A study of the radiative corrections to Bhabha scattering with QEDPS Monte Carlo

Keijiro TOBIMATSU, Kiyoshi KATO[1], Tomo MUNEHISA[2], Yoshimasa KURIHARA[3], Junpei FUJIMOTO[3] and Yoshimitsu SHIMIZU[3]

The Center for Information Science, Kogakuin University, Tokyo 163-8677

[1]*Department of Physics, Kogakuin University, Tokyo 163-8677*

[2]*Faculty of Engineering, Yamanashi University, Kofu 400-8511*

[3]*High Energy Accelerator Research Organization(KEK), Tsukuba, 305-801*

Japan

Abstract

An event generator for Bhabha scattering with QEDPS Monte Carlo has been developed. The energy scale $|t|$ is used for the Q^2 evolution in the QED cascade. The QEDPS parton shower is applied to the effective Born cross section which includes one-loop electroweak corrections. Comparisons of the results of the generator to the calculations with full $O(\alpha)$ corrections are given.

1 Introduction

In this article we report the result from the program of the QEDPS Monte Carlo for Bhabha scattering by which not only we can calculate the cross section with the radiative corrections for small and large angle scatterings but also generate the corrected events. Using the technique developed in the previous researches[1, 2, 3, 4] by the authors, we have constructed a new generator in order to give a more precise evaluation of the cross section and to generate events with the higher order corrections to the process by the combination of these calculations, *i.e.*, including all the required one loop electroweak corrections to the process effectively and all the leading-log QED corrections by QEDPS.

The QEDPS Monte Carlo enables us to generate multiple emissions of photons both from the initial e^+/e^- and the final e^+/e^- by using the Sudakov factor. Concerning the photon emissions from the initial e^+/e^- for t-channel dominant processes, it is reasonable to take the Lorentz invariant variable t for Q^2, the maximum virtuality in the exponent of the Sudakov factor. We propose that the squared momentum transfer t should be used as the scale in the QEDPS model for such processes. The Bhabha scattering is a typical and the most simple process for testing the validity of the model. In the next section we give a general discussion on the QEDPS-t model.

All the electroweak corrections, at least in the one loop level, are indispensable for the calculation around the Z^0-pole and in the higher energies. We evaluate the corrections from the photon and Z^0 vacuum polarization and their mixing effectively up to the higher orders and also the contributions from weak vertices and weak box diagrams and we define the effective Born cross section. In section 3 we compare the total effective Born cross sections to those obtained by ALIBABA[6].

In section 4 we give results and discussion in which we show the cross sections obtained by the new generator and compare them to those results with the full $O(\alpha)$ corrections[7] at the typical C.M. energies for TRISTAN and LEP. Further the distribution on the acollinearity angle for e^+ and e^- shows the comparison between the result and those for the radiative Bhabha scattering at the LEP II energy. Concluding remarks are given in the last section.

2 The QEDPS-t Model

The QEDPS model uses the Sudakov factor,

$$\Pi(K^2, Q^2_{min}) \quad = \quad \exp\left[-\frac{\alpha}{2\pi}\int_{Q^2_{min}}^{K^2}\frac{dQ^2}{Q^2}\int_0^{1-\epsilon}dx\frac{1+x^2}{1-x}\right] \tag{1}$$

where $\epsilon = Q_0^2/Q^2$ and Q_0^2 is a cutoff. This factor is regarded as the non-branching probability for the electron and the positron in the range of the virtuality $Q^2_{min} < Q^2 < K^2$. The range of K^2 is limited in the region $Q^2_{min} < K^2 < Q^2_{max}$, where Q_{max} is the maximum momentum allowed by the process. The factor is utilized to produce multiple emissions of photon by the Monte Carlo method.

