Super-p-brane actions from interpolating dualisations*

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Abstract

We review a recently proposed method for constructing super-p-brane world-volume actions. In this approach, starting from a democratic choice of world-volume gauge-fields guided by p-brane intersection rules, the requirements of κ-symmetry and gauge invariance can be used to determine the corresponding action. We discuss the application of the method to some cases of interest, notably the dyonic (p,q) 5-branes of type-IIB string theory in a manifestly S-duality covariant formulation.

Introduction

With the derivation of the κ-symmetric actions for the Dp-branes [1–5] and the M5-brane [6, 7], the programme of constructing world-volume actions for all the points on the 'revisited brane-scan' of ref. [8] was essentially completed by early 1997[1]. However, the case of the NS5-brane of the type-IIB theory was not satisfactorily resolved by these efforts, and, moreover, since then a number of more or less 'exotic' extended objects have been predicted from a detailed analysis of the relevant supertranslation algebras [10] and/or U-duality considerations (see ref. [11] and references therein). These facts have provided motivation for subsequent work on completing and refining the formulations of the world-volume dynamics for extended objects in string and M theory.

One line of further development has been the construction of manifestly S-duality covariant actions for the BPS multiplets of branes in the type-IIB theory, in particular for the dyonic (p,q)-strings [12] and for the SL(2)-singlet D3-brane [13]. This programme, whose extension to the dyonic (p,q)-5-branes was given in ref. [14] and will be reviewed below, is based on a particular type of action in which the brane tension is generated by an auxiliary world-volume gauge field. More specifically (leaving the issue of SL(2) covariance aside until later), the actions we will consider are of the generic form

$$S = \int \mathrm{d}^{p+1}\xi\sqrt{-g}\,\lambda\,[1+\Phi(\{F_i\})-(*F_{p+1})^2]\,, \tag{1}$$

where $\{F_i\}$ and F_{p+1} denote a set of gauge-invariant abelian world-volume field strengths, each of the form $F = \mathrm{d}A - C - \{\text{composite terms}\}$; we generically use C to denote (the superfield extension of) a supergravity gauge potential (here pulled back to the world-volume). Moreover, the composite terms are wedge products of lower-rank F's and C's required for gauge invariance. As indicated, the $(p{+}1)$-form field strength F_{p+1} plays a special role: the equation of motion for its non-propagating gauge potential A_p has the solution $\lambda\,{*F_{p+1}} = \text{constant}$, a constant which can be identified with the tension. In addition to A_p, the action contains a second auxiliary field, λ, which serves as a Lagrange multiplier for the constraint $\Upsilon = 1+\Phi(\{F_i\})-(*F_{p+1})^2 \approx 0$.[2]

In general, the choice of brane is determined by the leading background gauge potential C_{p+1} appearing in the tension field strength, while the remaining world-volume gauge fields correspond to other branes that can end on or intersect the world-volume. As we shall see, it is possible to construct covariant actions of the type (1) in which world-volume potentials appear together with their Poincaré duals; the problem of preserving the correct degree-of-freedom counting can be resolved in a rather elegant and natural way. An original motivation for considering such formulations—beside their potential usefulness in studies of target-space brane configurations from a world-volume perspective—stems from the observation [16] that relating the world-volume actions for branes which transform into each other under S-duality involves performing Poincaré-duality transformations on the world-volume gauge fields. As a recipe for constructing

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[1] For the M5-brane case, note, however, the caveat discussed in ref. [9].

[2] As a special case, restricting the world-volume gauge-field content to the Born–Infeld field strength $F_2 = \mathrm{d}A_1 - B_2$ (in addition to the tension field-strength) leads to an alternative formulation of the D-branes [15]. The standard formulation, in which the action consists of the sum of a DBI and a WZ term, can be regained by eliminating A_p and λ.

new brane actions from known ones, however, the latter approach has its limitations; the basic problem is that in order to express an action $S[F]$ in terms of the Poincaré-dual field strength $\tilde{F} \sim \frac{\delta S[F]}{\delta F}$, one has to solve an algebraic equation whose order increases with the world-volume dimension. In practice, this obstacle has, e.g., prevented the construction of a κ-symmetric action for the type-IIB NS5-brane from the D5-brane action by means of a combined background $\mathbb{Z}_2$ S-duality transformation and a world-volume Poincaré transformation of the Born–Infeld field strength F_2.[3] By employing a formulation where both field strengths appear simultaneously we are able to circumvent these problems, obtaining in the process a continuous family of actions interpolating between Poincaré-dual limits.

