tsu.ru**

where $C$'s are the same constants as in (1,2). The existence of these covariant derivatives $\mathcal{D}_\alpha$ (3) forming together with the local rotations $S_a$ the Lie algebra of the group $G$ (1,2) is a characterizing property of a coset space $M = G/H$.

Let us turn to the superfield formulations of free dynamics of higher spin fields in $N = 1$ AdS superspace $M^{4|4} = Osp(1|4)/SL(2,C)$ [6, 2]. We denote $4+4$ coordinates in superspace $M^{4|4}$ by $z$ and the covariant derivatives by $\mathcal{D}_A = \{\mathcal{D}_{\alpha\dot\alpha},\ \mathcal{D}_\alpha,\ \bar{\mathcal{D}}_{\dot\alpha} \equiv (\mathcal{D}_\alpha)^\dagger\}$.[2] They have the following algebra ($osp(1|4)$):

$$\begin{array}{rcll} \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} & = & -2i\mathcal{D}_{\alpha\dot\alpha}, & [\mathcal{D}_{\alpha\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] = -2\bar\mu\mu(\varepsilon_{\alpha\beta}\bar{M}_{\dot\alpha\dot\beta} + \varepsilon_{\dot\alpha\dot\beta}M_{\alpha\beta}), \\ \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} & = & -4\bar\mu M_{\alpha\beta}, & [\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}] = i\bar\mu\varepsilon_{\alpha\beta}\bar{\mathcal{D}}_{\dot\beta} \end{array} \tag{5}$$

and the complex conjugated relations. Here $\mu$ is the curvature of AdS superspace, $M_{\alpha(2)}$ and $\bar{M}_{\dot\alpha(2)}$ are local Lorentz rotations, e.g. $M_{\alpha(2)}\psi_\beta = \varepsilon_{\beta\alpha}\psi_\alpha$. More information about the geometry of AdS superspace can be found in [7] and in the references therein.

We will use here only so-called transversal formulation for half-integer superspin $s+1/2$ and longitudinal one for integer superspin $s$ [8, 6], denoting them as $S^\perp_{s+1/2}$ and $S^\parallel_s$. It is this choice that is necessary for constructing $N = 2$ supersymmetry transformations [9] and the generating formulation [2]. The actions $S^\perp_{s+1/2}$ and $S^\parallel_s$ are expressed in terms of the following dynamical superfields[3]

$$\mathcal{V}^\perp_{s+1/2} = \{\, H(s,s),\ \Gamma(s-1,s-1),\ \bar\Gamma(s-1,s-1)\,\}, \qquad s \geq 1 \tag{6}$$

$$\mathcal{V}^\parallel_s = \{\, H'(s-1,s-1),\ G(s,s),\ \bar{G}(s,s)\,\}, \qquad s \geq 1 \tag{7}$$

where $H$ and $H'$ are arbitrary real superfields while $\Gamma$ and G are so called transversal linear and longitudinal linear superfields respectively satisfying the following relations

$$(\bar{\mathcal{D}}^2 - 2(s+1)\mu)\Gamma(s-1,s-1) = 0, \quad (\bar{\mathcal{D}}^2 + 2s\mu)G(s,s) = 0, \tag{8}$$

where $\bar{\mathcal{D}}^2 = \bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}$. The gauge parameters are superfields of the same kind as the dynamical fields: the action $S^\perp_{s+1/2}$ is gauge invariant w.r.t. transformations with a longitudinal linear parameter $g(s,s)$ and the action $S^\parallel_s$ is gauge invariant w.r.t. the ones with a transversal linear parameter $\gamma(s-1,s-1)$.

To construct the generating formulation let us consider an $SO(3,1)$ four-vector, subjected to the two-sheeted hyperboloid, $r^a r_a = -1$, $r^{\alpha\dot\alpha}r_{\alpha\dot\alpha} = 2$. In [2] it is proposed to consider spin-tensors (6,7) as the coefficients of expansion of analytic functions in power series in $r$:

$$\phi(r) = \sum_{s=0}^{\infty} \phi_{\alpha(s)\dot\alpha(s)} r^{\alpha(s)\dot\alpha(s)}, \qquad r^{\alpha(s)\dot\alpha(s)} = \underbrace{r^{\alpha\dot\alpha}\ldots r^{\alpha\dot\alpha}}_{s \text{ times}}. \tag{9}$$