The maximum energy scale Q^2_{max} should be predetermined so as to obtain most efficiently a proper description of the process of interest. Such an energy scale in case of Bhabha scattering can be given by an absolute value of the momentum transter t since the cross section is dominated by the t-channel diagrams even for the large angle scattering (not for the backward scattering). We determine the scale $|t|$ by the approximated Bhabha Born cross section:

$$\frac{d\sigma_0}{dt} = \frac{4\pi\alpha^2}{t^2}\ , \quad \sigma_0 = \int_{|t|_{min}}^{|t|_{max}} \frac{d\sigma_0}{dt} dt = 4\pi\alpha^2\Delta\ , \qquad \Delta = \frac{1}{|t|_{min}} - \frac{1}{|t|_{max}}. \tag{2}$$

We propose the following expression as the differential cross section for the t-channel dominant e^+e^- processes with multiple photons being emitted from both the initial electron and the initial positron:

$$\frac{d\sigma}{dt}(s,t) = \int_0^1 dx_1 D_{e^-}(x_1,|t|) \int_0^1 dx_2 D_{e^+}(x_2,|t|) \frac{d\hat{\sigma}}{dt}(x_1 x_2 s, t)\ , \tag{3}$$

where $D_{e^-}(x_1,|t|)$ and $D_{e^+}(x_2,|t|)$ are the structure functions of the electron and the positron. x_1 and x_2 are their energy fractions. For the Bhabha scattering $d\hat{\sigma}/dt$ expresses the differential cross section of the elastic process $e^+e^- \to e^+e^-$. The equation can be used in the model by the following way:

$$\begin{aligned}
\sigma &= \int_{|t|_{min}}^{|t|_{max}} \frac{d\sigma_0}{dt} d|t| \int_0^1 dx_1 D_{e^-}(x_1,|t|) \int_0^1 dx_2 D_{e^+}(x_2,|t|) \times \frac{d\hat{\sigma}}{dt}(x_1x_2s,t) \Big/ \frac{d\sigma_0}{dt} \\
&= \sigma_0 \int_0^1 d\eta \int_0^1 dx_1 D_{e^-}(x_1,|t|) \int_0^1 dx_2 D_{e^+}(x_2,|t|) \times \frac{d\hat{\sigma}}{dt}(x_1x_2s,t) \Big/ \frac{d\sigma_0}{dt} \\
&= \frac{\sigma_0}{N} \times \sum_{j=1}^{N} W(x_{1j}, x_{2j}, t_j)\ ,
\end{aligned} \tag{4}$$

where N is the number of the generated events. The scale $|t| = |t_j|$ can be determined by the random number $\eta = \eta_j$ for each event $j = 1, \cdots, N$ according to the relation:

$$|t| = \frac{1}{1/|t|_{min} - \eta\Delta}\ . \tag{5}$$

For each energy scale we develop the QED parton showers both from the initial electron and the positron such that their energy fractions are fixed so as to obey the probability distributions $D_{e^-}(x_1,|t_j|)$ and $D_{e^+}(x_2,|t_j|)$, respectively. They are denoted by x_{1j} and x_{2j} and we obtain the weight for each energy scale:

$$W(x_{1j}, x_{2j}, t_j) \equiv \frac{d\hat{\sigma}(x_{1j}x_{2j}s, t_j)/dt}{d\sigma_0(t_j)/dt}\ . \tag{6}$$

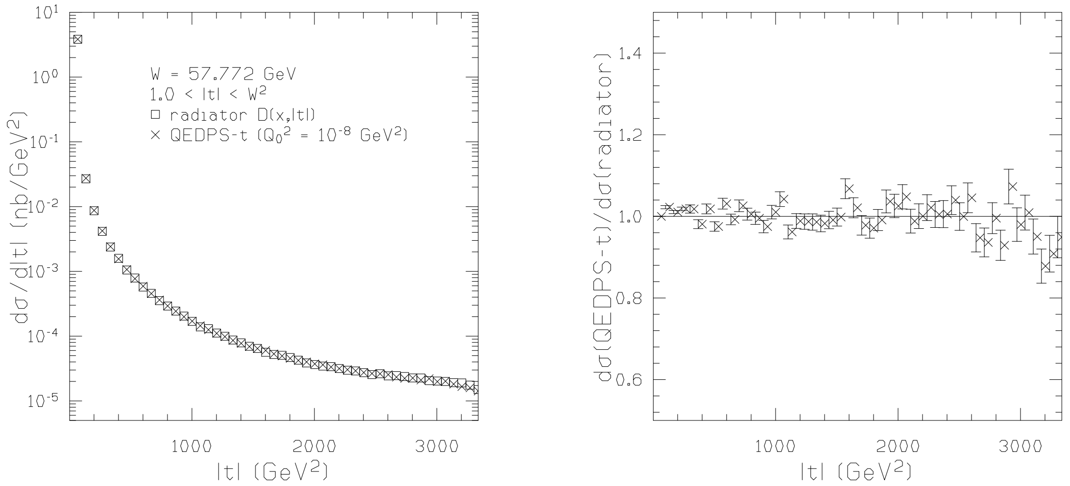
The toal cross section σ can be calculated as the product of the approximated cross section σ_0 and the average of the weights. For the model we need the input parameters, $|t|_{min}$ and $|t|_{max}$, both of which are given by the experimental conditions, *i.e*, the total CM energy and the angular acceptances θ_{min} and θ_{max}.