As it turns out, this method can be systematised to the extent that it becomes an essentially algorithmic procedure for constructing κ-symmetric world-volume actions. Led by the available background gauge potentials, one constructs a set of gauge-invariant world-volume field strengths. Except for the tension field strength, they all come in world-volume dual pairs and each such pair has an associated interpolation parameter. Inserting these field strengths in an Ansatz of the form (1), the requirement of κ-symmetry turns out to be sufficient to determine the action. Before we go on to discuss more complicated applications, we shall first illustrate the method using a simple and transparent example; relevant references are the papers [13,14,18,19] where more detailed accounts can be found.

A simple example: the M2/D2-brane

We consider the well-known case of the S-dual pair of the M2- and type-IIA D2-branes,[4] omitting some more technical points; for more details see ref. [14]. Let us begin by recalling some relevant facts about type-IIA supergravity. Among other fields, the bosonic sector of this theory contains the gauge-invariant field strengths R_2, H_3 and R_4 (the subscript indicating the form degree) which satisfy the Bianchi identities

$$\mathrm{d}R_2 = 0\,, \qquad \mathrm{d}H_3 = 0\,, \qquad \mathrm{d}R_4 = H_3\,R_2\,. \tag{2}$$

The first two relations imply that $R_2 = \mathrm{d}C_1$ and $H_3 = \mathrm{d}B_2$, whereas the third one allows for some freedom in defining R_4; more precisely, $R_4 = \mathrm{d}C_3 + x\,B_2\,R_2 - (1-x)\,C_1\,H_3 = \mathrm{d}(C_3 + x\,C_1\,B_2) - C_1\,H_3$, the parameter x thus arising from an ambiguity in the definition of C_3 corresponding to the field redefinition $C_3 \to C_3 + x\,C_1\,B_2$. Although changing the value of x does not change the physics, the effect on the form of the world-volume theory can be significant.

As mentioned above, gauge invariance is one of the guiding principles in the construction of the actions. At the supergravity level, the background field strengths are invariant under the gauge transformations

$$\delta C_1 = \mathrm{d}L_0\,, \qquad \delta B_2 = \mathrm{d}L_1\,, \qquad \delta C_3 = \mathrm{d}L_2 - x\,R_2\,L_1 - (1-x)\,H_3\,L_0\,. \tag{3}$$

Each of the background gauge potentials couple minimally to a world-volume gauge field, and the gauge transformations (3) thus induce transformations on the world-volume. In order for the world-volume field strengths to be gauge invariant, the latter have to be cancelled by transformations of the world-volume gauge potentials, possibly after the inclusion of sub-leading composite terms. For the case under consideration, the outcome of this analysis is the following set of world-volume field strengths:

$$F_1 = \mathrm{d}A_0 - C_1\,, \qquad F_2 = \mathrm{d}A_1 - B_2\,, \qquad F_3 = \mathrm{d}A_2 - C_3 + x\,B_2\,F_1 - (1-x)\,C_1\,F_2\,. \tag{4}$$

These field strengths are gauge invariant and satisfy the Bianchi identities

$$\mathrm{d}F_1 = -R_2\,, \qquad \mathrm{d}F_2 = -H_3\,, \qquad \mathrm{d}F_3 = -R_4 + x\,F_1\,H_3 - (1-x)\,F_2\,R_2\,. \tag{5}$$

Thus, although essentially trivial at the supergravity level, the field redefinition parameterised by x has a significant impact on the formulation of the world-volume theory, as can be seen by the x-dependent expression for the tension-form field strength F_3. For $x = 0$, F_1 decouples and the sub-leading terms in F_3 can be recognised as the negative of the WZ-form in the D2-brane action, while for $x = 1$ the

[3] With motivation coming from T-duality considerations of type-IIA KK monopoles, it has been suggested [17] that such a Poincaré-duality transformation is not needed in this particular case.