It turned out [2] that the joint content of superfield sets $\mathcal{V}^\perp_{s+1/2}$ and $\mathcal{V}^\parallel_s$ for all values $s \geq 1$, plus two scalar superfields describing superspins 0 and 1/2, is represented by the one complex $V(z,r)$, and two real $Y(z,r)$, $Z(z,r)$ functions, whose tensor components at $s$th power in expansion in $r^{\alpha\dot\alpha}$ are

$$V(s,s) = \Gamma(s,s) + G(s,s), \quad Y(s,s) = H'(s,s) - H(s,s), \quad Z(s,s) = (s+1)(H'(s,s) + H(s,s)). \tag{10}$$

The gauge parameters $\gamma(s,s)$, $g(s,s)$ are also join into a complex function $\lambda(z,r)$. Note that the dependence of $U$, $Y$, $X$ and $\lambda$ on their arguments is arbitrary (except for analyticity in $r$).

To derive an action for these superfields, the Lorentz-invariant trace is introduced in [2] to the algebra of analytic functions of vector $r$:

$$\mathrm{tr}\ \phi = \phi(0,0) \implies \mathrm{tr}\ (\phi \cdot \psi) = \sum_{s=0}^{\infty} \frac{1}{s+1}\ \phi^{\alpha(s)\dot\alpha(s)}\psi_{\alpha(s)\dot\alpha(s)}, \tag{11}$$

---

[2]Here $\dagger$ means usual complex conjugation in Grassman algebra.

[3]We consider only Lorentz tensors symmetric in their undotted and dotted indices. A tensor of type $(k,l)$ with $k$ undotted and $l$ dotted indices can be equivalently represented as $\psi(k,l) \equiv \psi_{\alpha(k)\dot\alpha(l)} \equiv \psi_{\alpha_1\ldots\alpha_k\dot\alpha_1\ldots\dot\alpha_l} = \psi_{(\alpha_1\cdots\alpha_k)(\dot\alpha_1\cdots\dot\alpha_l)}$. The indices, which are denoted by one and the same letter are supposed to be symmetrized, see also [1].

It is also convenient to use the following scalar operators:

$$\mathcal{Q} = ir^{\alpha\dot\alpha}\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}, \quad \bar{\mathcal{Q}} = -ir^{\alpha\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha = -\mathcal{Q}^\dagger, \quad \nabla = -i(\mathcal{Q}-\bar{\mathcal{Q}})/4 = -ir^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}/2 \tag{12}$$

$$\mathcal{P} = \frac{\mathcal{D}^2}{2\bar\mu} = \frac{1}{2\bar\mu}\mathcal{D}^\alpha\mathcal{D}_\alpha, \qquad \bar{\mathcal{P}} = \frac{\bar{\mathcal{D}}^2}{2\mu} = \mathcal{P}^\dagger \tag{13}$$

It was noted in [2] that the real superfield $Y$ can be gauged away. It is more convenient to split the superfield $V$ into its real and imaginary parts $V = \gamma + iX$. Then the joint action of superfields of all superspins (including nongauge massless fields of low spins 0, 1/2) takes a simple form:

$$\begin{aligned} S(\gamma, Z, X) \;\; = \;\; & \frac{1}{2}\mathrm{tr}\int \mathrm{d}^8z\, E^{-1}\Big\{ -\mu\bar\mu Z\mathcal{R}Z - 2iZ\mathcal{R}\nabla X + X\mathcal{R}X + \\ & + \frac{i}{2}Z\mathcal{R}(\mathcal{Q}+\bar{\mathcal{Q}})\gamma + i\gamma\mathcal{R}(\mathcal{P}-\bar{\mathcal{P}})X - \gamma(\mathcal{P}+\bar{\mathcal{P}}-1)\mathcal{R}\gamma\Big\}. \end{aligned} \tag{14}$$

where $\mathrm{d}^8z\, E^{-1}$ is AdS superinvariant measure and the operation $\mathcal{R}$ is the reflection of the vector $r^{\alpha\dot\alpha}$, $\mathcal{R}\phi(r,z) = \phi(-r,z)$. The action is invariant w.r.t. the following gauge transformations with real gauge parameter $\varepsilon(z,q)$:

$$\delta Z = -i(\mathcal{P}-\bar{\mathcal{P}})\varepsilon, \qquad \delta X = -\frac{i}{2}(\mathcal{Q}+\bar{\mathcal{Q}})\varepsilon, \qquad \delta\gamma = -2i\nabla\varepsilon. \tag{15}$$