We use the formula of QED Bhabha scattering as the elastic differential cross section given by

$$\frac{d\hat{\sigma}}{dt}(s,t) = \frac{4\pi\alpha^2}{t^2}\left(1 + 2\frac{t}{s} + 3\left(\frac{t}{s}\right)^2 + 2\left(\frac{t}{s}\right)^3 + \left(\frac{t}{s}\right)^4\right) \tag{7}$$

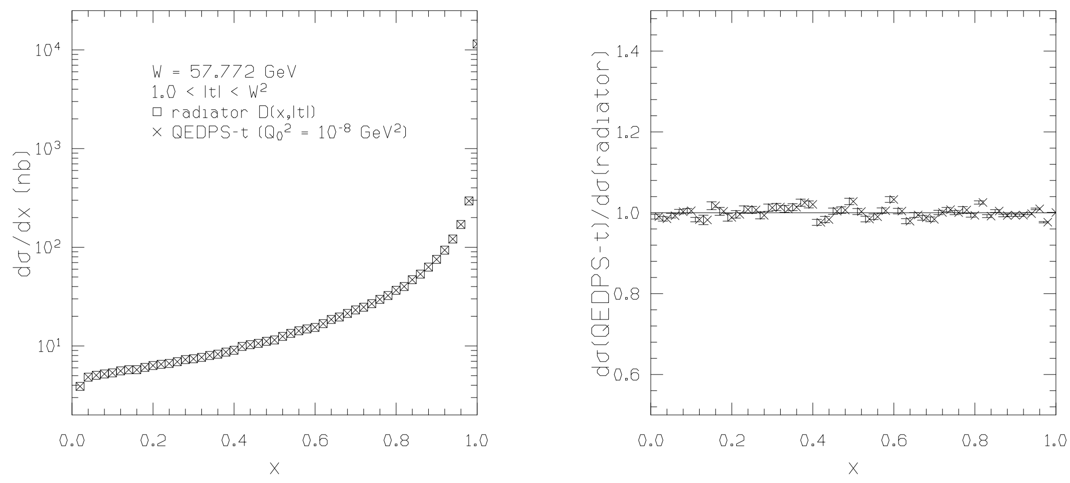
for testing the model and by which we can obtain the differential cross section for the QED Bhabha scattering with multiple photons emitted from the initial electron and the initial positron. We show the results of $|t|$ and x distributions for the electron in Fig.1 and Fig.2, respectively, at the center of mass energy of 57.772 GeV.

Eq.(3) can be also evaluated by using the solution of the structure function $D_e(x, Q^2)$ up to $O(\alpha^2)$ whose analytic formula is given in the reference[1]. The distributions thus obtained are also given in both figures. They show good agreement between the two methods and prove the validity of the model.



(a) (b)

Figure 1: (a) Comparison of $d\sigma/d|t|$ distribution between the QEDPS-t model and the radiator $D_e(x, |t|)$ calculated by Eq.(3). t is the squared momentum transfer. (b) The ratio $d\sigma(\text{QEDPS-}t)/d|t|$ to $d\sigma(\text{radiator})/d|t|$ with statistical error bars. A typical C.M. energy of TRISTAN experiment, $W = 57.772$ GeV and the cutoff parameter for the Sudakov factor, $Q_0 = 10^{-4}$ GeV, are used.