[4] Actually, we will be working exclusively in $D = 10$, so by the M2-brane we here mean the brane obtained by direct dimensional reduction on S^1 of the $D = 11$ supermembrane. In other words, we consider the situation where the S-duality transformation from the eleven-dimensional background theory down to ten dimensions has already been performed.

tension field strength is the one appropriate for the description of the dimensionally reduced M2-brane. For intermediate values, on the other hand, F_1 and F_2 are present simultaneously, and consequently there is one bosonic degree of freedom too many. Proceeding next to construct an action for the general case, we shall find that the resolution of this problem is inherent in the formalism [13]. We use the Ansatz[5]

$$S = \int \mathrm{d}^3\xi \sqrt{-g}\, \lambda \left[1 + \Phi(F_1, F_2) - (*F_3)^2\right] . \tag{6}$$

The equation of motion for A_2 is $\mathrm{d}[\lambda * F_3] = 0$, whereas those for the dynamical gauge potentials A_0 and A_1 read

$$\mathrm{d}\big[\lambda\{*\frac{\delta\Phi}{\delta F_1} + 2x\, B_2 *F_3\}\big] = 0\,, \qquad \mathrm{d}\big[\lambda\{*\frac{\delta\Phi}{\delta F_2} - 2(1{-}x)\, C_1 *F_3\}\big] = 0\,. \tag{7}$$

By using the Bianchi identities for F_1 and F_2 together with the equation of motion $\mathrm{d}[\lambda * F_3] = 0$, the explicit dependence on the background fields can be eliminated from these equations with the result

$$\mathrm{d}\big[\lambda\{*\frac{\delta\Phi}{\delta F_1} - 2x\, F_2 *F_3\}\big] = 0\,, \qquad \mathrm{d}\big[\lambda\{*\frac{\delta\Phi}{\delta F_2} + 2(1{-}x)\, F_1 *F_3\}\big] = 0\,. \tag{8}$$

It is thus consistent with the equations of motion and the Bianchi identities to impose the duality relations

$$-2x *F_3 *F_2 = K_1 := \frac{\delta\Phi}{\delta F_1}\,, \qquad 2(1{-}x) *F_3 *F_1 = K_2 := \frac{\delta\Phi}{\delta F_2}\,, \tag{9}$$

where Φ is yet to be determined. The effect of these relations is to reduce the total number of degrees of freedom contained in A_1 and A_2 by half, thus compensating for the doubling of gauge fields in the action. Since for the limiting value $x = 0$ ($x = 1$), F_1 (F_2) decouples, the degree-of-freedom counting works out correctly. It is important to note that the duality relations must not be substituted into the action; rather, they supplement the equations of motion derived from the latter.

The duality relations (9) also play a crucial role in implementing κ-symmetry—the other main guiding principle of the procedure—because they allow us to determine the variation of the action (6) (or, equivalently, that of the constraint $\Upsilon = 1 + \Phi - (*F_3)^2 \approx 0$) under the κ-transformations

$$\begin{aligned} \delta_\kappa g_{ij} &= 2\, E_{(i}{}^{a}\, E_{j)}{}^{B} \kappa^\alpha\, T_{\alpha B a}\,, \quad \delta_\kappa \phi = \kappa^\alpha \partial_\alpha \phi\,, \quad \delta_\kappa F_1 = -i_\kappa R_1\,, \\ \delta_\kappa F_2 &= -i_\kappa H_3\,, \quad \delta_\kappa F_3 = -i_\kappa R_4 + x\, F_1\, i_\kappa H_3 - (1{-}x)\, F_2\, i_\kappa R_2\,, \end{aligned} \tag{10}$$