The action has an explicit $N=1$ supersymmetry because it is formulated in terms of the $Osp(1|4)$-covariant derivatives and measure. But as was noted in [2] it is also invariant w.r.t. an additional global symmetry that is parametrized by scalar and spinor constant parameters and extends superalgebra $osp(1|4)$ to $osp(2|4)$. It is more convenient and elegant to express the corresponding second supersymmetry in explicitly $N=1$ invariant form, using so-called Killing scalar parameter $t(z)$ [2]:

$$t = \bar t, \qquad (\mathcal{D}^2 - 4\bar\mu)t = 0, \qquad \mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}t = 0, \tag{16}$$

containing one spinor and one scalar constant components. It is remarkable that the manifold $M^{7|4} = Osp(1|4)/SU(2)$ parametrized by the coordinates $(z,r)$ admits an action not only of the $N=1$ AdS supergroup, but also its $N=2$ extension. Actually we shall show that $M^{7|4}$ coincides with the coset space $Osp(2|4)/\mathcal{H}$ for $N=2$ AdS supergroup. The action of the second supersymmetry is given by the vector fields parametrized by that Killing scalar $t$:

$$\mathcal{M} = 2ir^{\alpha\dot\alpha}(t_\alpha\bar{\mathcal{D}}_{\dot\alpha} + \bar t_{\dot\alpha}\mathcal{D}_\alpha) + 2it\nabla, \qquad \mathcal{M}^\dagger = \mathcal{M}, \tag{17}$$

where $t_\alpha = 2\mathcal{D}_\alpha t$. These vector fields (for different $r$'s) commute on the Killing vector field in $N=1$ AdS space that gives the action of $osp(1|4)$.

Let us turn to $N=2$ supersymmetry transformations of the action (14). It was shown in Ref.[9] that their algebra closes off-shell. Nevertheless, this algebra was broken by the terms proportional to gauge transformations (see [2] for the generating form of this algebra). In [1] it has been observed that the algebra can be closed eventually by adding certain gauge transformation. The result is

$$\begin{gathered} \delta_t\gamma = \mathcal{M}\gamma, \quad \delta_t Z = -nX + it(\mathcal{P}-\bar{\mathcal{P}})\gamma, \quad \delta_t X = \mu\bar\mu n Z + \tfrac{i}{2}t(\mathcal{Q}+\bar{\mathcal{Q}})\gamma, \\ n = [\mathcal{P}+\bar{\mathcal{P}}, t] = \mathcal{N} + 4t, \qquad \mu\bar\mu\mathcal{N} = 2(\mu t^\alpha\mathcal{D}_\alpha + \bar\mu\bar t_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}) \end{gathered} \tag{18}$$

Here $\mathcal{M}$ is a vector superfield of $osp(2|4)$ superalgebra (17). Let us emphasize that the superfield $\gamma$ transforms in itself by the supervector operator $\mathcal{M}$. The whole representation of the superalgebra $osp(2|4)$ on superfields $\gamma$, $Z$ and $X$ has the structure of a semidirect sum. Surprisingly, it is possible to split it completely by adding variations proportional to the equations of motion. Then we can define new variations $\bar\delta_t$ with the same algebra:

$$\begin{aligned} \bar\delta_t\gamma \;&=\; \delta_t\gamma = \mathcal{M}\gamma, \\ \bar\delta_t Z \;&=\; \delta_t Z - t\mathcal{R}\,(\delta S/\delta X) = -(\mathcal{N}+6t)X + 2it\nabla Z, \\ \bar\delta_t X \;&=\; \delta_t X - t\mathcal{R}\,(\delta S/\delta Z) = \mu\bar\mu(\mathcal{N}+6t)Z + 2it\nabla X. \end{aligned} \tag{19}$$

Unfortunately, the new transformations leave invariant only the equations of motion and not the action (14). The reason is that to maintain the invariance of the action, the difference $\bar{\delta}-\delta$ of transformations should be a composition of an antisymmetric matrix with the action variations. However, in (19) a symmetric matrix is used.