(a) (b)

Figure 2: (a) Comparison of $d\sigma/dx$ distributions between the QEDPS-t model and the radiator $D_e(x, |t|)$ calculated by Eq.(3). x is the energy fraction of initial e^-. (b) The ratio $d\sigma(\text{QEDPS-}t)/dx$ to $d\sigma(\text{radiator})/dx$ with statistical error bars. A typical C.M. energy of TRISTAN experiment, $W = 57.772$ GeV and the cutoff parameter for the Sudakov factor, $Q_0 = 10^{-4}$ GeV, are used.

3 The effective Born scheme

The QEDPS model gives pure QED corrections to the process which come from the real photon emissions and also the virtual process through the non-emission probability, which includes the QED vertex correction in the leading-log order. To take other electroweak corrections in the model we have to modify the expression of the elastic differential cross section $d\hat{\sigma}/dt$.

Among all the one-loop electroweak corrections to the process[1] those due to the vacuum polarizations of photon and Z^0 are most important and they should be effectively summed up to higher orders. To do this we modify the propagator functions of photon and Z^0 in the electroweak Born cross section.

We use the propagator function of photon given by

$$D_{AA}(q^2) = \frac{1}{q^2 - \tilde{a}_A(q^2)} \ , \tag{8}$$

here $\tilde{a}_A(q^2)$ expresses the renormalized one-loop photon self-energy in which we use the function given by Burkhardt *et al.*[5] for quark loops.

We use the following expression of the s-channel propagator function of Z^0:

$$D_{ZZ}(s) = D^{(0)}_{ZZ}(s)\left[1 + \Delta\Pi^{(1)}_{ZZ}(s)D^{(0)}_{ZZ}(s) + \left(\Delta\Pi^{(1)}_{ZZ}(s)D^{(0)}_{ZZ}(s)\right)^2\right] \tag{9}$$

here we have introduced the s-dependent width in $D^{(0)}_{ZZ}(s)$,

$$D^{(0)}_{ZZ}(s) = \frac{1}{s - M_Z^2 + is\Gamma_Z/M_Z}. \tag{10}$$

M_Z and Γ_Z are the Z^0 mass and the full width of Z^0 both being the input parameters of the present calculation. $\Delta\Pi^{(1)}_{ZZ}(s)$ is defined as

$$\Delta\Pi^{(1)}_{ZZ}(s) = \tilde{a}_Z(s) + is\Gamma^{(0)}_Z/M_Z \ , \tag{11}$$

where $\tilde{a}_Z$ and $\Gamma^{(0)}_Z$ are the one-loop Z^0 self energy and the lowest decay width of Z^0, respectively. The Z^0 propagator function in the t-channel exchange is given by

$$D_{ZZ}(t) = \frac{1}{t - M_Z^2}\left[1 + \frac{\tilde{a}_Z(t)}{t - M_Z^2}\right] \tag{12}$$

Further the photon-Z^0 mixing term is introduced by

$$D_{AZ}(q^2) = \tilde{a}_{AZ}(q^2)D_{AA}(q^2)D_{ZZ}(q^2), \tag{13}$$

where $\tilde{a}_{AZ}(q^2)$ is the one-loop photon-Z^0 mixing self-energy.

We include the contributions from the weak vertex and the weak box corrections. In the effective Born cross section all propagators are replaced by those defined above except inside loops. The complete expression will appear in the reference[9].

Throughout the paper we use the mass parameters, $M_Z = 91.187$ GeV, $\Gamma_Z = 2.49$ GeV, $M_W = 80.41$ GeV, $\Gamma_W = 2.06$ GeV, $M_t = 174$ GeV and $M_H = 300$ GeV. In Table 1 the total effective Born cross sections at four C.M. energies, 57.772, 70.0, 91.187 and 189.0 GeV are given and compared to those obtained by ALIBABA[6]. Both results show very good agreement in 0.1% level in the worst case.

4 Results and Discussion

We show the results of our new generator. Note that it does not only take account of the initial radiation but also the final radiation and the kinematics at each branch for $e \to e\gamma$ is treated as in the references[2, 3, 4]. A generated event is processed by an analysis routine to see the condition of the acceptance. Those events which do not satisfy the condition are counted in the number of generated events but have zero