in spite of the fact that we do not know what Φ is from the outset (the background fields are now the superfield extensions of their bosonic counterparts). To see how this works [13, 18], note that the scalar function Φ is formed out of contractions between the world-volume field strengths and the metric only, and so a simple scaling argument shows that the variation of Υ can be written[6]

$$\begin{aligned} \delta_\kappa \Upsilon = \; & K^i\, \delta_\kappa F_i + \tfrac{1}{2!} K^{ij}\, \delta_\kappa F_{ij} + \tfrac{2}{3!} F^{ijk}\, \delta_\kappa F_{ijk} - \big[\tfrac{1}{2} K^{(i} F^{j)} + \tfrac{1}{2!} K^{(i}{}_l F^{j)l} \\ & + \tfrac{3}{2} \cdot \tfrac{2}{3!} F^{(i}{}_{lm} F^{j)lm}\big]\, \delta_\kappa g_{ij} + \big[\tfrac{3}{4} K_1 \cdot F_1 - \tfrac{1}{2} K_2 \cdot F_2 + \tfrac{2}{4} F_3 \cdot F_3\big]\, \delta_\kappa \phi\,. \end{aligned} \tag{11}$$

By imposing the duality relations (9), K_1 and K_2 get replaced by explicit expressions in the world-volume field strengths. Inserting the transformations (10) with the supergravity on-shell constraints imposed on the background superfields and demanding that the irreducible components of the resulting variation vanish, then allows us to deduce the duality relations and subsequently the action. The final result is

$$S = \int \mathrm{d}^3\xi \sqrt{-g}\, \lambda \left[1 + x\, F_1 \cdot F_1 + (1{-}x)\, F_2 \cdot F_2 + x\,(1{-}x)\, F_1 \cdot F_1\; F_2 \cdot F_2 - (*F_3)^2\right] . \tag{12}$$

From this action one derives both the equations of motion (cf. (7)) and the supplementary duality relations (cf. (9)). Another important object determined in the process is the κ-symmetry projection operator

$$2 *F_3\, P_\pm = *F_3\, \mathbb{1} \mp \left[\Xi - x *F_1 \cdot \gamma_2 \gamma_{11} + (1{-}x) *F_2 \cdot \gamma_1 \gamma_{11}\right] . \tag{13}$$

[5] In this note we will suppress the dilaton dependence; this dependence, which can be determined from the dilaton-scaling of the supergravity constraints, is of importance when determining the κ-variation of the action.

[6] The part of this expression proportional to the dilaton variation was derived by temporarily instating the dilaton-dependence of the action.

Let us at this point summarise the generic features of the procedure which carry over to the more complicated cases to be discussed next: starting from a world-volume gauge-field content motivated by target-space considerations we have, by employing the principles of gauge invariance and supersymmetry (at this level manifested as κ-symmetry), found a one-parameter family of actions for the D2-brane interpolating between two limiting Poincaré-dual cases. For intermediate parameter values the number of gauge fields in the action are doubled, the associated doubling of degrees of freedom being resolved by supplementing the equations of motion with duality relations inherent in the formalism.

The Type-IIB NS5-brane

On shell, the type-IIB NS5-brane—like its S-dual partner, the D5-brane—has $(1,1)$ supersymmetry in $D=6$ and is described by a vector multiplet [20]. Whether the κ-symmetric action for the NS5-brane should contain a one-form gauge-potential (like the one for the D5-brane) or should be formulated in terms of a dual three-form potential (as experience from other S-dual brane pairs would suggest) does however not follow from this fact. Our formulation allows us to by-pass this question by introducing both potentials together with an associated interpolation parameter. These one- and three-form potentials, $\tilde{A}_1$ and A_3, can also be motivated by the fact (see, e.g., ref. [21]) that D-strings and D3-branes can end on the NS5-brane world-volume. Since the same goes for D5-branes, we also introduce an extra five-form gauge potential A_5, in addition to the tension potential $\tilde{A}_5$.