**2. Structure of supercovariant derivatives.** Now we pass to explicitly $N=2$ supersymmetric description of free higher-spin equations of motion in the superspace $Osp(2|4)/U(2)$. For this purpose we express covariant derivatives in this space in terms of the usual $N=1$ covariant derivatives in AdS superspace (5). We use the fact that in all the coset spaces at hand there are covariant derivatives of the form (3) satisfying the algebra (4) equivalent to the algebra of the global symmetry of the space (compare with (1,2)). In the process of expression of $N=2$ covariant derivatives via the $N=1$ ones, which we are going to consider, the following sequence of derivatives arises:
1. The covariant derivatives $\mathcal{D}_A=\{\mathcal{D}_{\alpha\dot\alpha},\mathcal{D}_\alpha,\mathcal{D}_{\dot\alpha}\}$ in $N=1$ AdS superspace $Osp(1|4)/SL(2,C)=M^{4|4}$.
2. $N=1$ covariant derivatives $\Sigma_I=\{\Sigma_{\pm\pm a(2)},\Sigma_{\pm a},\Sigma\}$ with the smaller local group $SU(2)$ in the bigger coset space $Osp(1|4)/SU(2)=M^{7|4}$ – the space where the generating model (14) is formulated.
3. $N=2$ covariant derivatives $\check{D}_I=\{\check{D}_{\pm\pm a(2)},\check{D}^{\pm}_{\mp a},\check{D}\}$ in the same superspace $Osp(2|4)/\mathcal{H}=M^{7|4}$ (which is now considered to be a coset space for $N=2$ AdS supergroup), the local group is enlarged to a supergroup $\mathcal{H}$.
4. The final covariant derivatives $D_I, D^{\pm}_{\pm a}$ in the superspace $Osp(2|4)/U(2)=M^{7|8}$, that are used for the $N=2$ superfield formulation.

So we have solved three problems of expression of covariant derivatives for one coset space in terms of those for another coset space. To our mind the solution of these problems may have a special significance for general superfield models.

**2.1 $Osp(1|4)/SU(2)$-covariant derivatives.** Firstly, we construct $N=1$ covariant derivatives with local group $SU(2)$ starting from the Lorentz ones $\mathcal{D}_A$. This gives us a possibility to reduce any covariant equations in the larger space to those in the smaller space. We denote the index of a two-component spinor transforming under the group $SU(2)$ by $a$, $b$, and so on. Then the generators of the algebra are $S_{a(2)}$,

$$[S_{a(2)},S_{b(2)}]=2\varepsilon_{ab}S_{ab},\quad (S_{a(2)})^\dagger=S^{a(2)}\quad [S_{a(2)},\psi_b]=\varepsilon_{ab}\psi_a. \tag{20}$$

To convert $SL(2,C)$ indices $\alpha$, $\dot\alpha$ into $SU(2)$ indices we introduce what we call 'small vielbeins' $e_a{}^\alpha(z,r)$ and $e_{a\dot\alpha}(z,r)=(e^a{}_\alpha)^{\dagger 4}$ subjected to certain Lorentz invariant constraints:

$$\varepsilon_{ab}=-e_a{}^\alpha e_b{}^\beta\varepsilon_{\alpha\beta}.\qquad e^{a\alpha}e_a{}^{\dot\alpha}=r^{\alpha\dot\alpha}, \tag{21}$$

Then we introduce 7+4 vector fields forming a complete basis in the tangent space at every point of the manifold $M^{7|4}$:

$$\begin{aligned}\nabla_{a(2)} &\equiv e_a{}^\alpha e_a{}^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}=-(\nabla^{a(2)})^\dagger,\qquad \nabla\equiv-\frac{i}{2}r^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}=-\nabla^\dagger,\\ \partial_{a(2)} &\equiv -ie_a{}^\alpha e_a{}^{\dot\alpha}\partial_{\alpha\dot\alpha}=-2ie_a{}^\alpha e_a{}^\alpha\vec{M}_{\alpha(2)}=(\partial^{a(2)})^\dagger,\\ \nabla_a &\equiv e_a{}^\alpha\mathcal{D}_\alpha,\qquad \bar\nabla_a\equiv e_a{}^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}=(\nabla^a)^\dagger\end{aligned} \tag{22}$$