E_{CM}(GeV)	θ_{min}(deg)	θ_{max}(deg)	Ours(A)(pb)	ALIBABA(B)(pb)	$(A-B)/A\times 100\%$
57.772	10	170	10365.	10366.	−0.002%
57.772	20	160	2350.0	2349.8	0.008%
57.772	30	150	919.48	919.33	0.017%
57.772	40	140	447.42	447.31	0.025%
70.0	10	170	7144.18	7144.22	0.000%
70.0	20	160	1641.11	1640.96	0.009%
70.0	30	150	650.897	650.794	0.016%
70.0	40	140	320.409	320.342	0.021%
91.187	10	170	6118.01	6120.64	−0.043%
91.187	20	160	2749.47	2749.23	0.009%
91.187	30	150	1975.48	1974.08	0.071%
91.187	40	140	1539.15	1537.35	0.117%
189.0	10	170	968.719	968.440	0.029%
189.0	20	160	208.127	208.000	0.061%
189.0	30	150	75.6410	75.5771	0.084%
189.0	40	140	33.7830	33.7475	0.105%

Table 1: Total Effective Born cross sections integrated over the angular range $\theta_{min} < \theta_{e^\pm} < \theta_{max}$. The results are compared with those obtained by ALIBABA[6] at four C.M. energies, 57.772, 70.0, 91.187 and 189.0 GeV. The mass parameters, $M_Z = 91.187$ GeV, $\Gamma_Z = 2.49$ GeV, $M_W = 80.41$ GeV, $\Gamma_W = 2.06$ GeV, $M_t = 174$ GeV and $M_H = 300$ GeV, are used.

values for their weights. In Table 2 we give the cross sections for the small angle scatterings and also those with the full $O(\alpha)$ corrections at the same C.M. energies as in Table 1. The differences between two become from 0.5% to about 1% with increasing C.M. energy. The major part of these differences can be expected because of the difference of the treatments of both the higher order effects due to the self energy of photon in the effective Born and the leading-log QED corrections in the QEDPS model. These higher order corrections are missing in the full $O(\alpha)$ corrections.

We give also the cross sections for the large angle scatterings in Table 3. From the results we can observe a similar trend of differences as seen in Table 2. However a sizeable discrepancy appears at the Z^0-pole and $E_{C.M.} = 189$ GeV. This shows how the corrections are large in this energy region. Especially we have large corrections at the large scattering angles. One should note that the radiative corrections strongly depend on the cuts.

Next we show the acollinearity distributions in Fig.3(a) ($10^\circ < \theta_{e^\pm} < 170^\circ$) and Fig.3(b) ($30^\circ < \theta_{e^\pm} < 150^\circ$) at the C.M. energy $W = 189$ GeV obtained by the generator. In both figures we plot two results of the generator, that obtained by the input parameters $|t|_{min}$ and $|t|_{max}$ for the range of the maximum energy scale just determined with the angle cuts and the other one which includes those events from an additional smaller range of the energy scale,*i.e.*, with the scattering angles smaller than θ_{min}. These results are compared to those distributions of the radiative Bhabha scattering in which soft photon and virtual corrections are taken into account in the first bin.

In Fig.3(a) a big bump around the acollinearity angle 70 degree is due to the Z^0 exchange. These two plots concide each other up to around 145 degree except for the small difference at around 10 degree. The peak at the maximum acollinearity comes from the s'-singulaity in which the virtuality of the photon takes its minimum, *i.e.,* almost $4m_e^2$ due to the initial radiation. The generator cannot reproduce the peak, since the present model does not include such a singularity.

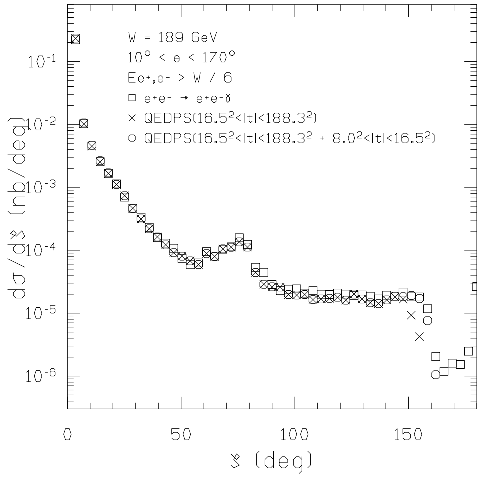
In Fig.3(b) we can see a clear Z^0 peak and find a serious difference between two plots in the full range of the acollinearity angle. It means that the range of the maximum energy scale of Q^2 is not sufficient if one determines it according to just the angle cuts as the process should not be treated as an elastic process. To take such an effect we should add the contribution from the smaller range of the maximum energy scale for the parton shower. It corresponds to those events in which both the electron and the positron are scattered into the acceptance region due to the photon emissions. Such consideration has also been taken to obtain the cross sections in Table 2 and Table 3.