Using the additional fact that fundamental strings cannot end on the NS5-brane, we require that the Born–Infeld field strength F_2 be absent from the world-volume action. This assumption fixes the parameterisation of the background field strengths, and we are led to RR background and world-volume field strengths that can be succinctly summarised in the expressions $R=e^B\,\mathrm{d}C$ and $e^{-B}F=\mathrm{d}A-C$, respectively. The latter expression is iterative and we have defined $F=F_0+\tilde{F}_2+F_4+F_6$, where $F_0=\mathrm{d}A_{-1}-C_0$ ($\mathrm{d}A_{-1}$ formally denoting a constant) is dual to F_6 and needed for gauge invariance (Peccei–Quinn symmetry). In the same compact notation the world-volume Bianchi identities read

$$\mathrm{d}F=-R+H_3\,F\,. \tag{14}$$

Furthermore, the gauge-invariant field strength of the NS-NS six-form gauge potential is chosen as

$$H_7=\mathrm{d}B_6-(1-y)\,C_6\,\mathrm{d}C_0+y\,C_0\,\mathrm{d}C_6-(1-x)\,C_2\,\mathrm{d}C_4+x\,C_4\,\mathrm{d}C_2\,, \tag{15}$$

an expression which preserves the Bianchi identity $\mathrm{d}H_7=R_7\,R_1-R_5\,R_3$ and leads to a lengthy expression for the tension field strength $\tilde{F}_6$ (given in ref. [19]) as well as a more compact one for its Bianchi identity:

$$\mathrm{d}\tilde{F}_6=-H_7-(1-y)\,R_7\,F_0+y\,R_1\,F_6-(1-x)\,R_3\,F_4+x\,R_5\,\tilde{F}_2+(1-x-y)\,H_3\,F_0\,F_4+(\tfrac{1}{2}-x)\,H_3\,\tilde{F}_2\,\tilde{F}_2\,. \tag{16}$$

Given the above field content, the Ansatz for the action (written in differential-form notation) takes the form

$$S=\int\lambda\left[*1+*\Phi(F_0,\tilde{F}_2,F_4,F_6)+\tilde{F}_6\,*\tilde{F}_6\right], \tag{17}$$

from which we can derive the following expressions for the duality relations compatible with the Bianchi identities (14):

$$\begin{aligned}
2y\,*\tilde{F}_6\,F_0&=*K_6:=\frac{\delta*\Phi}{\delta F_6}\,, &\qquad -2(1-y)\,*\tilde{F}_6\,F_6&=*K_0:=\frac{\delta*\Phi}{\delta F_0}\,,\\
-2(1-x)\,*\tilde{F}_6\,F_2&=*K_4:=\frac{\delta*\Phi}{\delta F_4}\,, &\qquad 2x\,*\tilde{F}_6\,F_4&=*K_2:=\frac{\delta*\Phi}{\delta F_2}\,.
\end{aligned} \tag{18}$$

Using the known on-shell constraints for type-IIB supergravity, one can then proceed to determine the action by imposing κ-symmetry. The calculations, as well as the general results, are rather lengthy and can be found in ref. [19]. For certain values of the interpolation parameters, however, the expressions simplify considerably. For instance, when $x=\frac{2}{3}$ and $y=0$, F_6 decouples and we have

$$\begin{aligned}
2*\tilde{F}_6\,P_\pm&=*\tilde{F}_6\,\mathbb{1}\pm\big[*\gamma_6\,K+F_0*\gamma_6\,J+\tfrac{2}{3}*\tilde{F}_2\cdot\gamma_4\,I\\
&\qquad-\tfrac{1}{3}\{F_0*F_4-\tfrac{1}{2}*(\tilde{F}_2\wedge\tilde{F}_2)\}\cdot\gamma_2\,K-\tfrac{1}{3}*F_4\cdot\gamma_2\,J\big]\,,
\end{aligned} \tag{19}$$

$$\begin{aligned}
*\Phi&=F_0*F_0+\tfrac{2}{3}\,\tilde{F}_2\wedge*\tilde{F}_2+\tfrac{1}{3}\,F_4\wedge*F_4-\tfrac{1}{3}\,F_0*F_4\wedge\tilde{F}_2\wedge\tilde{F}_2+\tfrac{1}{3}\,(F_0)^2\,F_4\wedge*F_4\\
&\quad+\tfrac{2}{9}\,(\tilde{F}_2\wedge F_4)*(\tilde{F}_2\wedge F_4)+\tfrac{1}{6}*(*F_4\wedge*F_4)\wedge\tilde{F}_2\wedge\tilde{F}_2+\tfrac{1}{12}*(\tilde{F}_2\wedge\tilde{F}_2)\wedge\tilde{F}_2\wedge\tilde{F}_2\,.
\end{aligned} \tag{20}$$