In these relations (e.g. in the connection parts of the covariant derivatives $\mathcal{D}_A$) the Lorentz rotations $M_{\alpha(2)}$ are everywhere replaced by the vector fields $\vec{M}_{\alpha(2)}=\frac{1}{2}r_\alpha{}^{\dot\alpha}\partial_{\alpha\dot\alpha}$, $\partial_{\alpha\dot\alpha}\equiv\frac{\partial}{\partial r^{\alpha\dot\alpha}}$ that act the same way on scalars $M_{\alpha(2)}\phi(r)=\vec{M}_{\alpha(2)}\phi(r)$. Now covariant derivatives in $M^{7|4}$ in a special basis can be constructed as follows

$$\mathcal{L}_{a(2)} = -\partial_{a(2)}-v_{a(2)}{}^{b(2)}S_{b(2)},\quad \mathcal{D}_{a(2)}=\nabla_{a(2)}+u_{a(2)}{}^{b(2)}S_{b(2)} \tag{23}$$

$$\Sigma = 2\nabla+w^{b(2)}S_{b(2)},\quad \mathcal{D}_a=i\nabla_a+iy_a{}^{b(2)}S_{b(2)}. \tag{24}$$

Here connections $v$, $u$, $w$ and $y$ are all independent combinations of the first-order derivatives of the small vielbein. It is convenient for us to pass to another basis as follows:

$$\Sigma_{\pm\pm a(2)}=\mathcal{D}_{a(2)}\pm\mathcal{L}_{a(2)},\quad \Sigma_{+a}=\frac{\bar{j}}{2}(\mathcal{D}_a+\bar{\mathcal{D}}_a),\quad \Sigma_{-a}=-\frac{j}{2}(\mathcal{D}_a-\bar{\mathcal{D}}_a) \tag{25}$$

[4] As usual, we raise and lower indices $\alpha$, $\dot\alpha$ and $a$ of the small vielbein with help of antisymmetric symbols $\varepsilon_{\alpha\beta}$, $\varepsilon_{\dot\alpha\dot\beta}$, $\varepsilon_{ab}$ and so on. For example, $e^a{}_\alpha=\varepsilon_{\alpha\beta}\varepsilon^{ab}e_b{}^\beta$.

where $j = e^{i\pi/4}$. One can show that the constructed covariant derivatives $\Sigma_I$ together with local rotations $S_{a(2)}$ form $N = 1$ AdS superalgebra $osp(1|4)$, that contains in addition to (20) the following nonzero commutators:

$$
\begin{aligned}
[S_{a(2)}, \Sigma_{\pm\pm b(2)}] &= 2\varepsilon_{ab}\Sigma_{\pm\pm ab}, \qquad [S_{a(2)}, \Sigma_{\pm b}] = \varepsilon_{ab}\Sigma_{\pm a}, & (26)\\
[\Sigma, \Sigma_{\pm\pm a(2)}] &= \pm 2\Sigma_{\pm\pm a(2)}, \qquad [\Sigma, \Sigma_{\pm a}] = \pm\Sigma_{\pm a}, & (27)\\
[\Sigma_{++a(2)}, \Sigma_{--b(2)}] &= 2\varepsilon_{ab}(2S_{ab} - \varepsilon_{ab}\Sigma), \quad [\Sigma_{\pm\pm a(2)}, \Sigma_{\mp b}] = 2\varepsilon_{ab}\Sigma_{\pm a}, & (28)\\
\{\Sigma_{\pm a}, \Sigma_{\pm a}\} &= -\Sigma_{\pm\pm a(2)}, \qquad \{\Sigma_{\pm a}, \Sigma_{\mp b}\} = -S_{ab} \pm \varepsilon_{ab}\Sigma/2. & (29)
\end{aligned}
$$

Here $\pm$ denote weights w.r.t. $\Sigma$. This algebra is the same as the covariant derivatives $\mathcal{D}_A$ together with Lorentz rotations $M_{\alpha\dot\alpha}$ satisfy (5), but in another basis and with explicit decomposition of $U(2)$ irreducible parts.

**2.2 $Osp(2|4)$-covariant derivatives in $M^{7|4}$.** Before constructing $N = 2$ covariant derivatives we write down the algebra, they have to form, that is the superalgebra $osp(2|4)$. We decompose the generators w.r.t. a subgroup $SU(1)\otimes U(1)\otimes U(1)$. Then all the generators are five generators of this subgroup $S_{a(2)}$, $\Sigma$, $\Gamma$, six even generators $D_{\pm\pm a(2)}$ and eight odd generators $D^{\pm}_{\pm a}$, $D^{\pm}_{\mp a}$. Their commutation relations and conjugation rules look as follows:

$$
[S_{a(2)}, S_{b(2)}] = 2\varepsilon_{ab}S_{ab}, \quad [S_{a(2)}, D^{\cdot\cdot}_{b}] = \varepsilon_{ab}D^{\cdot\cdot}_{a}, \quad [S_{a(2)}, D^{\cdot\cdot}_{\cdot b(2)}] = 2\varepsilon_{ab}D^{\cdot\cdot}_{\cdot ab} \tag{30}
$$

$$
[\Sigma, D^{\cdot}_{\pm}] = \pm D^{\cdot}_{\pm}, \qquad [\Gamma, D^{\pm}_{\dots}] = \pm D^{\pm}_{\dots} \tag{31}
$$

$$
[D_{++a(2)}, D_{--b(2)}] = 4\varepsilon_{ab}(S_{ab} - \tfrac{1}{2}\varepsilon_{ab}\Sigma) \tag{32}
$$

$$
[D_{\pm\pm a(2)}, D^{+}_{\mp b}] = 2\varepsilon_{ab}D^{+}_{\pm b}; \quad [D_{\pm\pm a(2)}, D^{-}_{\mp b}] = 2\varepsilon_{ab}D^{-}_{\pm b} \tag{33}
$$

$$
\{D^{+}_{+a}, D^{-}_{+b}\} = D_{++(ab)}, \quad \{D^{+}_{-a}, D^{-}_{-b}\} = D_{--(ab)} \tag{34}
$$

$$
\{D^{+}_{+a}, D^{-}_{-b}\} \equiv S_{(ab)} + \varepsilon_{ab}(\Gamma - \tfrac{1}{2}\Sigma), \quad \{D^{+}_{-a}, D^{-}_{+b}\} \equiv S_{(ab)} + \varepsilon_{ab}(\Gamma + \tfrac{1}{2}\Sigma). \tag{35}
$$

$$
S^{\dagger}_{a(2)} = S^{a(2)}, \quad S^{\dagger} = -S, \quad \Gamma^{\dagger} = -\Gamma, \quad \Sigma^{\dagger}_{++a(2)} = -\Sigma^{a(2)}_{--}, \quad (D^{\pm a}_{-})^{\dagger} = D^{\mp}_{+a} \tag{36}
$$

To decompose the basis into two subsets, the covariant derivatives and the local rotations, it is necessary to introduce the following linear combinations:

$$
S = \Gamma - \Sigma/2, \qquad D = (\Gamma - \Sigma)/2 \tag{37}
$$

We define the local supergroup $\mathcal{H}$ of the manifold $M^{7|4}$ to be the subgroup of $Osp(2|4)$ generated by $S$, $S_{a(2)}$, $D^{\pm}_{\pm a}$. Then note, that we have to consider two different coset spaces for $Osp(2|4)$, $M^{7|4}$ and $M^{7|8}$, with different numbers of covariant derivatives. We mark by additional emphasis '$\check{}$' the covariant derivatives in the space $M^{7|4}$. Our notations are summarizeded in the following tabulation:

| Generators of the algebra | counterparts for $M^{7\|4}$ | counterparts for $M^{7\|8}$ |
|---|---|---|
| $D_{\pm\pm a(2)}$, $D^{\pm}_{\mp a}$, $D$ | covar. deriv. $\check{D}_{\pm\pm a(2)}$, $\check{D}^{\pm}_{\mp a}$, $\check{D}$ | covar. deriv. $D_{\pm\pm a(2)}$, $D^{\pm}_{\mp a}$, $D$ |
| $D^{\pm}_{\pm a}$ | local rotations $S^{\pm}_{\pm a}$ | covariant derivatives $D^{\pm}_{\pm a}$ |
| $S_{a(2)}$, $S$ | local rotations $S_{a(2)}$, $S$ | local rotations $S_{a(2)}$, $S$ |

The superalgebra $osp(1|4)$ embedded into $osp(2|4)$ is spanned by the generators $S_{a(2)}$, $D_{\pm\pm a(2)}$, and the combinations $\Sigma$ and $\Sigma_{\pm a}$:

$$
\Sigma = S - 2D, \quad \Sigma_{\pm a} = \pm(-D^{\mp}_{\pm a} + D^{\pm}_{\pm a})/\sqrt{2} \tag{38}
$$