E_{CM}(GeV)	θ_{min}(deg)	θ_{max}(deg)	$O(\alpha)$(A)(nb)	QEDPS(B)(nb)	$(B-A)/A\times100\%$
57.772	1	3	893.06(0.12)	897.29(0.38)	0.47%
57.772	3	5	68.628(0.012)	68.950(0.030)	0.47%
57.772	5	7	18.290(0.004)	18.393(0.008)	0.56%
57.772	7	10	9.731(0.002)	9.782(0.004)	0.52%
70.0	1	3	609.02(0.10)	612.03(0.30)	0.49%
70.0	3	5	46.774(0.010)	47.003(0.023)	0.49%
70.0	5	7	12.466(0.003)	12.548(0.008)	0.65%
70.0	7	10	6.642(0.002)	6.682(0.003)	0.59%
91.187	1	3	359.64(0.06)	361.44(0.41)	0.50%
91.187	3	5	27.633(0.006)	27.773(0.018)	0.51%
91.187	5	7	7.379(0.002)	7.412(0.004)	0.46%
91.187	7	10	3.952(0.001)	3.968(0.002)	0.40%
189.0	1	3	84.203(0.019)	84.716(0.090)	0.61%
189.0	3	5	6.446(0.002)	6.495(0.003)	0.77%
189.0	5	7	1.712(0.001)	1.727(0.001)	0.86%
189.0	7	10	0.911(0.000)	0.919(0.000)	0.87%

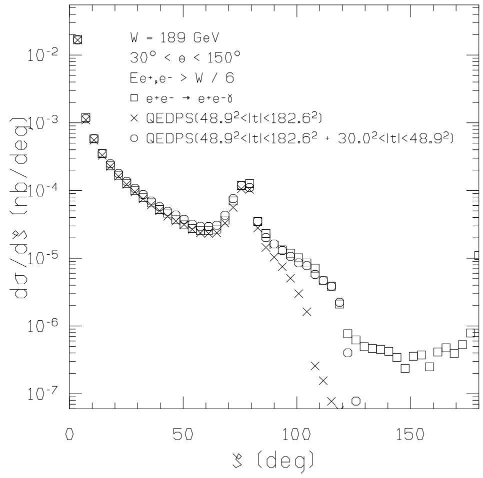
Table 2: The cross sections obtained by the generator for small angle Bhabha scatterings. The angular cuts $\theta_{min} < \theta_{e^\pm} < \theta_{max}$ and the energy cuts $E_{e^\pm} > E_{beam}/3$ and no acollinearity angle cut are imposed. The results are compared with those obtained by the calculations with the $O(\alpha)$ corrections[7]. The C.M. energies and the mass parameters are the same as used in Table 1.

E_{CM}(GeV)	θ_{min}(deg)	θ_{max}(deg)	$O(\alpha)$(A)(pb)	QEDPS(B)(pb)	$(B-A)/A\times100\%$
57.772	10	170	9979.3(5.0)	10026.1(2.64)	0.47 %
57.772	20	160	2259.6(1.2)	2273.4(0.64)	0.61 %
57.772	30	150	878.71(0.45)	884.41(0.26)	0.65 %
57.772	40	140	423.94(0.21)	426.81(0.13)	0.68 %
70.0	10	170	6866.1(3.7)	6902.4(1.7)	0.53 %
70.0	20	160	1573.4(0.9)	1582.2(0.6)	0.55 %
70.0	30	150	619.36(0.33)	623.32(0.19)	0.64 %
70.0	40	140	302.01(0.16)	303.91(0.10)	0.63 %
91.187	10	170	5676.8(2.7)	5569.7(11.1)	−1.89 %
91.187	20	160	2386.4(1.0)	2290.4(5.0)	−4.02 %
91.187	30	150	1635.6(0.7)	1557.2(2.8)	−4.79 %
91.187	40	140	1237.6(0.5)	1176.3(1.7)	−4.95 %
189.0	10	170	925.93(0.68)	934.04(0.31)	0.88 %
189.0	20	160	199.31(0.16)	201.65(0.10)	1.17 %
189.0	30	150	72.205(0.056)	73.418(0.045)	1.68 %
189.0	40	140	31.958(0.025)	32.632(0.025)	2.11 %