From the latter expression the equations of motion and the duality relations may readily be derived. Since the action is κ-symmetric and couples correctly to the NS-NS six-form potential we can conclude that it describes the type-IIB NS5-brane.

Manifestly SL(2)-covariant actions

As mentioned in the introduction, world-volume actions of the kind discussed above that are manifestly covariant under the SL(2,$\mathbb{Z}$) S-duality group can be constructed for the type-IIB branes [12–14]. These constructions rely on the superspace formulation of type-IIB supergravity [22] where the U(1) R-symmetry has been gauged, allowing the scalars of the theory to transform linearly under SL(2,$\mathbb{R}$). More specifically, the scalars form a 2×2 matrix

$$\begin{pmatrix} \mathcal{U}^1 & \bar{\mathcal{U}}^1 \\ \mathcal{U}^2 & \bar{\mathcal{U}}^2 \end{pmatrix} \tag{21}$$

on which SL(2,$\mathbb{R}$) acts from the left and U(1) locally from the right, both group actions preserving the constraint $\frac{i}{2}\epsilon_{rs}\mathcal{U}^r\bar{\mathcal{U}}^s = 1$ (here $\epsilon_{12} = -1$). The two physical scalars of the theory—the dilaton ϕ and the axion C_0—are encoded in (21) as $\mathcal{U}^1(\mathcal{U}^2)^{-1} = -C_0 + i\,e^{-\phi}$. The SL(2) doublet $\mathcal{U}^r$ links fields transforming in the fundamental representation of SL(2) and SL(2)-invariant fields charged under the gauged U(1) R-symmetry, as the world-volume field strengths for the (p,q) five-branes illustrate [13, 14]:

$$\begin{aligned}
&\mathcal{F}_2 = \mathcal{U}^r \mathrm{d}A_{1;r} - \mathcal{C}_2\,, \qquad F_4 = \mathrm{d}A_3 - C_4 + \tfrac{1}{2}\,\mathrm{Im}(\mathcal{C}_2\,\bar{\mathcal{F}}_2)\,,\\
&\mathcal{F}_6 = \mathcal{U}^r \mathrm{d}A_{5;r} - \mathcal{C}_6 + x\,\mathcal{C}_2\,F_4 - (1{-}x)\,C_4\,\mathcal{F}_2 + \tfrac{1}{2}(\tfrac{2}{3}{-}x)\,\mathrm{Im}(\mathcal{C}_2\,\bar{\mathcal{F}}_2)\,\mathcal{F}_2 + \tfrac{1}{2}(\tfrac{1}{3}{-}x)\,\mathrm{Im}(\mathcal{C}_2\,\bar{\mathcal{F}}_2)\,\mathcal{C}_2\,.
\end{aligned} \tag{22}$$

These fields are all both gauge and SL(2) invariant. Whereas the four-form field strength F_4 is also U(1) neutral, the complex two- and six-form field strengths have U(1) charge +1, a fact which we indicate by the use of calligraphic letters (complex conjugation, indicated with a bar, reverses the sign of the U(1) charge). Analogously, $\mathcal{C}_2$, C_4 and $\mathcal{C}_6$ are SL(2)-invariant versions of the supergravity gauge potentials, and $\mathcal{H}_3$, H_5 and $\mathcal{H}_7$ in eq. (23) below the corresponding field strengths (of which $\mathcal{H}_7$ depends on the interpolation parameter x). The precise expressions for the world-volume field strengths were determined in the usual fashion by gauge invariance. They satisfy the Bianchi identities (D is the U(1)-covariant derivative and P is a covariantly constant one-form of U(1) charge +2 [22])