It appears that we have already known the action on $M^{7|4}$ of all the generators from the basis of the $N = 2$ AdS superalgebra. Note that this whole superalgebra can be spanned by the generators of $N = 1$ subalgebra and generators $S$, $D^{\pm}_{\pm a}$, that turn into local rotations $S$, $S^{\pm}_{\pm a}$ in the space $M^{7|4}$, whose action on the fields taking values in a representation is given by the definition of the representation. We have already constructed operators $\Sigma_I$, that form the basis in $N = 1$ subalgebra. Thus, we know the action of a set of generators, spanning the space of the superalgebra $osp(2|4)$. Therefore we can express the desired covariant derivatives as a linear combinations of the known operators:

$$\check{D}_{\pm\pm a(2)} = \Sigma_{\pm\pm a(2)}, \quad \check{D} = -\Sigma/2 + S/2, \quad \check{D}^{\pm}_{\mp a} = \pm\sqrt{2}\Sigma_{\mp a} + S^{\mp}_{\mp a} \tag{39}$$

These covariant derivatives have the standard structure – differential operator plus connection in a certain basis, but the connection has a special structure (local supergroup $H$ is broken to $SU(2)$).

**2.3** $Osp(2|4)$**-covariant derivatives in** $M^{7|8}$**.** Now we are going to express covariant derivatives on $M^{7|8}$ in terms of those on $M^{7|4}$, both for the same $N = 2$ AdS supergroup. This will enable us to produce the component analysis in an explicitly $N = 2$ supersymmetric way. We need to determine the dependence of the covariant derivatives on four additional odd coordinates. For this aim we introduce purely odd coset superspace $\mathcal{H}/U(2)$ with coordinates $\theta$ and covariant derivatives $D^{\pm}_{\pm a}$ in this space. We identify the space $M^{7|8}$ with the manifold parametrized by old coordinates $z^A$, $r^{\alpha\dot{\alpha}}$ of $M^{7|4}$ and odd coordinates $\theta$. Then it can be shown [1] that the covariant derivatives in $M^{7|8}$ can be found as a unique solution of a certain differential equation (derived from the specific algebra) with the following initial conditions:

$$D_I(z,r,\theta)|_{\theta=0} = \check{D}_I(z,r)|_{S^{\pm}_{\pm}\to D^{\pm}_{\pm}} \tag{40}$$

where in the r.h.s. stand the derivatives in $M^{7|4}$ constructed on the previous step (with odd local rotations replaced by the covariant derivatives on the coset space $\mathcal{H}/U(2)$).

This gives us the following rules for component analysis. Consider an arbitrary superfield $\Phi_p(z,r,\theta)$ in superspace $M^{7|8}$ with definite weight w.r.t. $S$. Its component content is given by the following 16 superfields in $\bar{M}^{7|4}$:

$$\begin{gathered}\Phi_p|, \quad D^{\pm}_{\pm a}\Phi_p|, \quad D^{\pm a}_{\pm}D^{\pm}_{\pm a}\Phi_p| \equiv (D^{\pm}_{\pm})^2\Phi_p|, \quad [D_{+a}, D_{-b}]\Phi_p|,\\ D^{\mp}_{\mp b}(D^{\pm}_{\pm})^2\Phi_p|, \quad \{(D^{+}_{+})^2, (D^{-}_{-})^2\}\Phi_p|.\end{gathered} \tag{41}$$

The initial conditions (40), together with Eqs.(39) and the fact that connections there at the odd local rotations $S^{\pm}_{\pm a}$ are constant superfields, give us the following component rules:

$$(D_{\pm\pm a(2)}\Phi_p)| = \Sigma_{\pm\pm a(2)}(\Phi_p|), \quad (D^{\pm}_{\mp a}\Phi_p)| = \pm\sqrt{2}\Sigma_{\mp a}(\Phi_p|) + (D^{\mp}_{\mp a}\Phi_p)|, \tag{42}$$

$$(D\Phi_p)| = -\tfrac{1}{2}\Sigma(\Phi_p|) + \tfrac{1}{2}(S\Phi_p)|, \qquad S\Phi_p = (p/2)\ \Phi_p \tag{43}$$

where $(D^{\mp}_{\mp a}\Phi_p)|$ in (42) is another independent component of the superfield $\Phi_p$.