Table 3: The cross sections obtained by the generator for large angle Bhabha scatterings. The angular cuts $\theta_{min} < \theta_{e^\pm} < \theta_{max}$ and the energy cuts $E_{e^\pm} > E_{beam}/3$ and no acollinearity angle cut are imposed. The results are compared with those obtained by the calculations with the $O(\alpha)$ corrections[7]. The C.M. energies and the mass parameters are the same as used in Table 1.



(a)



(b)

Figure 3: Acollinearity distributions for outgoing e^+ and e^-. The same mass parameters as Table 1 and a typical C.M. energy of LEP II, $W = 189$GeV, are used. The energy cut for e^+ and e^- is $(1/3)E_{beam}$ for both figures. The angle acceptance for the scattered e^+ and e^- are taken to be (a) $10° < \theta_{e^\pm} < 170°$ and (b) $30° < \theta_{e^\pm} < 150°$. For both figures (a) and (b), we have three results, the result of the radiative Bhabha scattering which contains virtual corrections in the first bin, the QEDPS result which has been obtained by using the range of maximum energy scale determined by the acceptance for the scattering angle for e^+ and e^- and finally that which includes the contribution of those events from the smaller range of the maximum energy scale.

5 Concluding Remarks

We have developed a new generator for Bhabha scatterings. One of the progress is that we use a formula for the effective Born cross section which includes all the required one loop corrections, particularly the corrections from the vacuum polarization of γ and Z^0 are effectively summed up to the higher orders. It is shown that our effective Born scheme gives quite close values to those by ALIBABA for the integrated total effective cross section.

The second point to note is we have proposed the QEDPS-t model which is applicable to all the t-channel dominant e^+e^- processes and tested in Bhabha scattering to show the validity of the model[8]. The generator needs input parameters for the range of the maximum energy scale which can be given in principle by the acceptance for the scattering angles of e^+ and e^-. However, it is shown that an additional smaller region of the maximum energy scale also contributes to the cross section for the given acceptance. It means that we cannot ignore those events which come into the acceptance region due to the photon emissions.

The present model does not include both effects from the s-channel initial-final interference and the t-channel electron-positron interference. The next-to-leading order QED corrections is also neccesary to equip. Recently one of the authers, T.M., developed the NLL QEDPS[10]. These corrections should be taken into account in the coming.

References

[1] J. Fujimoto, M. Igarashi, N. Nakazawa, Y. Shimizu and K. Tobimatsu, Prog. Theor. Phys. 100(1990)1.

[2] J. Fujimoto, Y. Shimizu and T. Munehisa, Prog. Theor. Phys. 90(1993)177.

[3] J. Fujimoto, Y. Shimizu and T. Munehisa, Prog. Theor. Phys. 91(1994)333.

[4] T. Munehisa, J. Fujimoto, Y. Kurihara and Y. Shimizu, Prog. Theor. Phys. 95(1996)375.

[5] H. Burkhardt, F. Jegerlehner, G. Penso and C. Verzegnassi, Zeit. Phys. C−Particles and Fields 43(1989)497.

[6] W. Beenakker, F.A. Berends and S.C. van der Marck, Nucl. Phys. B349(1991)323.
S. Odaka, private communications.

[7] K. Tobimatsu and Y. Shimizu, Prog. Theor. Phys. 74(1985)567, 76(1986)Err. and 75(1986)905.
K. Kuroda, T. Kamitani, S. Kawabata, K. Tobimatsu and Y. Shimizu, Comput. Phys. Commun. 48(1988)335.

[8] Y. Kurihara, J. Fujimoto, Y. Shimizu, K. Kato, K. Tobimatsu and T. Munehisa, KEK-CP-088.

[9] K. Tobimatsu, K. Kato, T. Munehisa, Y. Kurihara, J. Fujimoto and Y. Shimizu, in preparation.

[10] T. Munehisa, will be published in Prog. Theor. Phys.