$$\begin{aligned}
&\mathrm{D}\mathcal{F}_2 + i\,\bar{\mathcal{F}}_2\,P + \mathcal{H}_3 = 0\,, \qquad \mathrm{d}F_4 + H_5 + \tfrac{1}{2}\,\mathrm{Im}(\bar{\mathcal{F}}_2\,\mathcal{H}_3) = 0\,,\\
&\mathrm{D}\mathcal{F}_6 + i\,\bar{\mathcal{F}}_6\,P + \mathcal{H}_7 - x\,\mathcal{H}_3\,F_4 + (1{-}x)\,H_5\,\mathcal{F}_2 + \tfrac{1}{2}(\tfrac{2}{3}{-}x)\,\mathcal{F}_2\,\mathrm{Im}(\mathcal{F}_2\,\bar{\mathcal{H}}_3) = 0\,.
\end{aligned} \tag{23}$$

The Ansatz for the action reads

$$S = \int \lambda\left[*1 + *\Phi(\mathcal{F}_2, \bar{\mathcal{F}}_2, F_4) + \mathcal{F}_6 * \bar{\mathcal{F}}_6\right]\,, \tag{24}$$

where λ is a Lagrange multiplier for the constraint $\Upsilon = 1 + \Phi(\mathcal{F}_2, \bar{\mathcal{F}}_2, F_4) - *\mathcal{F}_6 * \bar{\mathcal{F}}_6 \approx 0$, and Φ is required to have U(1) charge zero but is otherwise unconstrained. Compatibility between the equations of motion encoded in (24) and the Bianchi identities (23) require the duality relations to take the form [13]

$$-2x\,\mathrm{Re}(*\mathcal{F}_6 * \bar{\mathcal{F}}_2) = K_4 := \frac{\delta\Phi}{\delta F_4}\,, \qquad (1{-}x)*\mathcal{F}_6 * F_4 + \tfrac{i}{6}*[\mathrm{Re}(*\mathcal{F}_6\,\bar{\mathcal{F}}_2)\wedge\mathcal{F}_2] = \mathcal{K}_2 := \frac{\delta\Phi}{\delta\bar{\mathcal{F}}_2}\,. \tag{25}$$

The complicated structure of these relations, which can be traced back to the fact that the tension field strength is complex, makes the κ-symmetry analysis difficult and necessitates a perturbative approach. For an outline of this analysis and a more extensive discussion, we refer the reader to ref. [14], quoting here only the final results for the projection operator and the action valid up to fourth order in the world-volume field strengths (β is a free parameter in calculations up to this order):

$$2*\gamma_6\,P_\pm\,\zeta = *\gamma_6\,\zeta \mp \left[\tfrac{2i}{3}*F_4\cdot\gamma_2\,\zeta + \tfrac{i}{3}*\mathcal{F}_2\cdot\gamma_4\,\bar{\zeta} + *\mathcal{F}_6\,\bar{\zeta}\right] + \mathcal{O}(F^5)\,, \tag{26}$$

$$\begin{aligned}
S = \int \mathrm{d}^6\xi\,\sqrt{-g}\,\lambda\,\big[&1 + \tfrac{1}{3}\,\mathcal{F}_2\cdot\bar{\mathcal{F}}_2 + \tfrac{2}{3}\,F_4\cdot F_4 + \tfrac{1}{6}(\beta{-}2)*(\mathcal{F}_2\wedge F_4)*(\bar{\mathcal{F}}_2\wedge F_4) + \tfrac{1}{6}(\beta{-}\tfrac{2}{3})\,\mathcal{F}_2\cdot\bar{\mathcal{F}}_2\,F_4\cdot F_4\\
&+ \tfrac{1}{6}(1{-}\beta)\,(\mathcal{F}_2\wedge\bar{\mathcal{F}}_2)\cdot(*F_4\wedge *F_4) + \tfrac{1}{6}\,\beta\,(\mathcal{F}_2\wedge *F_4)\cdot(\bar{\mathcal{F}}_2\wedge *F_4) + \mathcal{O}(F^6) - *\mathcal{F}_6 * \bar{\mathcal{F}}_6\big]\,.
\end{aligned} \tag{27}$$

A somewhat peculiar feature worth noting, is that the interpolation parameter x gets fixed (to the value $\frac{2}{3}$) in the process.