**3. Equations of motion in** $M^{7|8}$ Supermanifold $M^{7|8}$ admits many kinds of off-shell constrained superfields. For example, for any spinor covariant derivative, e.g. $D^{+}_{-a}$, we can impose the conditions that a superfield $\phi$ in $M^{7|8}$ is annihilated by this derivative, $D^{+}_{-a}\phi = 0$. As the covariant derivative anti-commutes with itself, this equation does not imply additional constraints of the first order in derivatives. Therefore the superfield $\phi$ has four independent components that are arbitrary superfields in $M^{7|4}$. We call such fields weakly chiral. One can define also two types of strongly chiral superfields. The first ($\gamma$) can be real and must have zero weight w.r.t. $S$; the second ($\phi_p$) must be complex but its weight $p/2$ can be arbitrary.

$$1.\ D^{+}_{+a}\gamma = D^{-}_{-a}\gamma = 0 \tag{44}$$

$$2.\ D^{+}_{+a}\phi_p = D^{+}_{-a}\phi_p = 0, \qquad S\,\phi_p = (p/2)\phi_p. \tag{45}$$

These pairs of constraints do not contradict the algebra (30–35). Either strongly chiral superfield has only one independent component that is an unconstrained superfield in $M^{7|4}$. All the components except $\gamma|$ of the superfield $\gamma$ are equal to zero as a consequence of (44). All the components of the superfield $\phi_p$ can be expressed via its null component (which is an unconstrained superfield in $M^{7|4}$) with the aid of (45):

$$(D^{-}_{-a}\phi_p)| = -\sqrt{2}\Sigma_{-a}\phi_p|, \quad ((D^{-}_{-})^2\phi_p)| = -\sqrt{2}\Sigma^{a}_{-}(D^{-}_{-a}\phi_p)|. \tag{46}$$

To formulate the desired equations of motion we introduce one real strongly chiral superfield $\gamma$ and one complex strongly chiral superfield $\Phi_6$ of the first and second kinds respectively (44,45).

$$\gamma = \gamma^{\dagger}, \quad \Phi_6, \quad \bar{\Phi}_{-6} = (\Phi_6)^{\dagger} \tag{47}$$

Then every $N=1$ superspin enters the equations of motion for free massless higher-spin superfields, with multiplicity one: the equation look as follows:

$$D_+^{-a}D_{+a}^-\Phi_6 + D_-^{+a}D_{-a}^+\bar{\Phi}_{-6} = 8i([D_-^{+a}, D_{+a}^-]_- - 2)\gamma, \tag{48}$$
$$i(D-2)\Phi_6 = D_-^{+a}D_{-a}^+\gamma, \tag{49}$$

These equations have a gauge symmetry with a real strongly chiral gauge parameter $\varepsilon$ of the first kind,

$$\delta\Phi_6 = D_-^{+a}D_{-a}^+\varepsilon, \quad \delta\gamma = iD\varepsilon, \quad D_{+a}^+\varepsilon = D_{-a}^-\varepsilon = 0, \quad S\varepsilon \equiv 0 \tag{50}$$

These transformations are consistent with the constraints imposed on $\gamma$ and $\Phi_6$.

The equivalence of these equations of motion to those derived from the generating action $S(\gamma, Z, X)$ can be proven in components [1]. Superfields $\gamma$, $Z$, $X$ can be identified with the component fields of those chiral superfields as $\gamma| = \gamma$, $\Phi_6| = -4(Z+iX)$. Then the null components of the equations (48, 49) are equal to the equations of motionderived from the action $S(\gamma, Z, X)$ for $\gamma|$ and $\Phi_6|$ respectively. Other components of the equations (48, 49) give differential relations.

Note that because we do not use the harmonic superspace or the like, the number of auxiliary fields per single superspin is finite. Moreover, the constructed formulation with the explicit $N=2$ supersymmetry has neither auxiliary nor purely gauge components in addition to those of the $N=1$ superfield version. Next, to our knowledge, the question of AdS background in $N=2$ superfield supergravity still remains unresolved and $N=2$ superfield models in AdS superspace have not been considered before.

In conclusion let us discuss the question of an action for these equations of motion. The fact that the off-shell chiral superfields $\Phi_6$ and $\gamma$ are independent implies that the second version of $N=2$ supersymmetry (18) is explicit which has the structure of a direct sum. This version of supersymmetry does not leave the action invariant. This means that although the equations are Lagrangian and their action possesses $N=2$ supersymmetry, they probably have no *explicitly* $N=2$ supersymmetric action. However, the question remains whether the first version (19) of supersymmetry (that leaves the action invariant) can be realized, maybe in another coset superspace instead of $M^{7|8}$, explicitly.

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