Dilatonic 7-branes

It has been argued that there should exist certain exotic branes which do not show up in Hull's brane scan [10] based on an analysis of central charges in the $D = 11$ and $D = 10$ supertranslation algebras (for a recent discussion, see ref. [23]). Such branes—whose masses scale with the string coupling in a non-standard fashion as g_s^{-k}, with $k > 2$—are required to fill up U-duality multiplets for torus compactifications of M theory [11, 24, 25]. As the existence of a type-IIB NS7-brane whose mass goes as g_s^{-3} is generally accepted, it is natural to ask whether a similar object exists also on the type-IIA side. For every background supergravity potential it has been possible to construct an associated gauge-invariant WZ term and subsequently a world-volume action describing a brane which couples to the potential. The type-IIA supergravity theory in its doubled formulation indeed contains a field strength $H_9 = \mathrm{d}B_8 + \cdots$, where B_8 is dual to the dilaton, thus suggesting the existence of an associated 'dilatonic' NS7-brane. The method reviewed above lends itself well to the investigation of the possible existence of such an object from a constructive point of view. However, due to lack of space, we refer the reader to an expanded version of this note available on the hep-th archive for a more extensive discussion of this case.

Discussion

Let us end by summarising some of the main benefits of using actions with auxiliary duality relations. One point worth mentioning, is that this formulation allow for symmetries to be handled in a natural way, an important example being actions with manifest SL(2) covariance in the type-IIB theory. Another important aspect is the constructive nature of the approach: even in the cases where the form of the bosonic action is not known, one can determine the full action in an essentially algorithmic fashion using gauge invariance and κ-symmetry. The essential steps of the method are:

- Construct gauge-invariant world-volume field strengths and derive their Bianchi identities.
- Using the general Ansatz for the action, derive the associated duality relations compatible with the world-volume Bianchi identities.
- Derive the κ-variation of the action (or equivalently of the constraint Υ), eliminating all direct dependence of the unknown function Φ in the action Ansatz by means of the duality relations.
- Make an Ansatz for the κ-symmetry projection operator. (This is the only step which involves some guesswork; we would like to stress though that in all known cases the projection operator has a very natural form.)
- Insert the background supergravity constraints and expand the variation in its irreducible components. This leads to a set of constraints which can be used to deduce the duality relations and hence also the action. (One can also, at least in principle, use the requirement of κ-invariance to learn about the background supergravity constraints.)

Moreover, we would like to stress that in our approach all gauge symmetries are made manifest; other approaches lead to actions which do not have this property. Such examples are the dual D3- and D4-brane actions in ref. [26] as well as the type-IIB NS5-brane action in ref. [17] (whereas for the former two cases one can show that there is agreement with our approach at the level of the equations of motion, this has not been shown for the case of the NS5-brane).

For higher-dimensional cases the actions in their most general form tend to become complicated. For the D-branes it is always possible to choose the parameters so that all world-volume fields except F_2 decouple. In this limit the auxiliary duality relations become redundant and can be dropped, and the action becomes the standard one; here our construction is thus a natural generalisation of earlier work. For other cases (such as the NS5-brane) there is no choice of the parameters for which the auxiliary duality relations become superfluous. This fact seems to reflect an inherent obstruction to the construction of conventional actions with all gauge symmetries manifested (cf. the M5-brane, where no action with a chiral three-form exists [9], but where, when supplemented with its anti-chiral part ('doubled') and an auxiliary duality relation, a generalised action of the form discussed above can be constructed [18]). Thus, the price one has to pay for having manifest symmetries is the introduction of auxiliary duality relations.

Acknowledgements

The work of A.W. and N.W. was supported by the European Commission under contracts FMBICT972021 and FMBICT983302, respectively.

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